QCD Analysis of the combined HERA structure function data - HERAPDF2.0

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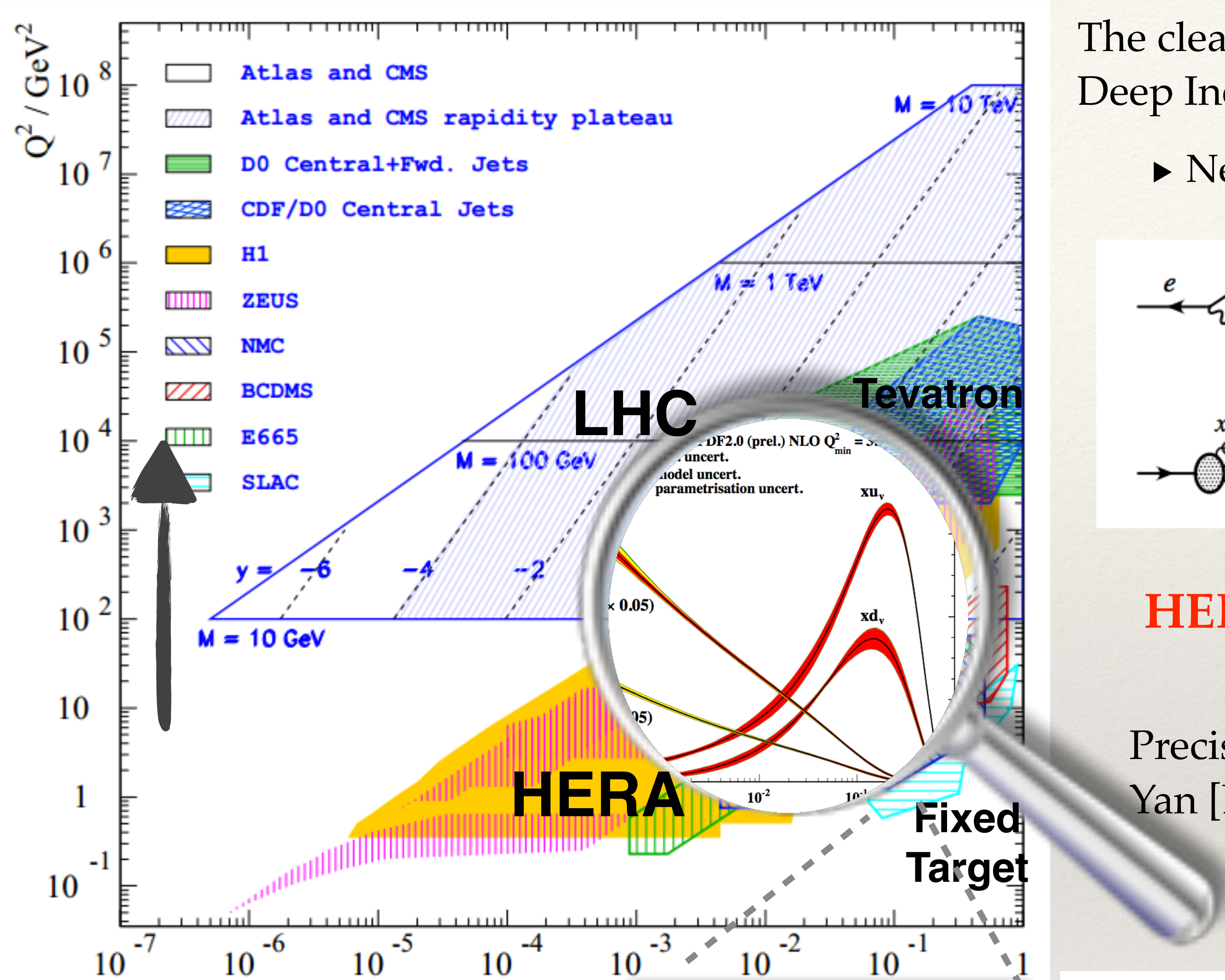
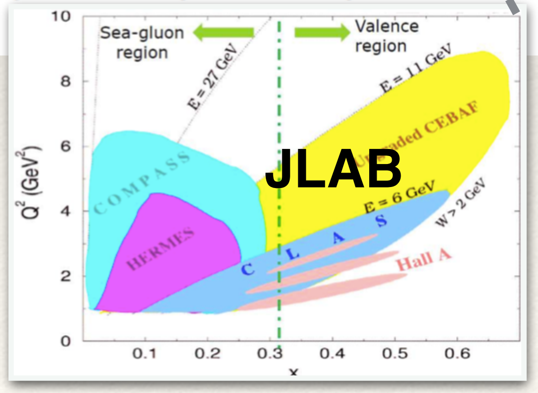
on behalf of H1 and ZEUS

Today’s data on proton structure

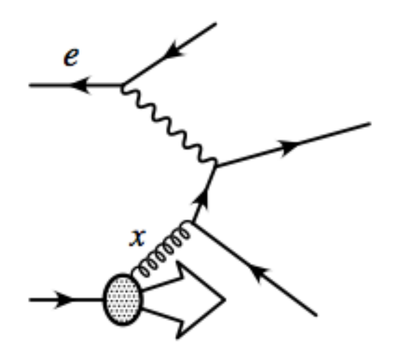


$Q^2 = -q^2 = -(k-k')^2$	Photon virtuality
$x = \frac{Q^2}{2p\cdot q}$	Bjorken variable
$y = \frac{p\cdot q}{p\cdot k}$	Inelasticity



The cleanest way to probe Proton Structure is via Deep Inelastic Scattering [DIS]:

- Neutrinos, muons, electrons

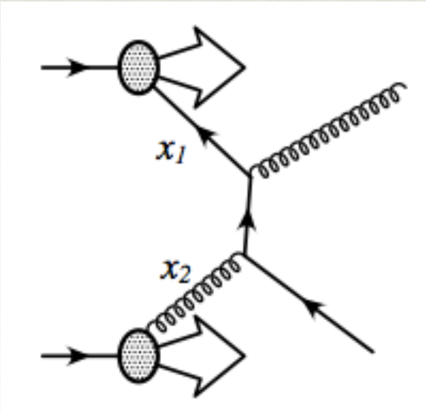


—> probes linear combination of quarks: sea quarks, gluon

HERA provides the basis of any PDFs

Precision of PDFs can be complemented by the Drell Yan [DY] processes at the collider experiments

$$\sigma_{hh\to X} = \boxed{f_{h\to a}} \otimes \hat{\sigma}_{ab\to X} \otimes \boxed{f_{h\to b}}$$

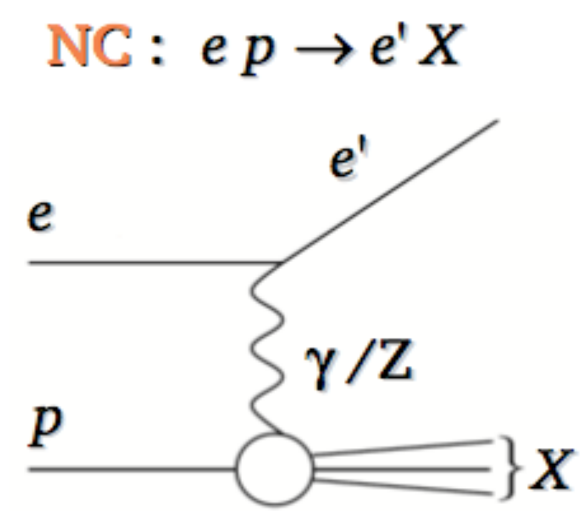


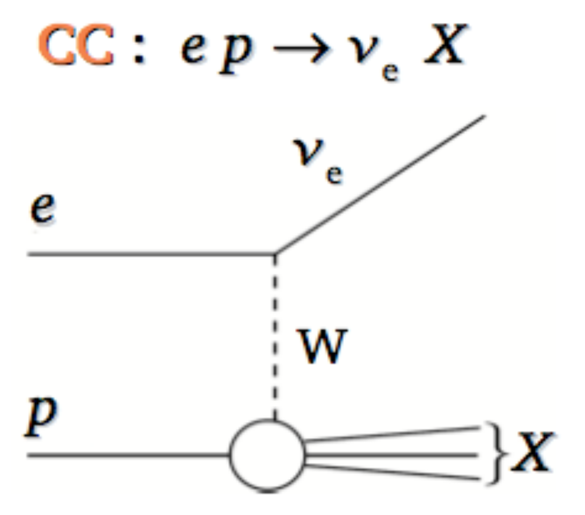
—> can provide flavour separation and more insight into gluons

—> probes bilinear combination of quarks

HERA ep collider (1992-2007) @ DESY

- H1 and ZEUS experiments at HERA collected ~1/fb of data
    - Ep=460/575/820/920 GeV and Ee=27.5 GeV
- 4 types of processes accessed at HERA: **Neutral Current and Charged Current e+p, e-p**





$$\frac{d\sigma^{\pm}_{NC}}{dxdQ^2} = \frac{2\pi\alpha^2}{x}\left[\frac{1}{Q^2}\right]^2\left[Y_+\tilde{F}_2 \mp Y_- x\tilde{F}_3 - y^2\tilde{F}_L\right]$$

$$\frac{d\sigma^{\pm}_{CC}}{dxdQ^2} = \frac{G_F^2}{4\,\pi\,x}\left[\frac{M_W^2}{M_W^2+Q^2}\right]^2\left[Y_+\tilde{W}_2^{\pm} \mp Y_- x\tilde{W}_3^{\pm} - y^2\tilde{W}_L^{\pm}\right]$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

$\tilde{F}_2 \propto \sum(xq_i + x\bar{q}_i)$ → dominant contribution (all Q^2 plane)

$x\tilde{F}_3 \propto \sum(xq_i - x\bar{q}_i)$ → significant contributions at high Q^2

$\tilde{F}_L \propto \alpha_s \cdot xg(x,Q^2)$ → high y

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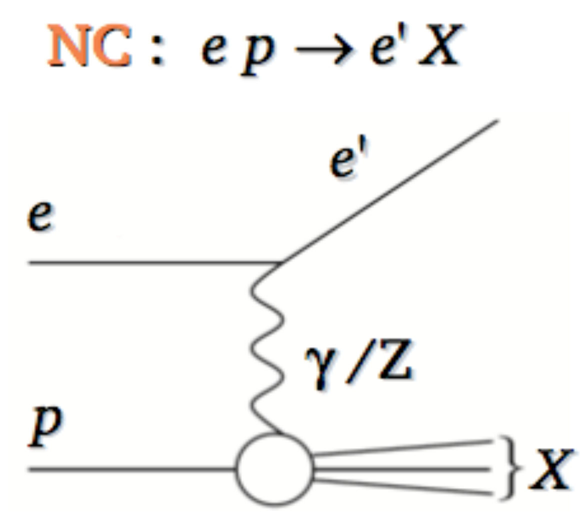

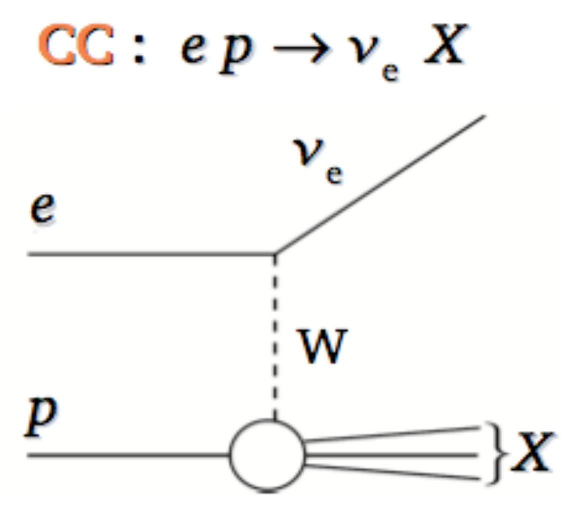

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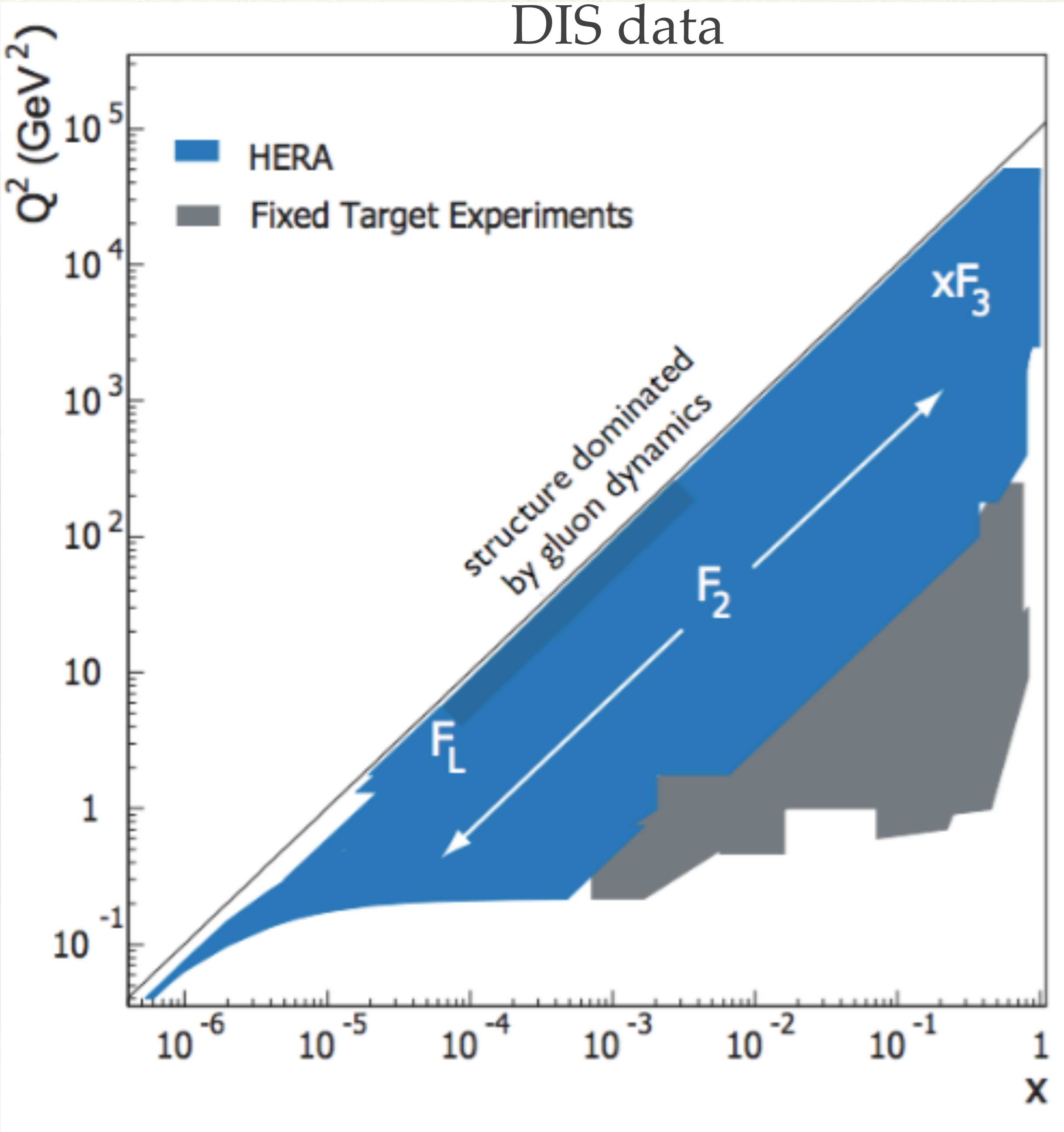
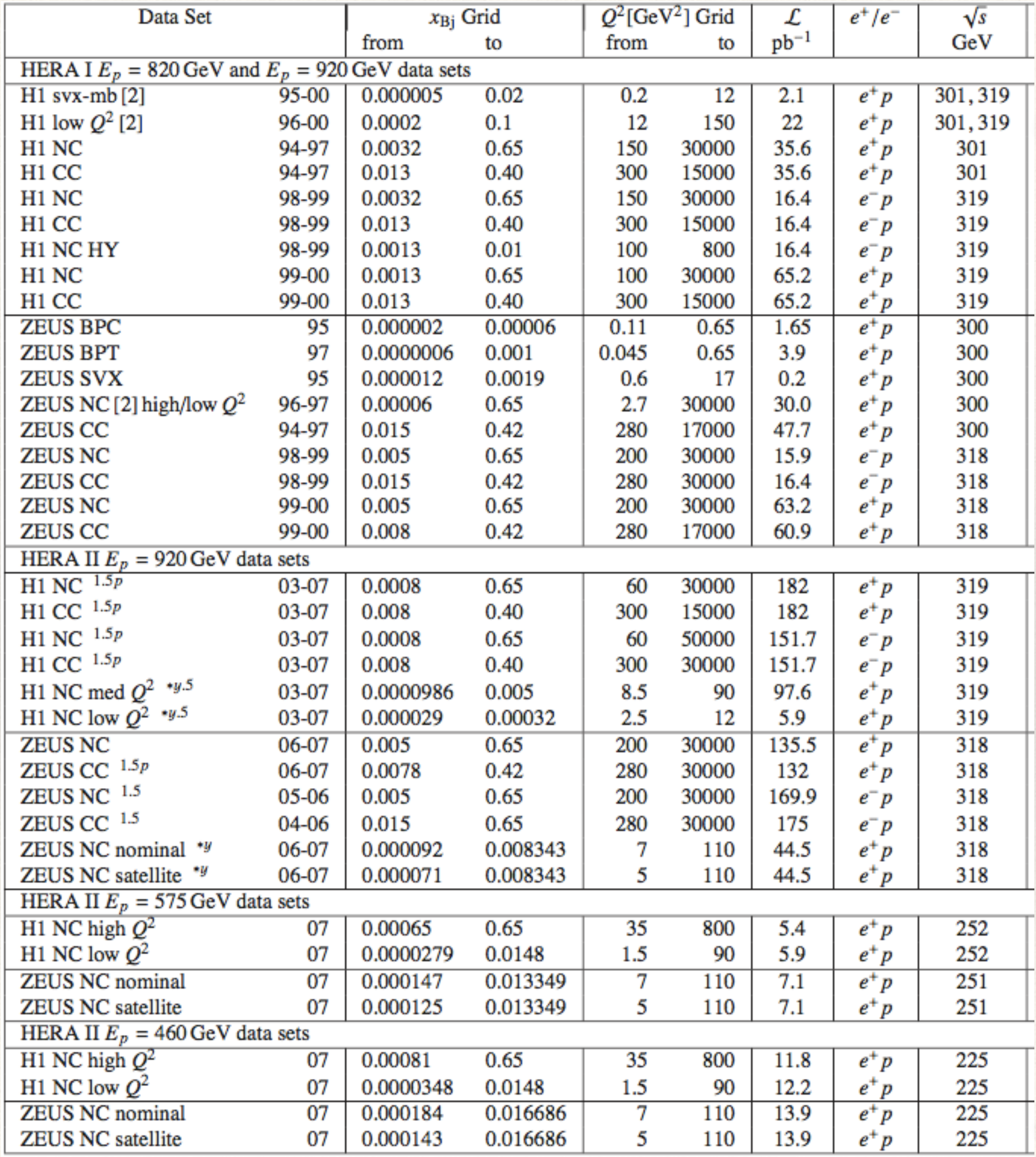
DIS data

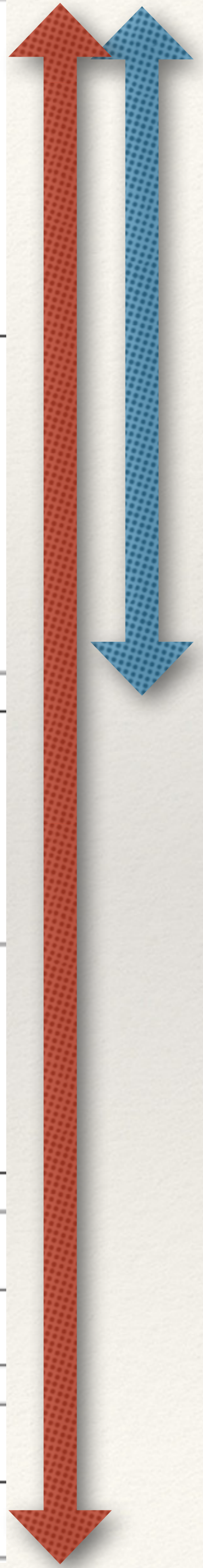


HERA ep collider (1992-2007) @ DESY



Data Set		$x_{\rm Bj}$ Grid		Q^2[GeV^2] Grid		$\mathcal{L}$	e^+/e^-	$\sqrt{s}$
		from	to	from	to	pb^{-1}		GeV
HERA I E_p = 820 GeV and E_p = 920 GeV data sets								
H1 svx-mb [2]	95-00	0.000005	0.02	0.2	12	2.1	e^+p	301, 319
H1 low Q^2 [2]	96-00	0.0002	0.1	12	150	22	e^+p	301, 319
H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p	301
H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p	301
H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p	319
H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p	319
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p	319
H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p	319
H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p	319
ZEUS BPC	95	0.000002	0.00006	0.11	0.65	1.65	e^+p	300
ZEUS BPT	97	0.0000006	0.001	0.045	0.65	3.9	e^+p	300
ZEUS SVX	95	0.000012	0.0019	0.6	17	0.2	e^+p	300
ZEUS NC [2] high/low Q^2	96-97	0.00006	0.65	2.7	30000	30.0	e^+p	300
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p	300
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p	318
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p	318
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p	318
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p	318
HERA II E_p = 920 GeV data sets								
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	30000	182	e^+p	319
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	15000	182	e^+p	319
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	50000	151.7	e^-p	319
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	30000	151.7	e^-p	319
H1 NC med Q^2 $^{*y.5}$	03-07	0.0000986	0.005	8.5	90	97.6	e^+p	319
H1 NC low Q^2 $^{*y.5}$	03-07	0.000029	0.00032	2.5	12	5.9	e^+p	319
ZEUS NC	06-07	0.005	0.65	200	30000	135.5	e^+p	318
ZEUS CC $^{1.5p}$	06-07	0.0078	0.42	280	30000	132	e^+p	318
ZEUS NC $^{1.5}$	05-06	0.005	0.65	200	30000	169.9	e^-p	318
ZEUS CC $^{1.5}$	04-06	0.015	0.65	280	30000	175	e^-p	318
ZEUS NC nominal ^{*y}	06-07	0.000092	0.008343	7	110	44.5	e^+p	318
ZEUS NC satellite ^{*y}	06-07	0.000071	0.008343	5	110	44.5	e^+p	318
HERA II E_p = 575 GeV data sets								
H1 NC high Q^2	07	0.00065	0.65	35	800	5.4	e^+p	252
H1 NC low Q^2	07	0.0000279	0.0148	1.5	90	5.9	e^+p	252
ZEUS NC nominal	07	0.000147	0.013349	7	110	7.1	e^+p	251
ZEUS NC satellite	07	0.000125	0.013349	5	110	7.1	e^+p	251
HERA II E_p = 460 GeV data sets								
H1 NC high Q^2	07	0.00081	0.65	35	800	11.8	e^+p	225
H1 NC low Q^2	07	0.0000348	0.0148	1.5	90	12.2	e^+p	225
ZEUS NC nominal	07	0.000184	0.016686	7	110	13.9	e^+p	225
ZEUS NC satellite	07	0.000143	0.016686	5	110	13.9	e^+p	225

- 41 data sets: 2927 data points are combined to1307 averaged measurements with 169 sources of correlated systematic uncertainties.



HERAPDF1.0

JHEP01 (2010) 109

HERAPDF1.5
(prelim)

HERAPDF2.0

[arxiv:1506.06042]

Combination of the H1 and ZEUS Measurements

[see O. Turkot]

FINAL HERA I+II inclusive data combination [arxiv:1506.06042]

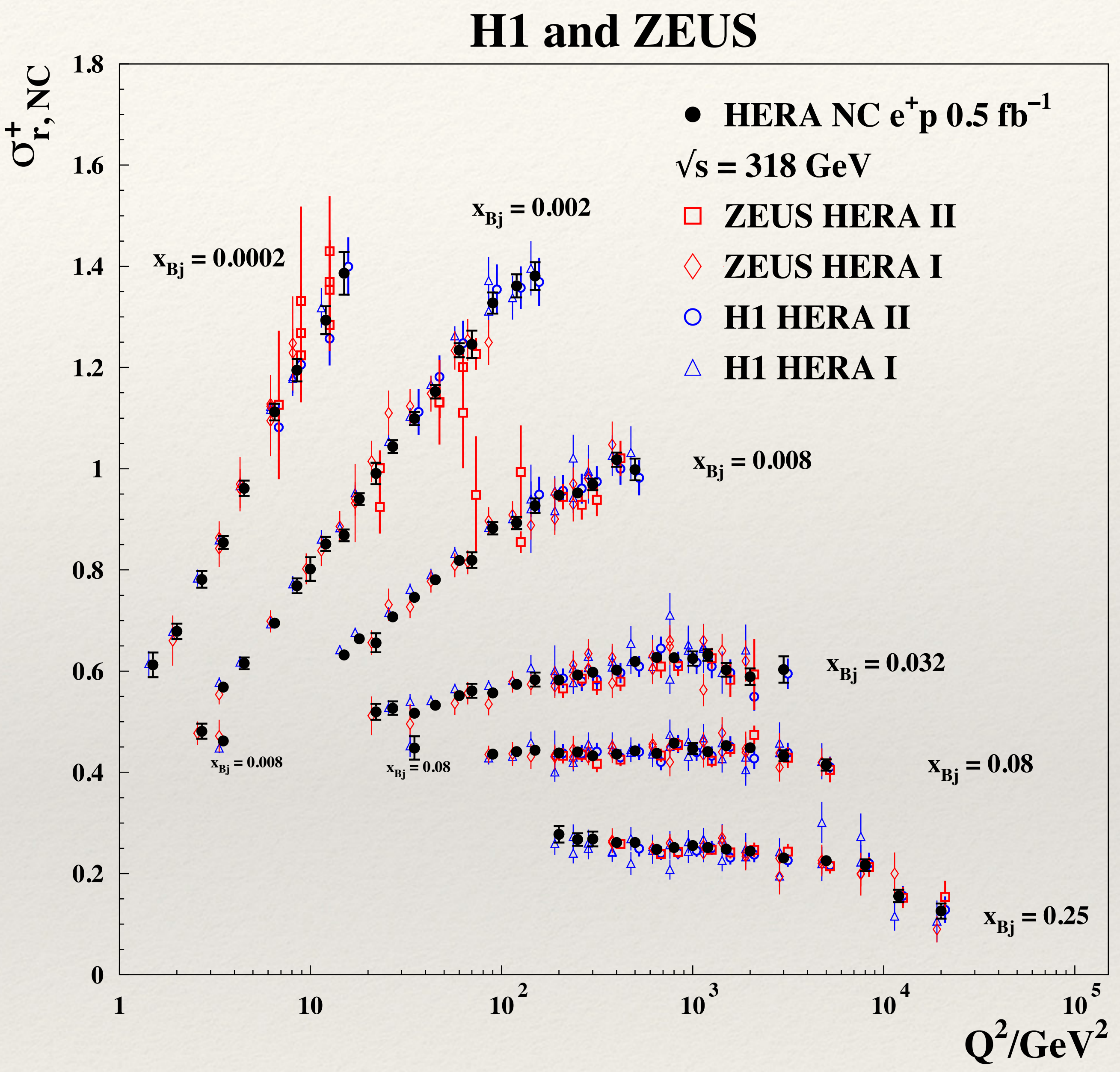
- Ultimate precision is obtained by combining the H1 and ZEUS measurements
- The combination procedure is performed before QCD analysis using χ^2 minimisation
  - χ^2 / dof =1687/1620

➢ Improvement on Statistical precision:

➢ Improvement of Systematic precision:

- ❖ H1 and ZEUS are different detectors and use different analysis techniques;
- ❖ The H1 and ZEUS cross sections have different sensitivities to similar sources of correlated systematic uncertainty.

—> total uncertainty < 1.3% for Q^2 up to 400 GeV^2



$0.045 < Q^2 < 50000$ GeV^2 $\quad$ $6.\ 10^{-7} < x_{Bj} < 0.65$

$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{\mathrm{d}^2\sigma^{e^\pm p}_{\mathrm{NC}}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2} \cdot \frac{Q^4 x_{\mathrm{Bj}}}{2\pi\alpha^2 Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+} x\tilde{F}_3 - \frac{y^2}{Y_+}\tilde{F}_{\mathrm{L}}$$

Combination of data is now actively used at LHC for ex W, Z for muon and electron channels

Extraction of PDFs through QCD fits

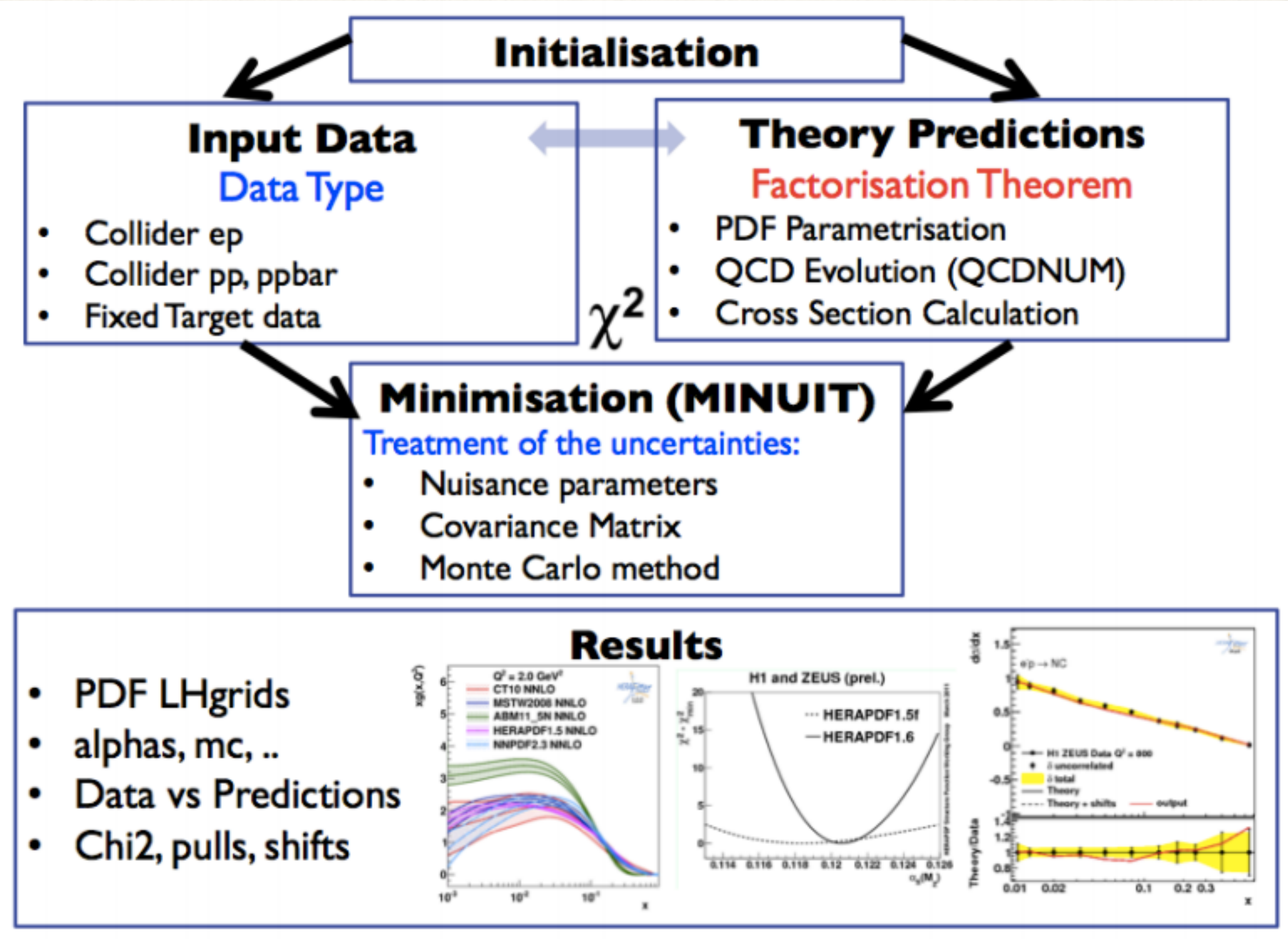
[see V.R. HERAFitter talk]

- Extraction of PDFs relies on the factorisation: $\sigma = \hat{\sigma} \otimes \mathrm{PDF}$

- Typical measurements sensitive to PDFs are precise, with statistical uncertainties < 10%, so they follow normal distribution —> use of χ^2 minimisation for PDF extraction.

Main Steps:

- Parametrise PDFs at a starting scale
- Evolve PDFs to the scale corresponding to data point
- Calculate the cross section
- Compare with data via χ^2
- Minimise χ^2 with respect to PDF parameters which takes about ~2000 iterations:



Initialisation

Input Data
Data Type
- Collider ep
- Collider pp, ppbar
- Fixed Target data

Theory Predictions
Factorisation Theorem
- PDF Parametrisation
- QCD Evolution (QCDNUM)
- Cross Section Calculation

χ^2

Minimisation (MINUIT)
Treatment of the uncertainties:
- Nuisance parameters
- Covariance Matrix
- Monte Carlo method

Results
- PDF LHgrids
- alphas, mc, ..
- Data vs Predictions
- Chi2, pulls, shifts

herafitter.org: open source QCD platform arxive:1503.05221

QCD Settings for HERAPDF2.0

The QCD settings are optimised for HERA measurements of proton structure functions:

PDFs are parametrised at the starting scale $Q_0^2=1.9\ \mathrm{GeV}^2$ as follows:

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g},\\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right),\\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}},\\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x),\\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

⬜ fixed or constrained by sum-rules

🟦 parameters set equal but free

NC structure functions

$$F_2 = \frac{4}{9}\left(xU + x\bar{U}\right) + \frac{1}{9}\left(xD + x\bar{D}\right)$$

$$xF_3 \sim xu_v + xd_v$$

CC structure functions

$$W_2^- = x(U+\overline{D})\,, \qquad W_2^+ = x(\overline{U}+D)$$

$$xW_3^- = x(U-\overline{D})\,, \qquad xW_3^+ = x(D-\overline{U})$$

Due to increased precision of data, more flexibility in functional form is allowed —> 14 free parameters

- PDFs are evolved via evolution equations (DGLAP) to NLO and NNLO ($\alpha_S(M_Z)=0.118$)
- Thorne-Roberts GM-VFNS for heavy quark coefficient functions – as used in MMHT
- χ^2 definition used in the minimisation [MINUIT] accounts for correlated uncertainties:

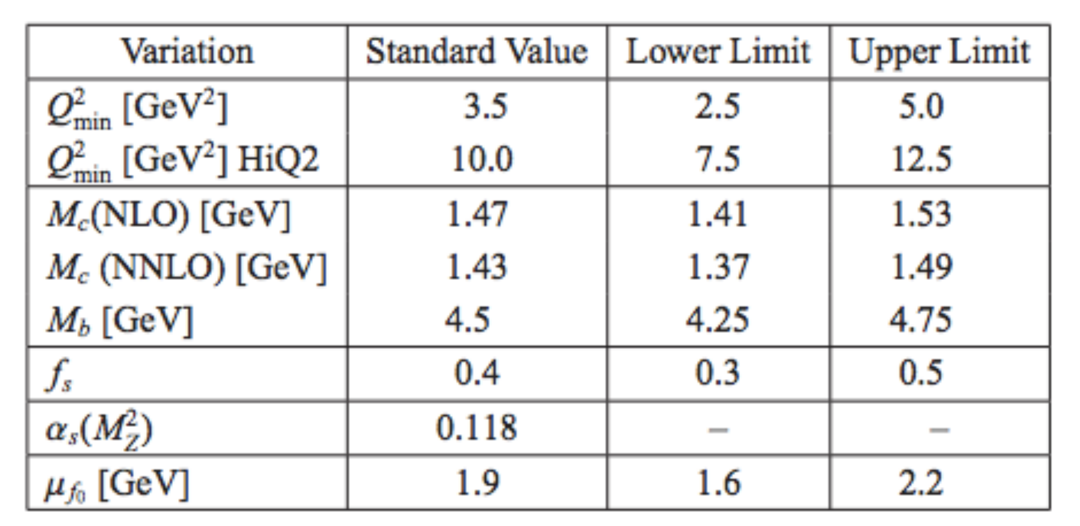
$$\chi^2_{\mathrm{exp}}(\boldsymbol{m},\boldsymbol{s}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i s_j - \mu^i\right]^2}{\delta^2_{i,\mathrm{stat}}\,\mu^i m^i + \delta^2_{i,\mathrm{uncor}}\,(m^i)^2} + \sum_j s_j^2 + \sum_i \ln \frac{\delta^2_{i,\mathrm{stat}}\mu^i m^i + (\delta_{i,\mathrm{uncor}} m^i)^2}{(\delta^2_{i,\mathrm{stat}} + \delta^2_{i,\mathrm{uncor}})(\mu^i)^2}$$

m - th prediction
μ - data
s - sys shift

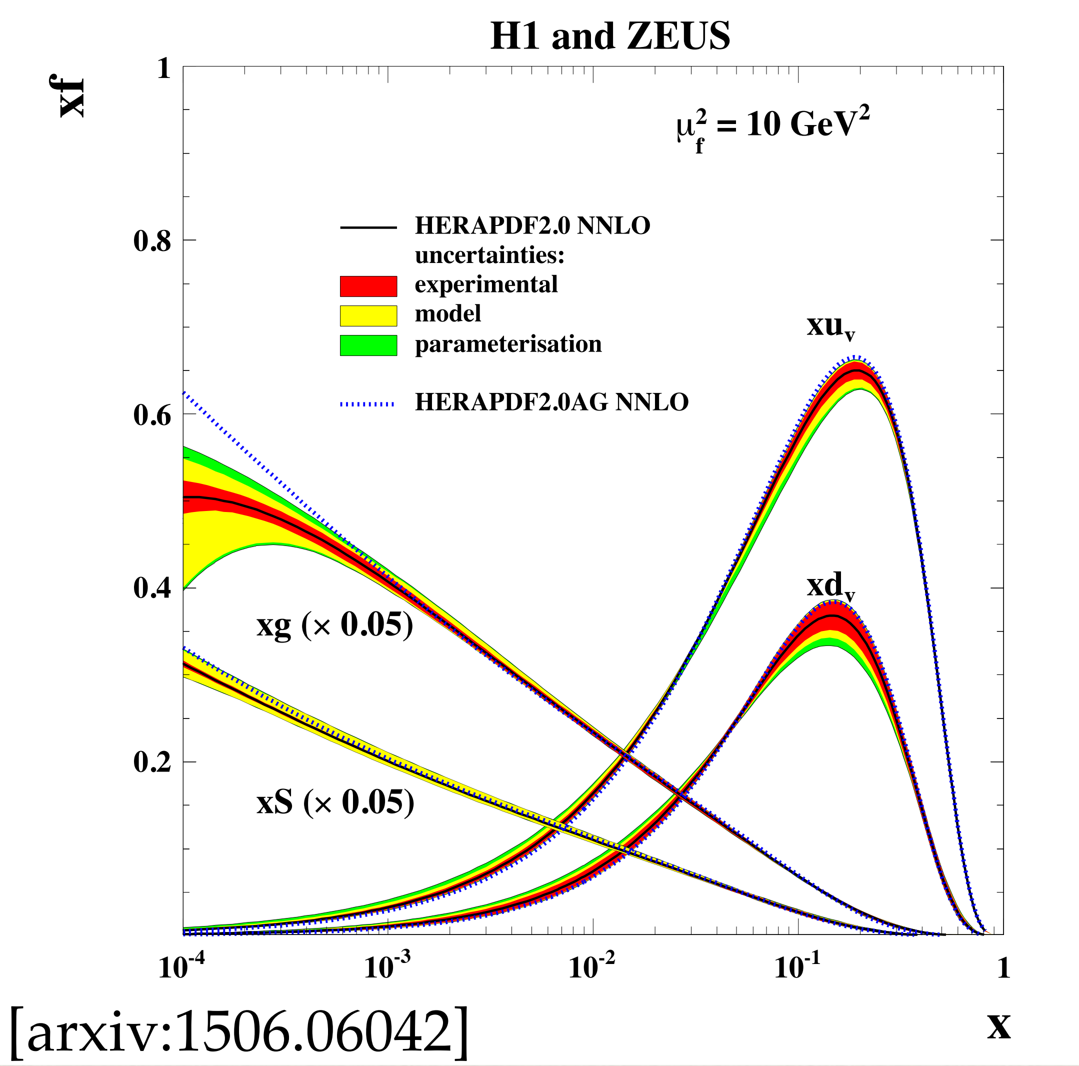
Modern understanding of PDFs

Different types of PDF uncertainties are considered:

- Experimental:
  - Hessian method used: MMHT, CT, …
    - Consistent data sets → use $\Delta\chi^2$=1
  - Monte Carlo Method: replicas of data(NNPDF)
- Model:
  - variations of all assumed input parameters in the fit



Variation	Standard Value	Lower Limit	Upper Limit
$Q^2_{\min}$ [GeV^2]	3.5	2.5	5.0
$Q^2_{\min}$ [GeV^2] HiQ2	10.0	7.5	12.5
M_c(NLO) [GeV]	1.47	1.41	1.53
M_c (NNLO) [GeV]	1.43	1.37	1.49
M_b [GeV]	4.5	4.25	4.75
f_s	0.4	0.3	0.5
$\alpha_s(M_Z^2)$	0.118	–	–
μ_{f_0} [GeV]	1.9	1.6	2.2

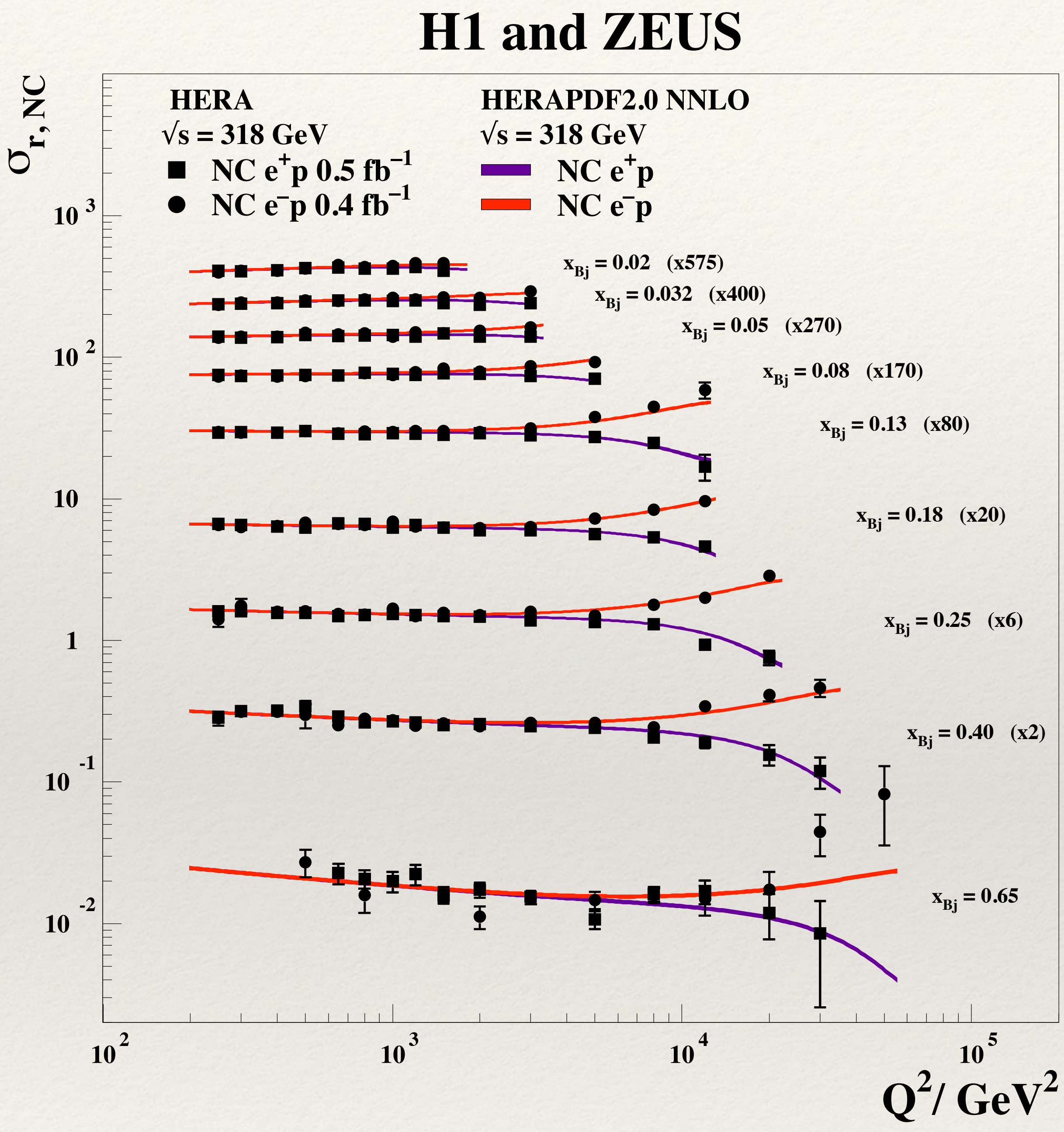


[arxiv:1506.06042]

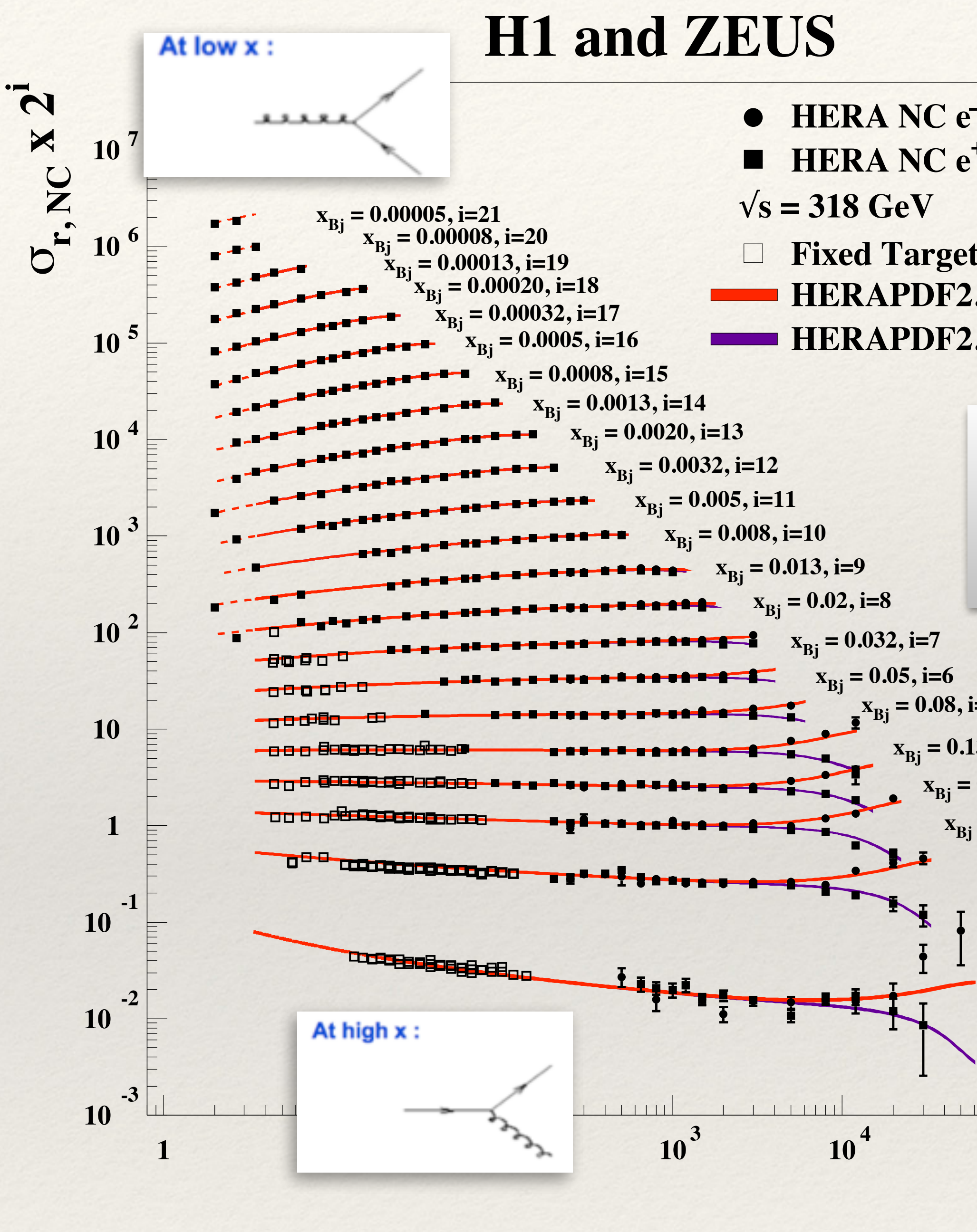
- Parametrisation: only HERAPDF includes this as an additional uncertainty
  - NNPDFs use neural network approach based on data driven regularisation

QCD scaling and EW effects

- EW effects clearly seen at high Q^2:



QCD scaling violations nicely seen:

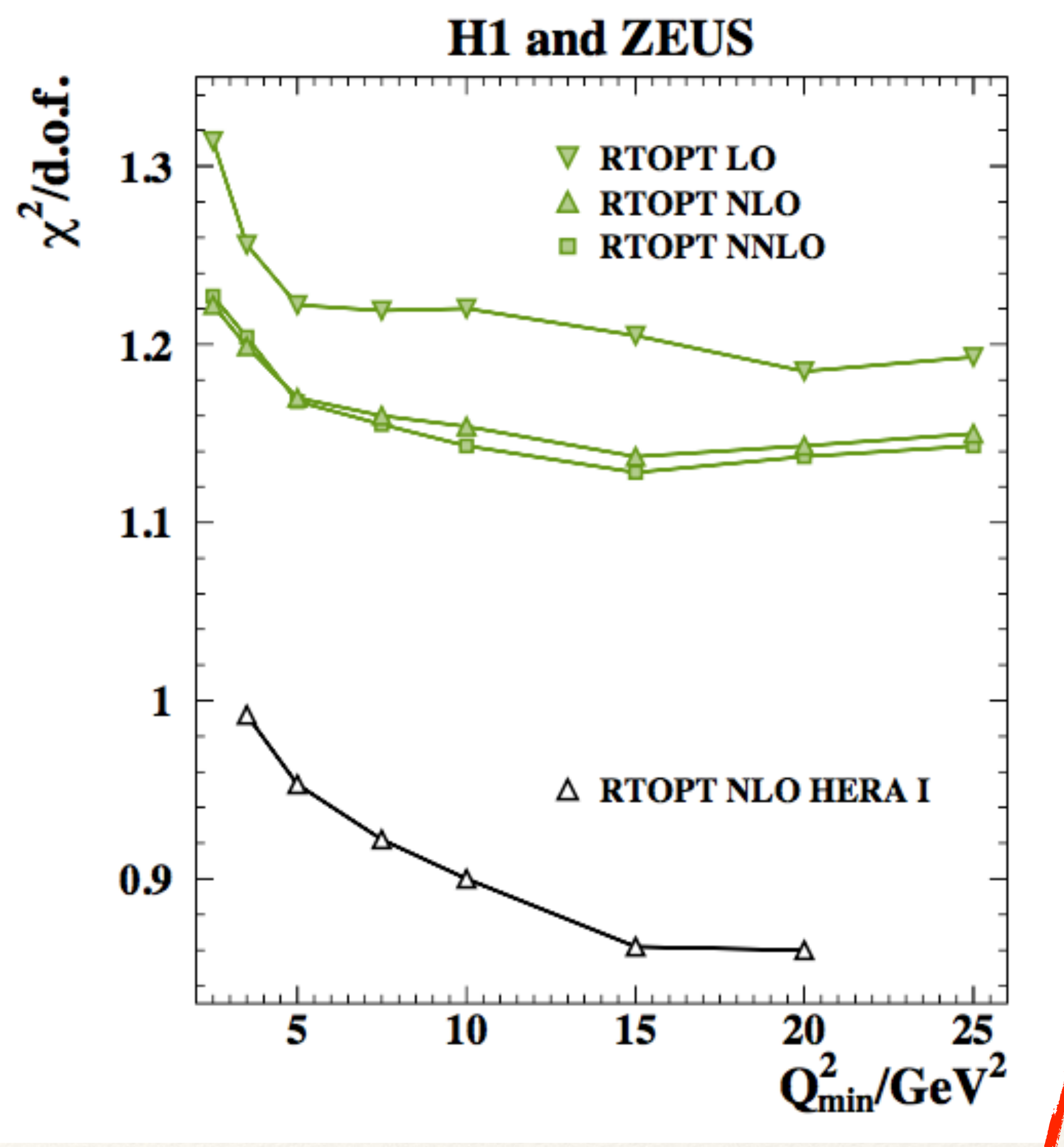


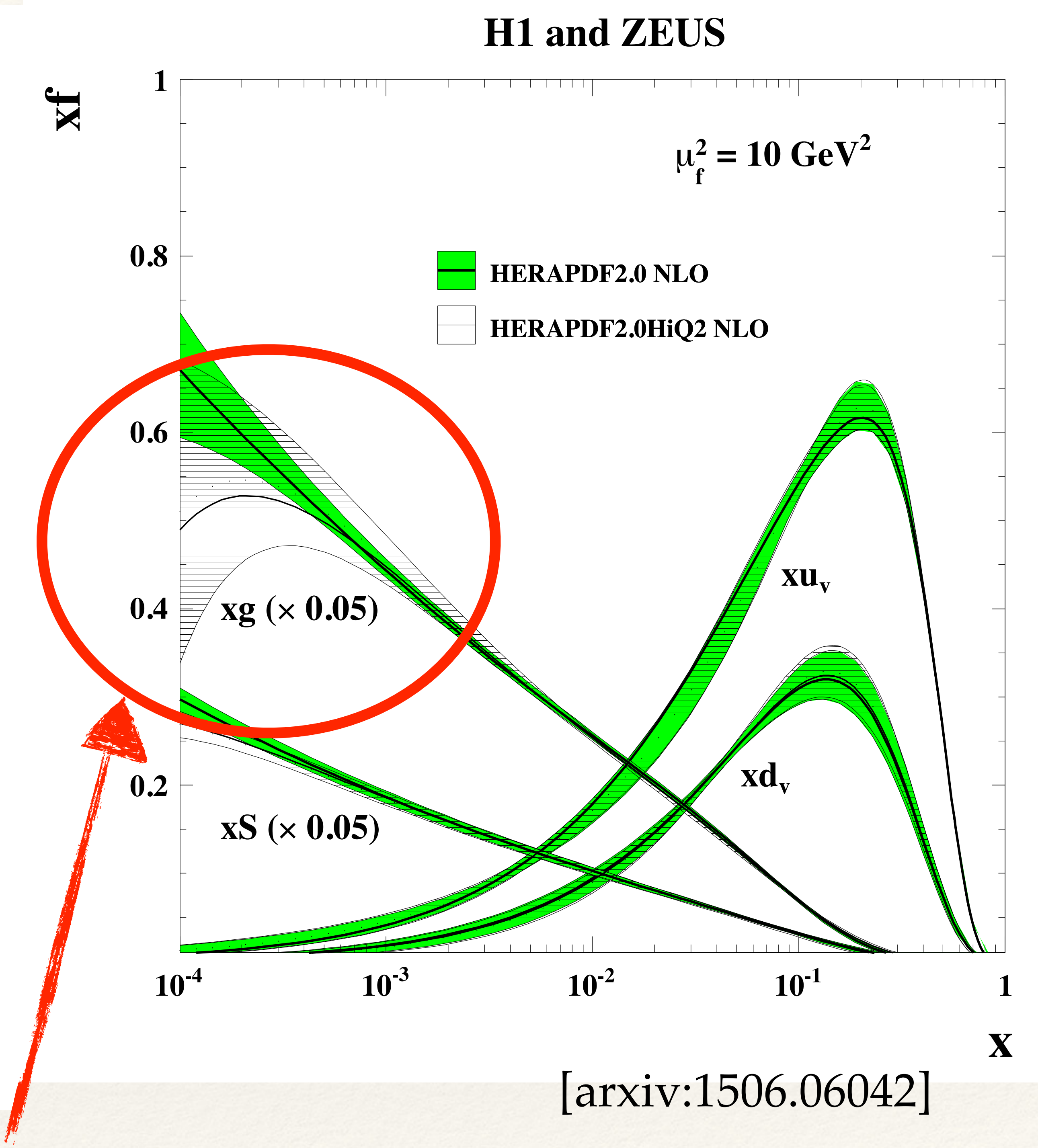
The x region relevant for the Higgs production is at 10-2.

$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{\mathrm{d}^2\sigma^{e^\pm p}_{\mathrm{NC}}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2}\cdot\frac{Q^4 x_{\mathrm{Bj}}}{2\pi\alpha^2 Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+}x\tilde{F}_3 - \frac{y^2}{Y_+}\tilde{F}_{\mathrm{L}}$$

Q^2 cut dependence on PDFs

- HERA data provides a unique access to the low x, low Q^2 region to investigate:
  - the validity of the DGLAP mechanism



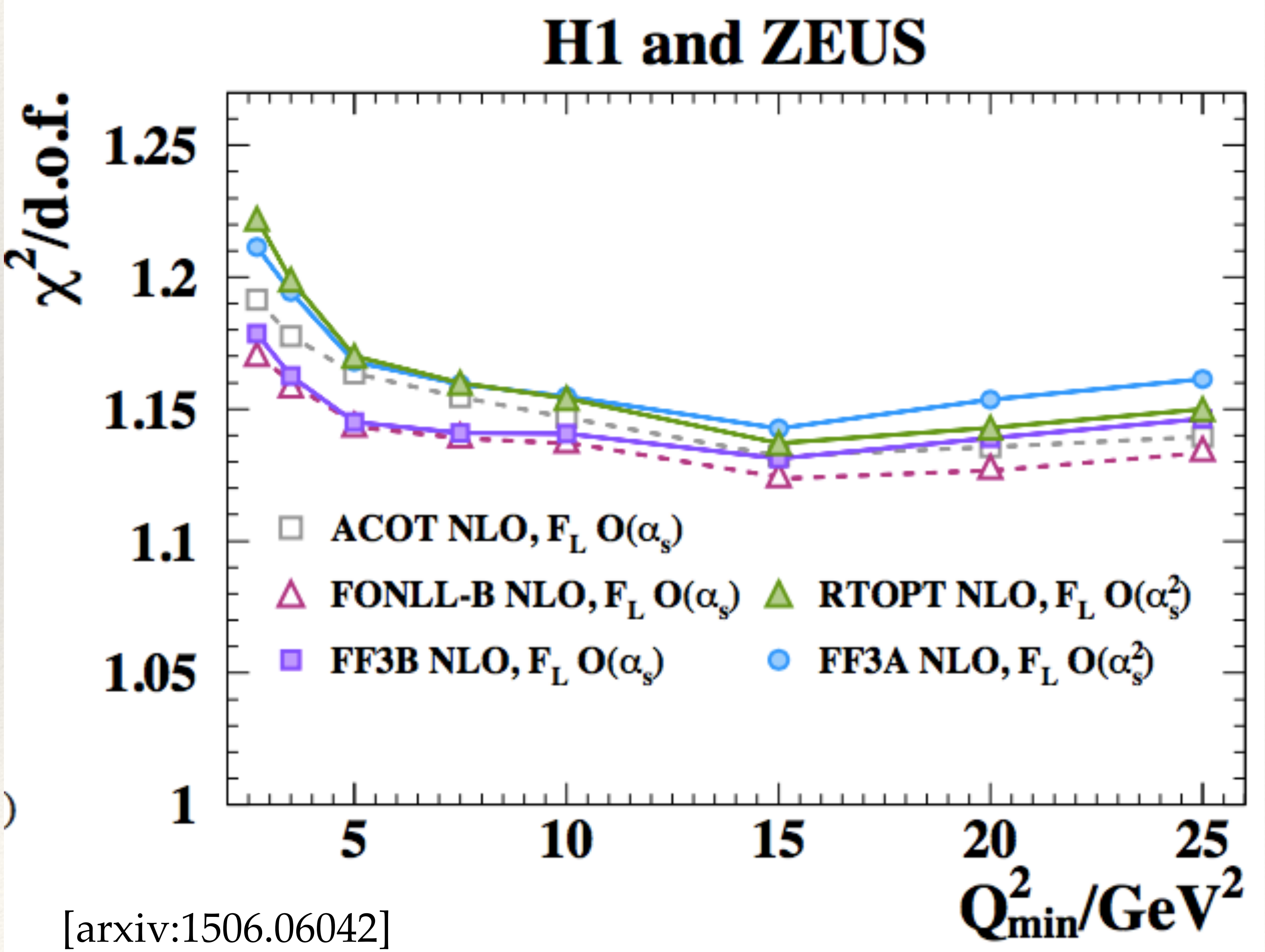


[arxiv:1506.06042]

low Q^2 data very important to constrain low x PDFs!

Q^2 cut dependence

- HERA data provides a unique access to the low x, low Q^2 region to investigate:
  - the validity of the DGLAP mechanism
  - the various scheme dependence (fixed vs variable flavours)



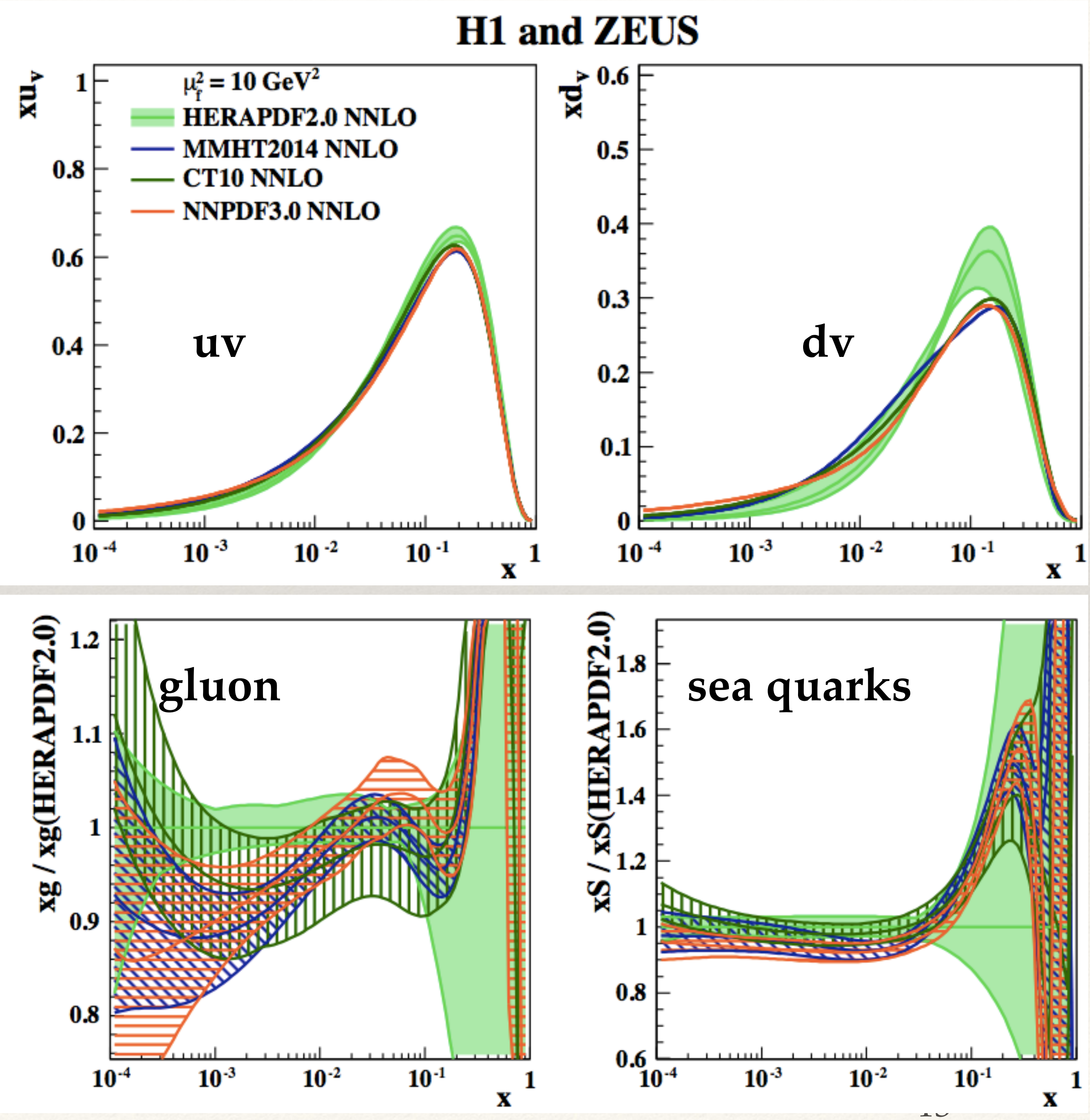
[arxiv:1506.06042]

ACOT -> as used by CT
RT -> as used by MMHT
FONLL ->as used by NNPDF
FF3A ->as used by ABM

Low Q^2 remains an interesting region to investigate (low x phenomenology)

HERAPDF2.0 vs other PDF sets

- HERAPDF sets are extracted solely from ep data and require no assumptions or corrections, hence provide an important cross check of PDF universality (process independence):

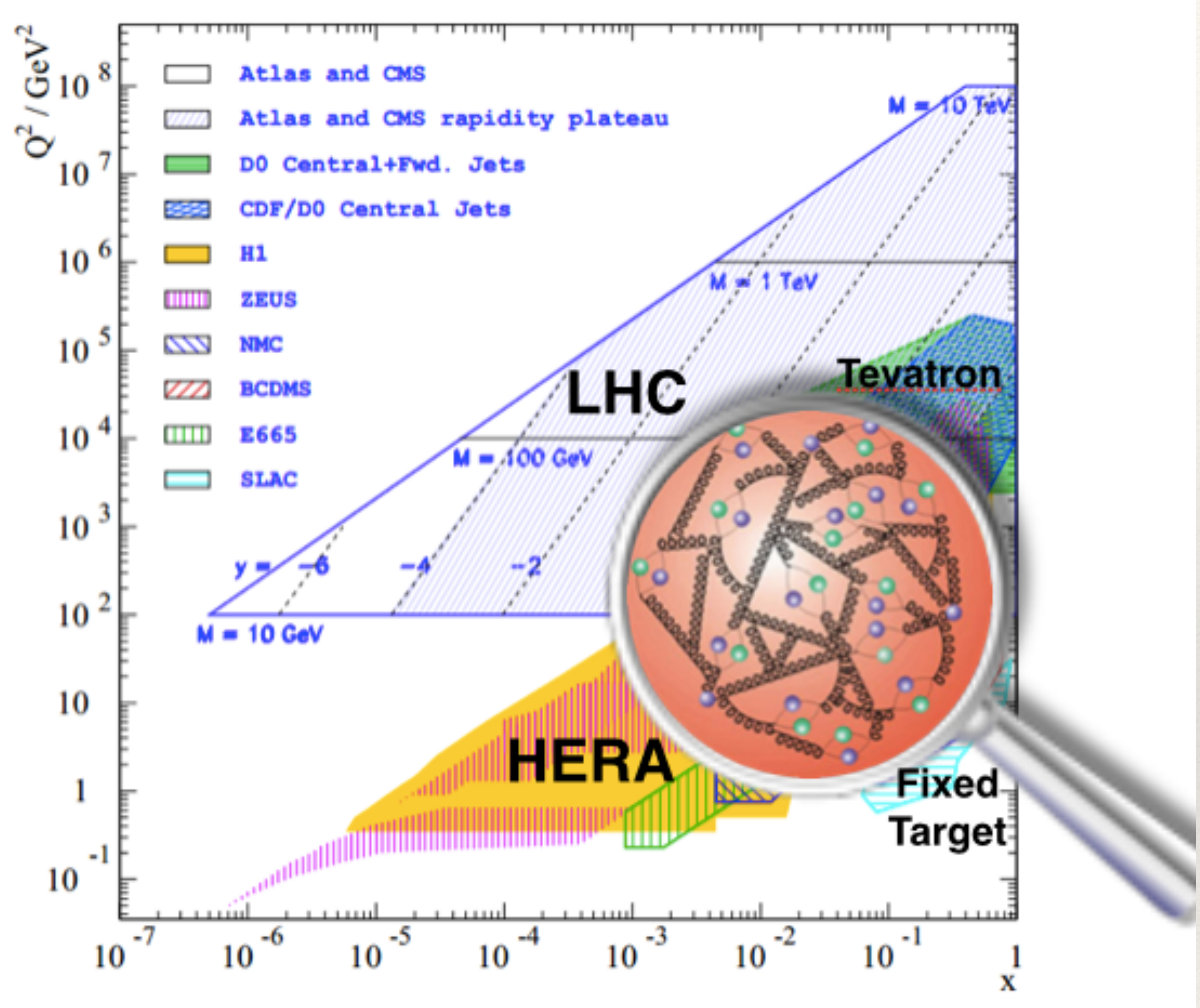


high x valence different:
new high- x data and use of proton target only

At NNLO gluon and sea quarks are both compatible with other PDFs

[arxiv:1506.06042]

Summary



PDFs are very important as they still limit our knowledge of cross sections whether SM or BSM.

- HERA has finalised its separate measurements relevant to PDFs and has combined them into final measurements to reach its ultimate precision:
    - PDFs, mc, mb, alphas

other related HERA talks at EPS:
- O. Turkot
- K. Wichmann
- A. Geiser

back-up slides

not necessarily useful …

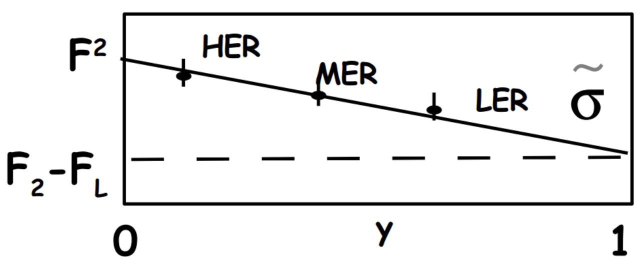
Longitudinal Structure Function

Longitudinal structure function FL is a pure QCD effect:

—> an independent way to probe sensitivity to gluon

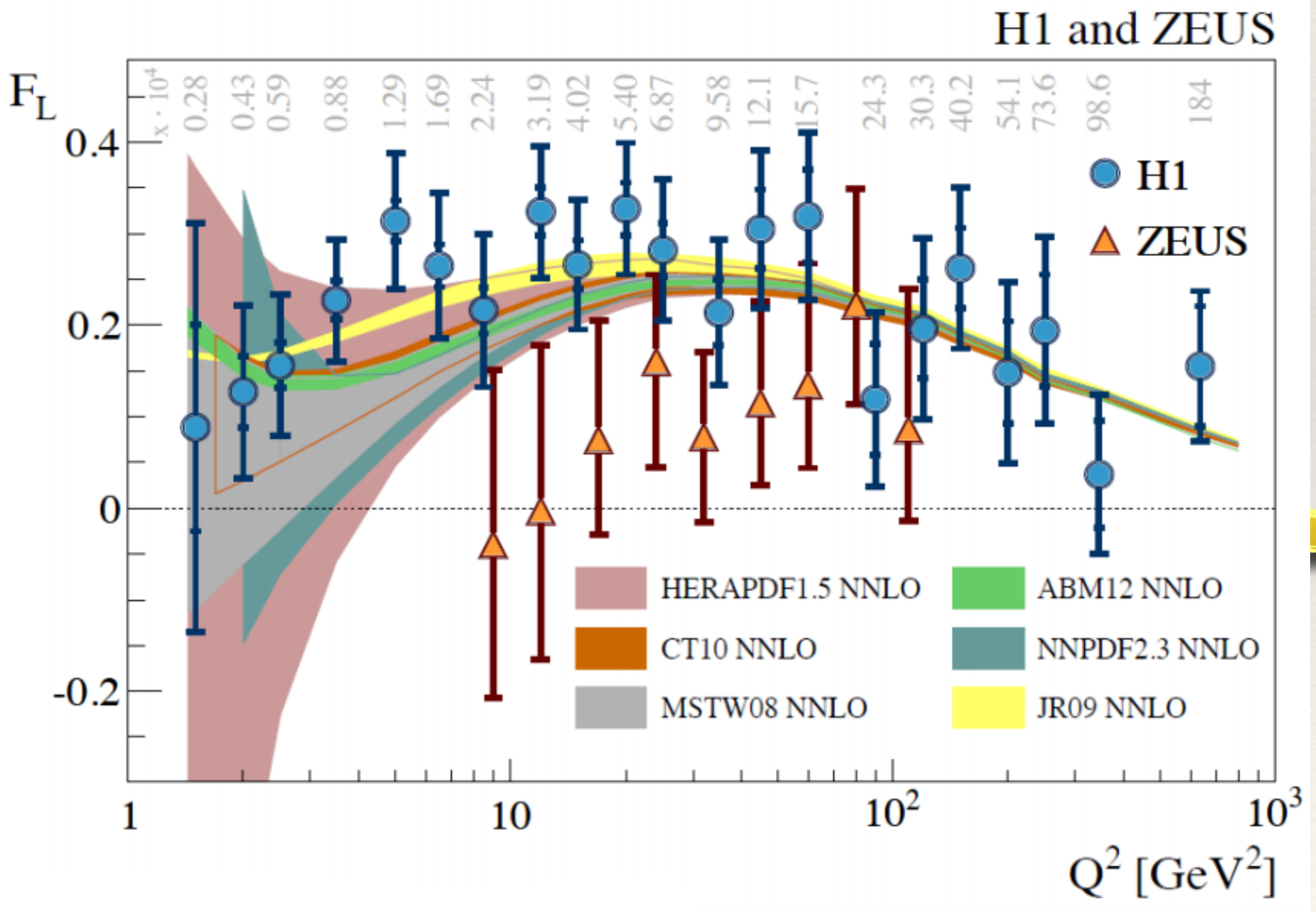
$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3}\left[\frac{16}{3}\underbrace{F_2}_{\text{quarks radiating a gluon}} + 8\sum_q \underbrace{e_q^2\left(1-\frac{x}{z}\right)zg(z)}_{\text{gluons splitting into quarks}}\right]$$

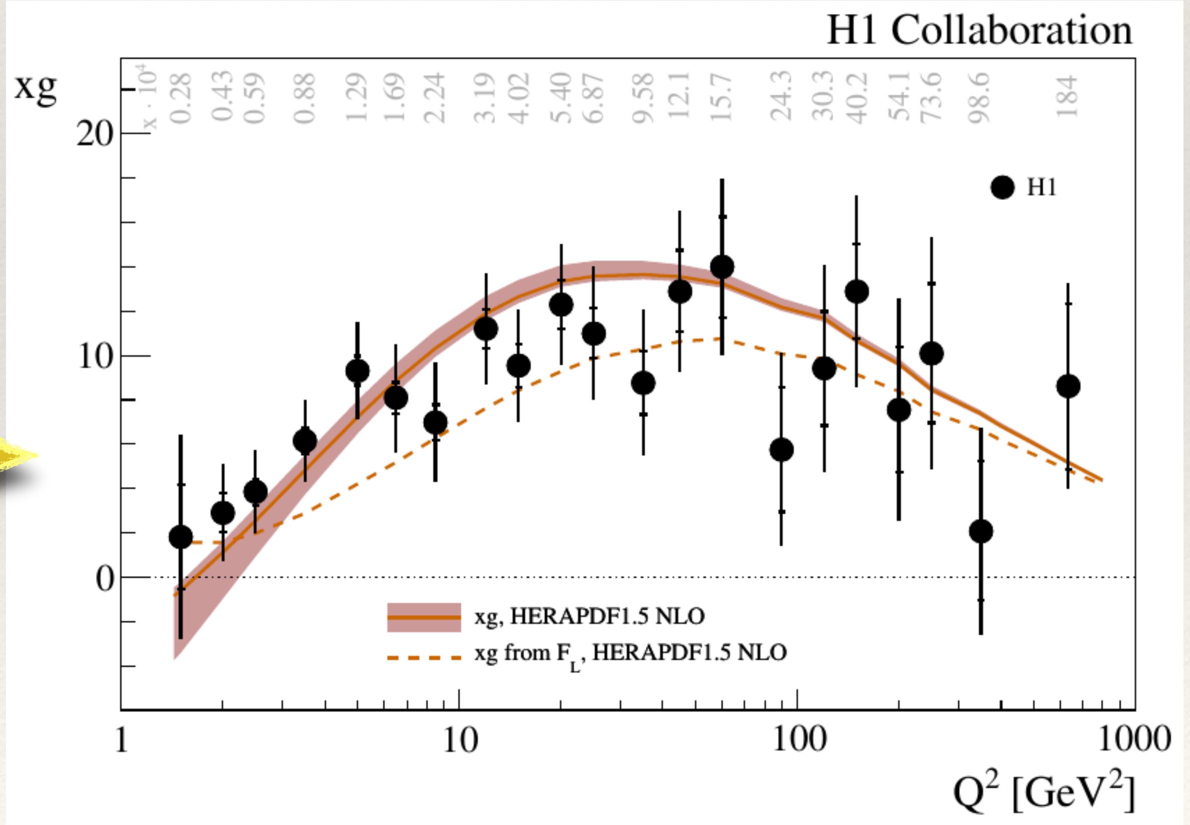
Direct measurement of FL at HERA required differential cross sections at same x and Q^2 but different y —> different beam energies: Ep= 460, 575, 920 GeV



$$\sigma_{NC}(x,Q^2,y) \propto F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$$

$$xg(x,Q^2) \approx 1.77\frac{3\pi}{2\alpha_S(Q^2)}F_L(ax,Q^2)$$





Eur. Phys. J. C 74 (2014) 2814 [arXiv:1312.4821]

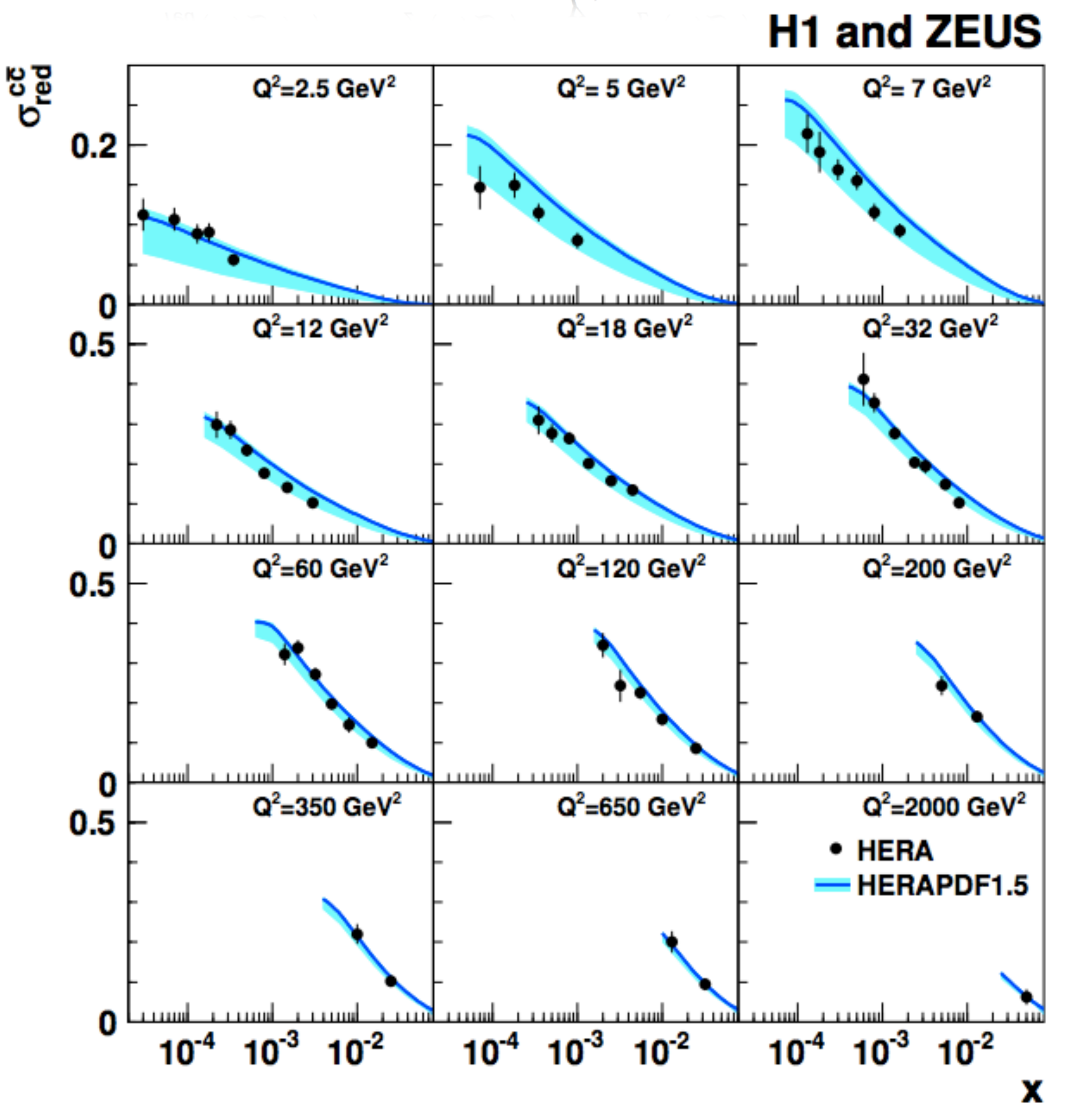
DESY-14-053, submitted to Phys. Lett. B [arXiv:1404.6376]

F2 charm Structure Function

EPJC 73 (2013) 2311

- Rates at HERA in DIS regime σ(b) : σ(c) ≈ O(1%) : O(20%) of σ_{TOT}
- Charm data combination is performed at charm cross sections level:
  - they are obtained from xsec in visible phase space and extrapolated to full space

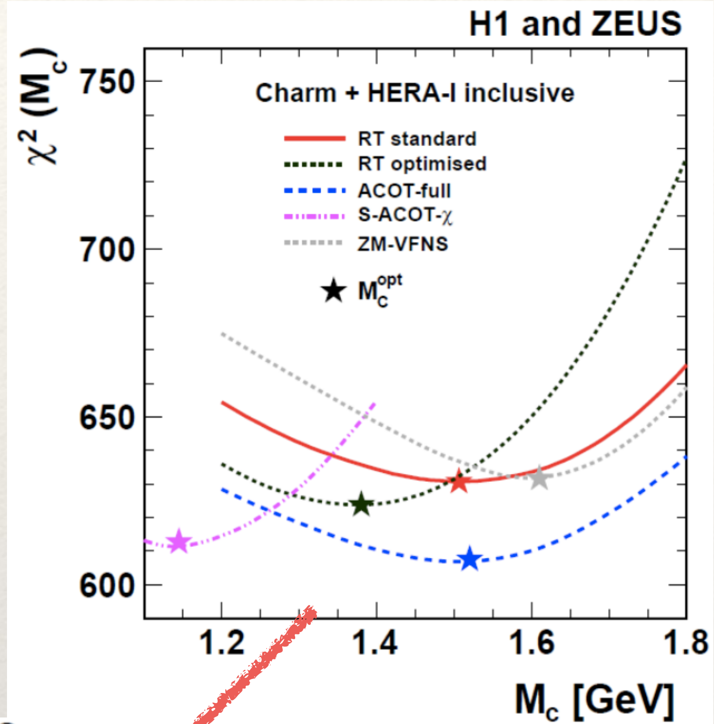
$$\sigma_{red}^{c\bar{c}}(x,Q^2,s) = F_2^{c\bar{c}}(x,Q^2) - \frac{y^2}{Y_+}F_L^{c\bar{c}}(x,Q^2)$$

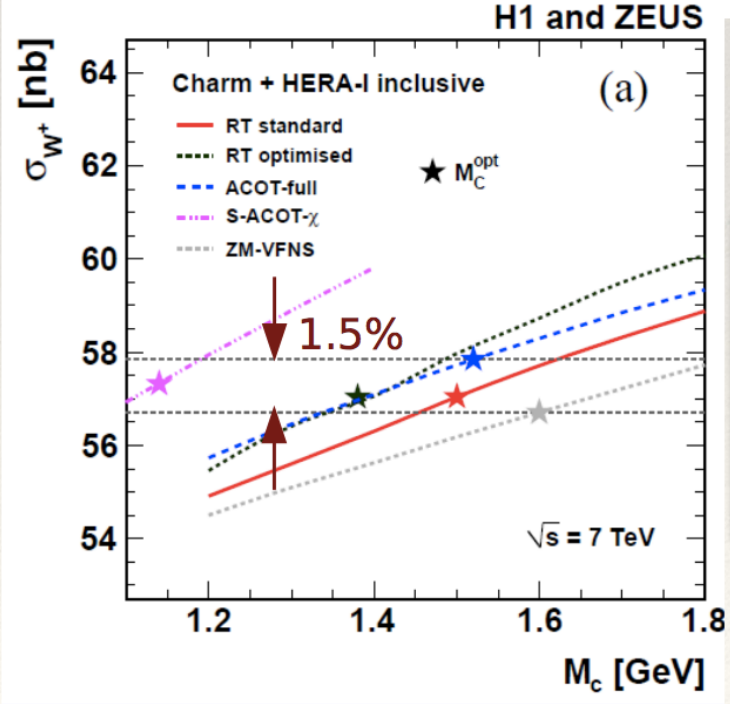


QCD Fits

HERA I+charm

Different calculation schemes prefer different Mc





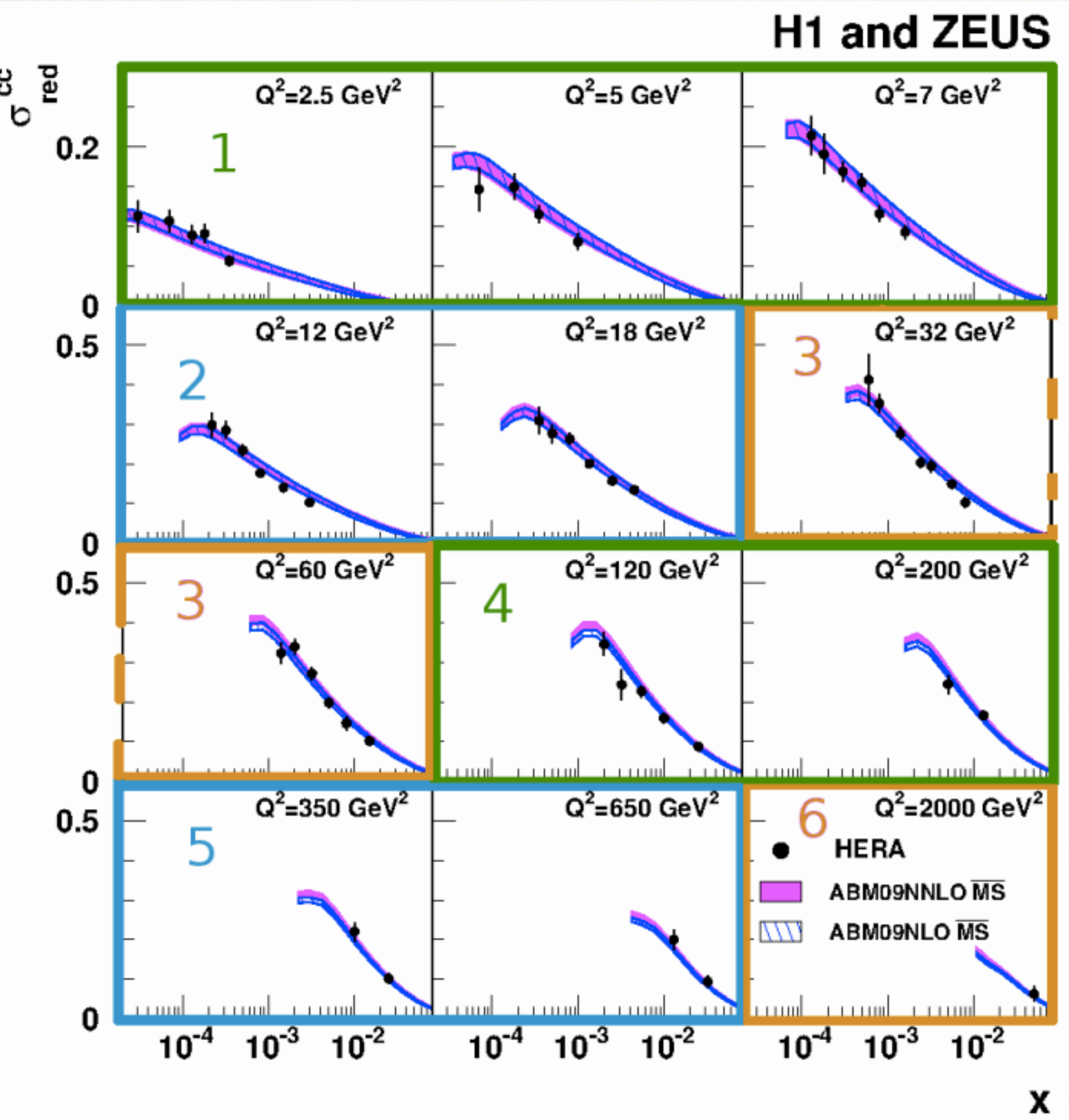
measurements help reduce uncertainties of predictions for the LHC

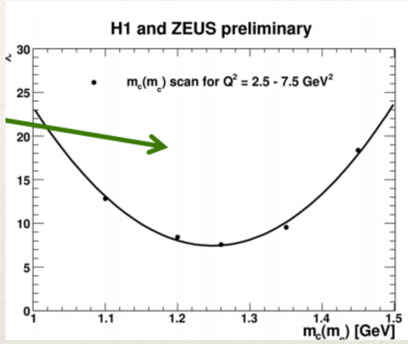
New Measurement of Charm Mass Running

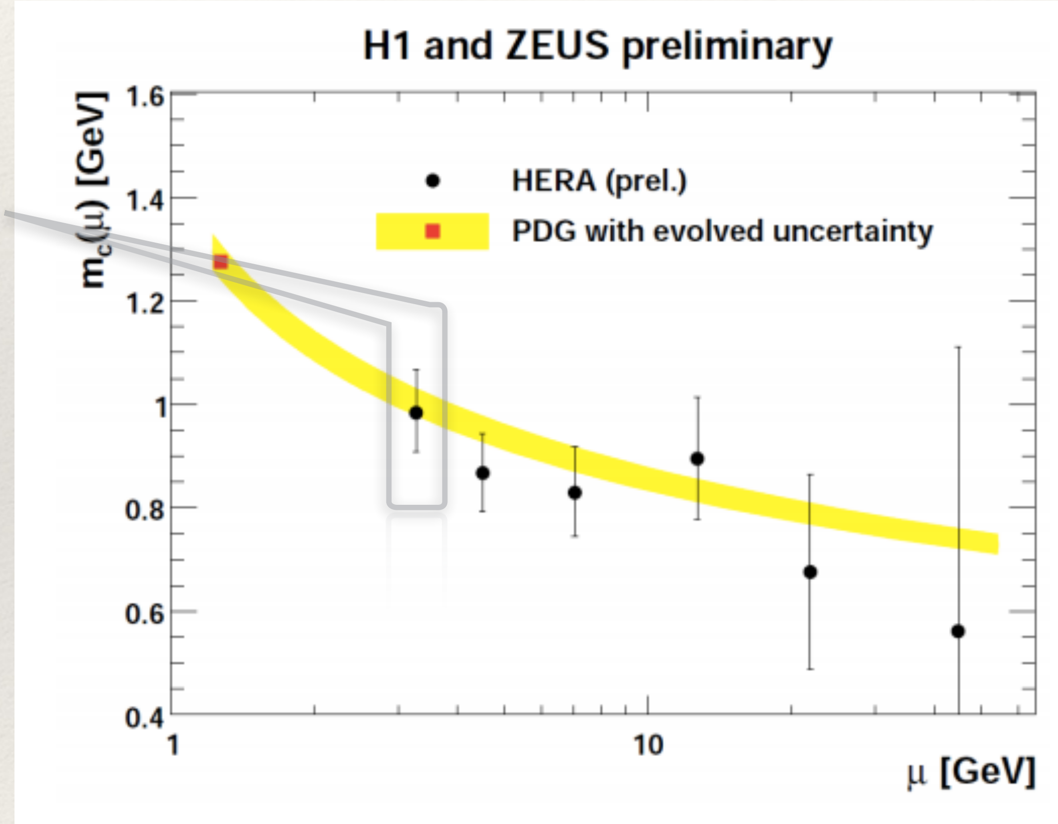
H1-prelim-14-071 ZEUS-prel-14-006 and S. Moch

The running of the charm mass in the $\overline{\text{MS}}$ scheme is measured for the first time from the same HERA combined charm data:

- Extract $m_c(m_c)$ in 6 separate kinematic regions
- Translate back to $m_c(\mu)$ [with $\mu=\sqrt{Q^2+4m_c^2}$] using OpenQCDrad [S.Alekhin's code].



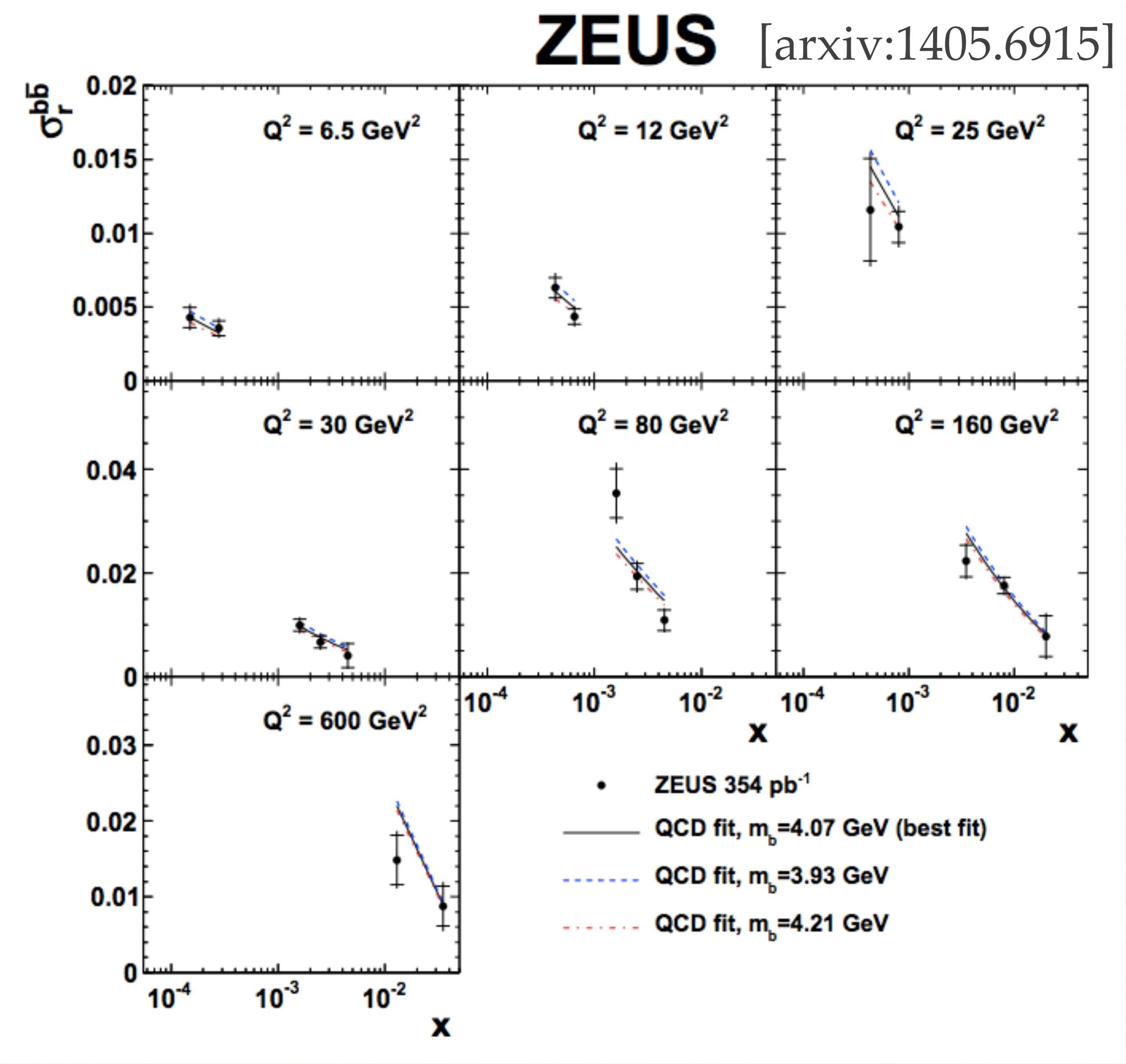




The scale dependence of the mass is consistent with QCD expectations

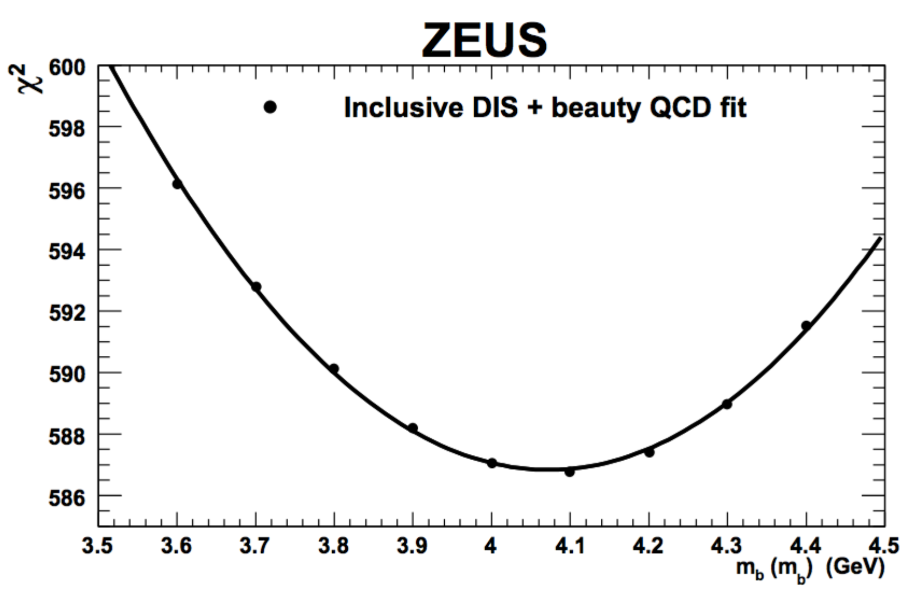
Running beauty mass from F2b

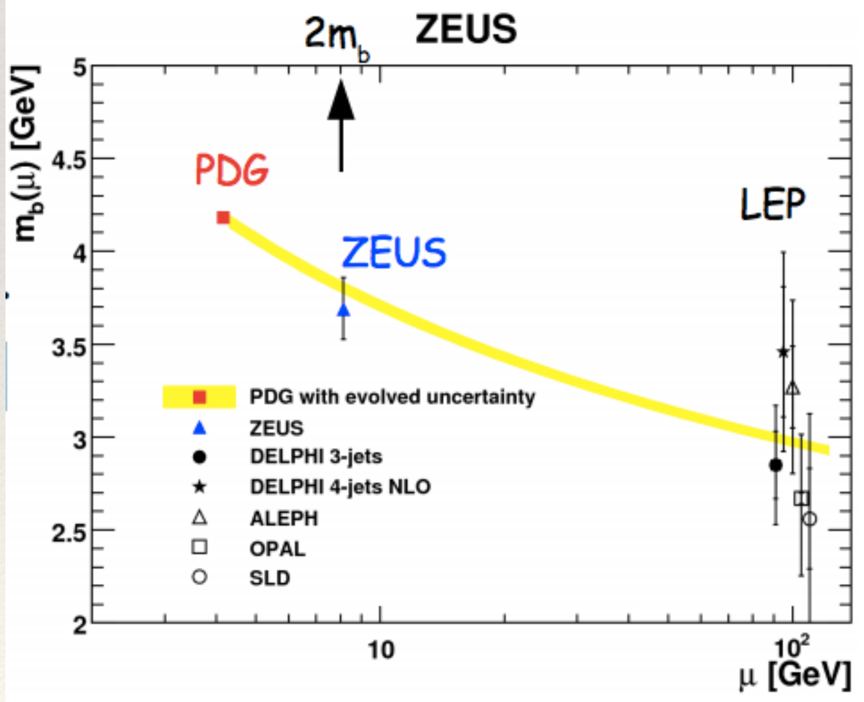
- The value of the running beauty mass is obtained using HERAFitter (via OPENQCDRAD):
  - chi2 scan method from QCD fits in FFN scheme to the combined HERA I inclusive data + beauty measurements, beauty-quark mass is defined in the $\overline{\text{MS}}$ scheme.


[arxiv:1405.6915]

QCD Fits

HERA I+beauty





The extracted $\overline{\text{MS}}$ beauty-quark mass is in agreement with PDG average and LEP results.

DIS Cross Sections

- Differential cross section is experimentally measured: theory meets the experiment
- Factorisable nature of interaction: Inclusive scattering cross section is a product of leptonic and hadronic tensors times propagator characteristic of the exchanged particle:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\sum_j \eta_j L_j^{\mu\nu} W_j^{\mu\nu}$$

For NC: j=γ, Z, γZ
For CC: j=W+, W-

$$\eta_\gamma = 1; \qquad \eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2+M_Z^2}\right); \qquad \eta_Z = \eta_{\gamma Z}^2;$$

$$\eta_W = \frac{1}{2}\left(\frac{G_F M_W^2}{4\pi\alpha}\frac{Q^2}{Q^2+M_W^2}\right)^2,$$

Leptonic tensor: related to the coupling of the lepton with the exchanged boson

- contains the electromagnetic or the weak couplings
- can be calculated exactly in the standard electroweak U(1) × SU(2) theory.

Hadronic tensor: related to the interaction of the exchanged boson with proton

- can't be calculated, but only be reduced to a sum of structure functions:

~m_{lepton}

$$W^{\alpha\beta} = -g^{\alpha\beta}W_1 + \frac{p^\alpha p^\beta}{M^2}W_2 - \frac{i\epsilon^{\alpha\beta\gamma\delta}p_\gamma q_\delta}{2M^2}W_3 + \frac{q^\alpha q^\beta}{M^2}W_4 + \frac{p^\alpha q^\beta + p^\beta q^\alpha}{M^2}W_5 + \frac{i(p^\alpha q^\beta - p^\beta q^\alpha)}{2M^2}W_6$$

$$\frac{d^2\sigma}{dxdQ^2} = A^i\left\{(1 - y - \frac{x^2y^2M^2}{Q^2})F_2^i + y^2xF_1^i \mp (y - \frac{y^2}{2})xF_3^i\right\}$$

A^i: process dependent