Measurement of perturbative QCD and hadronic final states at HERA



Leonid Gladilin (Moscow State University)





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Outline: QCD explorer HERA

Jets and α_s

Diffractive dijets

Prompt photons in (diffractive) photoproduction

Heavy quark production

Combined HERA D** Cross Sections in DIS

Summary & Prospects

Back-up: Extraction of m_c and m_b ratio of $D^{*\pm}$ PhP x-sections at different \sqrt{s} charged particle spectra in DIS

The HERA ep collider (1992 - 2007)

ep collider:

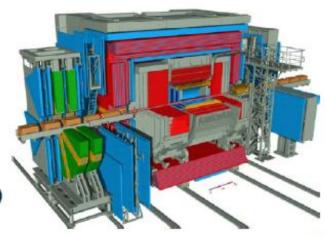
e[±] energy: 27.6 GeV

p energy: 920 GeV

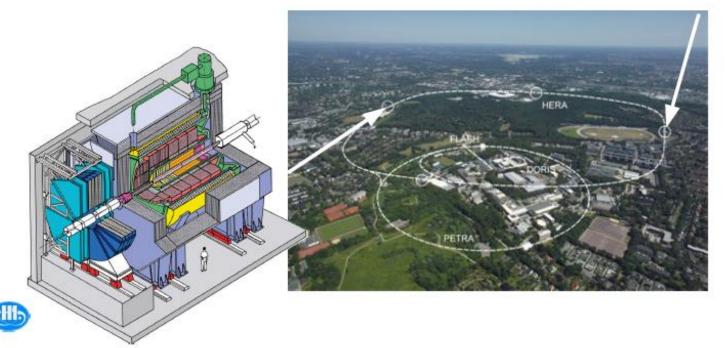
Center of mass energy: 319 GeV

2 collider experiments: H1 and ZEUS

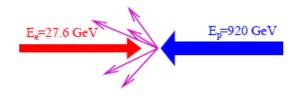
• Integrated luminosity: ~0.5 fb⁻¹ (per experiment)

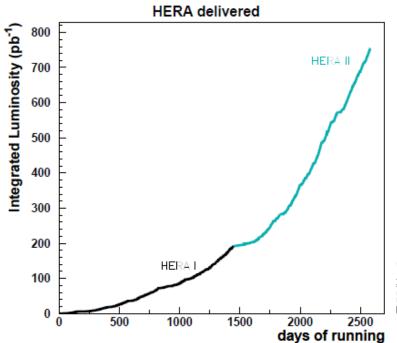






QCD explorer HERA





 $\sigma_{c\bar{c}} \approx 1 \,\mu b \Longrightarrow 10^9 \text{ events } (\mathcal{L}_{int} = 1 \,fb^{-1})$ $\sigma_{b\bar{b}} \approx 10 \text{ nb} \Longrightarrow 10^7 \text{ events } (\mathcal{L}_{int} = 1 \text{ fb}^{-1})$ HER.A

HERA II

1992-2000

2003-2007

320 (300)

 $320\,\mathrm{GeV}$

 $1.5 \cdot 10^{31}$

 $7 \cdot 10^{31} \, cm^{-2} \, s^{-1}$

0.13

 $0.37 \ fb^{-1}$

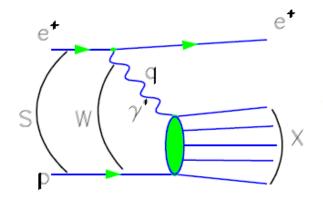
$$e(k) + p(P) \rightarrow e(k') + X$$
 $s = (P + k)^2$

$$s = (P+k)^2$$

$$Q^2 = -q^2 = -(k - k')^2$$

Photoproduction DIS

$$Q^2 \simeq 0 \, GeV^2 \ Q^2 > 1 \, GeV^2$$



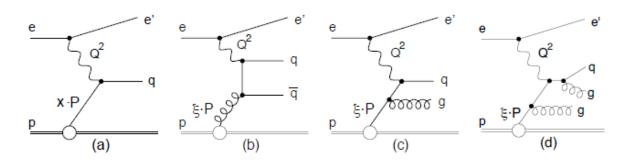
$$W^2 = (P+q)^2$$

$$y = \frac{qP}{kP} \cong \frac{W^2 + Q^2}{s}$$
 $x = \frac{Q^2}{2qP} \cong \frac{Q^2}{sy}$

$$x = \frac{Q^2}{2qP} \cong \frac{Q^2}{sy}$$

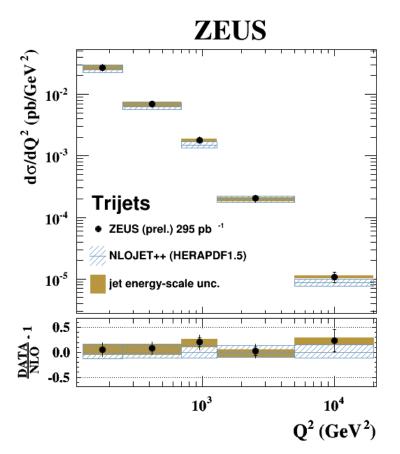
Trijet Production in DIS

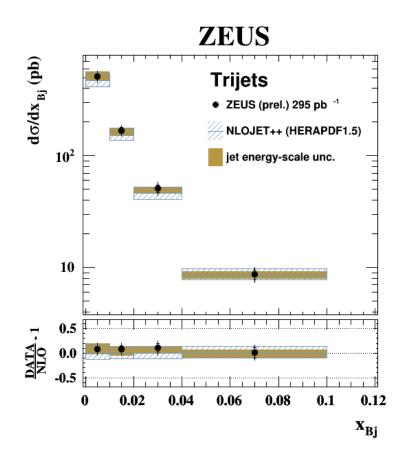
ZEUS-prel-14-008



125 < Q² < 20 000 GeV² 0.2 < y < 0.6

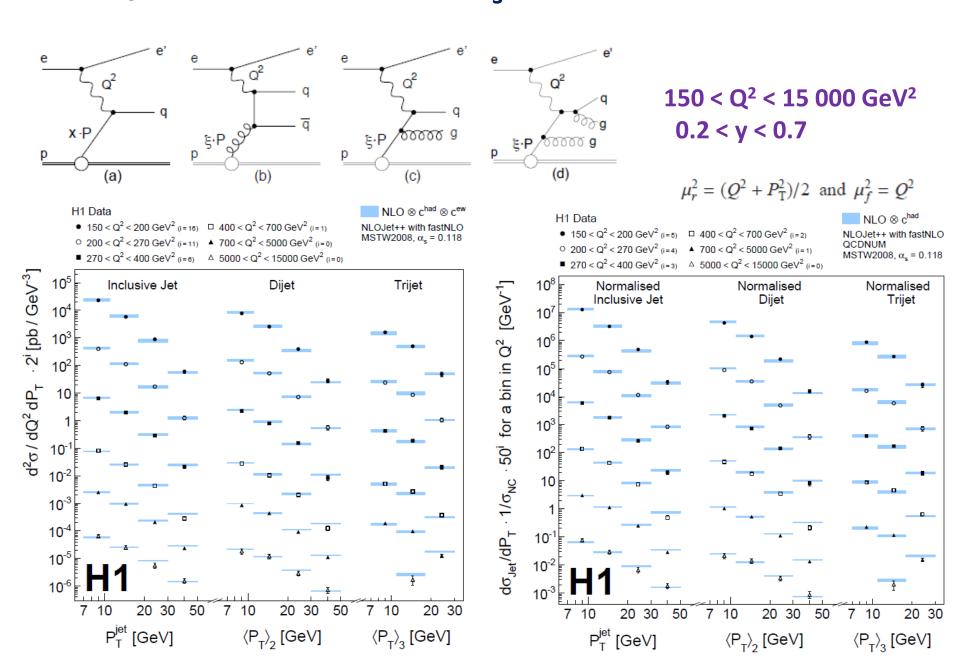
$$\mu_r^2 = (Q^2 + P_{\mathrm{T}}^2)/2$$
 and $\mu_f^2 = Q^2$



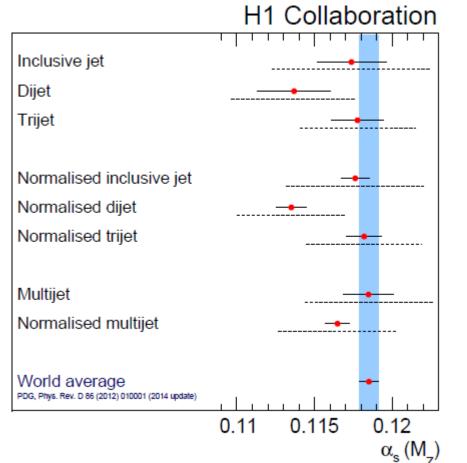


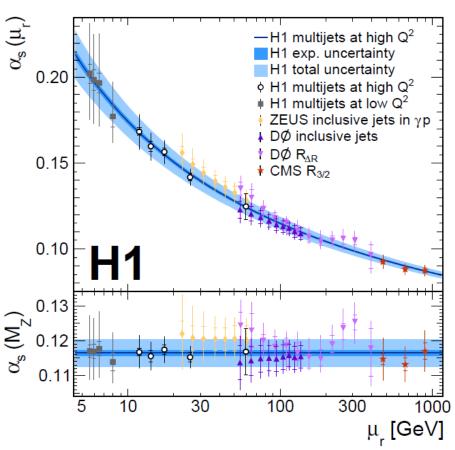
Multijet Production and α_s extraction

Eur. Phys. J. C75 (2015) 65 arXiv:1406.4709



Multijet Production and α_s extraction





 $\alpha_s = 0.1165 \pm 0.0008 exp \pm 0.0038 theo$

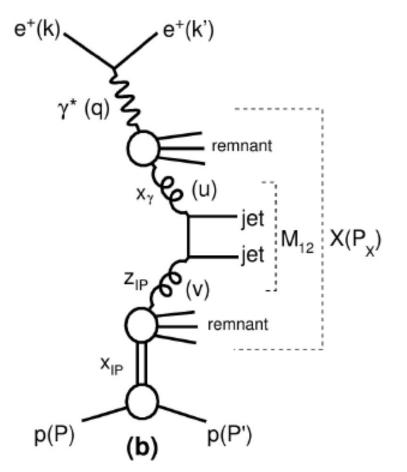
For $Q^2 > 400 \text{ GeV}^2$:

 $\alpha_s(M_Z)|_{k_T} = 0.1165 \ (8)_{exp} \ (5)_{PDF} \ (7)_{PDFset} \ (3)_{PDF(\alpha_s)} \ (8)_{had} \ (36)_{\mu_r} \ (5)_{\mu_f}$ $= 0.1165 \ (8)_{exp} \ (38)_{pdf,theo} \ .$

 $\alpha_s(M_Z)|_{k_T} = 0.1160 \ (11)_{\text{exp}} \ (32)_{\text{pdf,theo}}$

Diffractive Dijets in DIS and Photoproduction

JHEP 05 (2015) 056 arXiv:1502.01683



Measure scattered proton in VFPS (Very Forward Spectrometer)

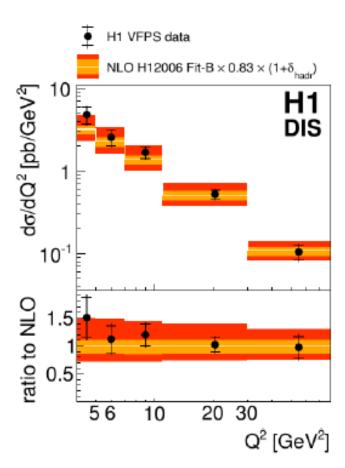
- VFPS is 220m from interatction point
- Complementary method to LRG method

PHP	DIS						
$Q^2 < 2 \mathrm{GeV^2}$	$4 \mathrm{GeV^2} < Q^2 < 80 \mathrm{GeV^2}$						
Common Cuts							
0.2 < y < 0.7							
$E_T^{*jet1} > 5.5 \text{GeV}$	$E_T^{*\text{jet2}} > 4.0\text{GeV}$						
$-1 < \eta^{ m jet1} < 2.5$	$-1 < \eta^{ m jet2} < 2.5$						
$ t < 0.6 {\rm GeV^2}$	$0.010 < x_{I\!\!P} < 0.024$						
$z_{I\!\!P} < 0.8$							

Diffractive Dijets in DIS and Photoproduction

Dijets in DIS

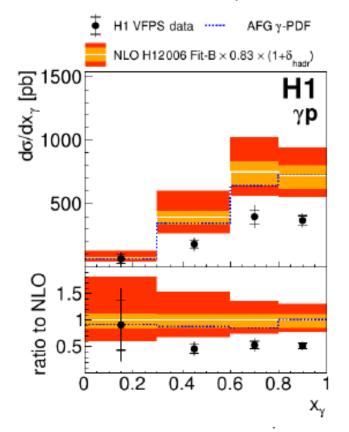
- · Single diff. cross sections
- · NLO by nlojet++
- H12006 Fit-B



Shape and normalisation well described by NLO

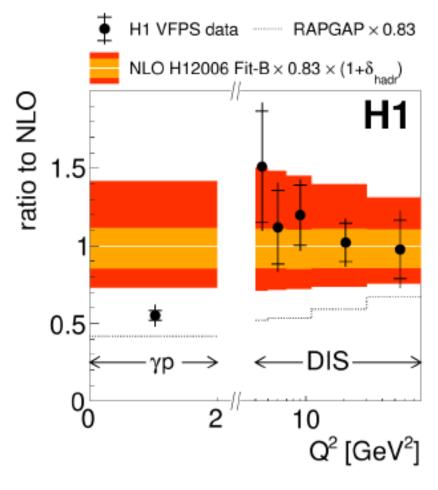
Dijets in yp

- · Single diff. cross sections
- · NLO by FKS (Frixione et al.)
- H12006 Fit-B
- GRV and AFG y-PDF



Shape well described by NLO, but normalisation is overestimated

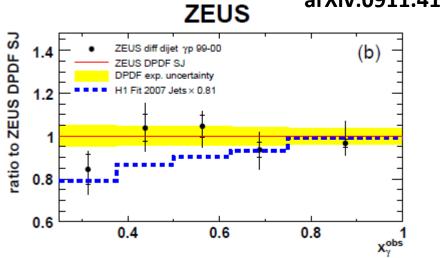
Diffractive Dijets in DIS and Photoproduction



New analysis confirms previous results from H1 with complementary experimental method

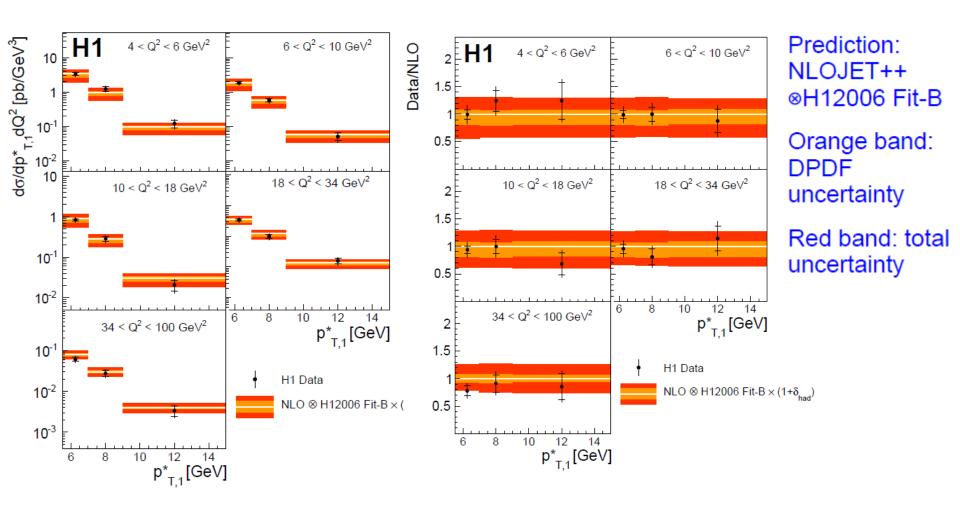
However, there is a disagreement with the ZEUS conclusion: NPB 831 (2010) 1

arXiv:0911.4119



Diffractive Dijets in DIS (LRG)

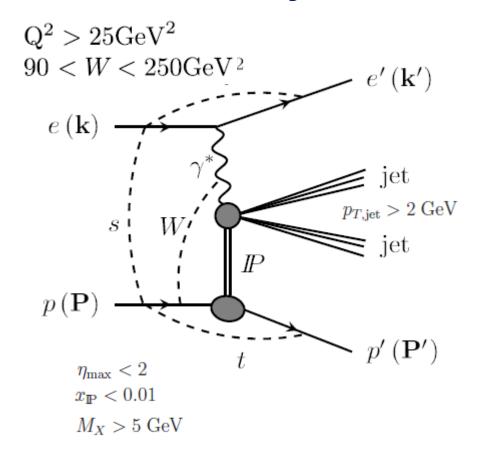
JHEP 03 (2015) 092 arXiv:1412.0928

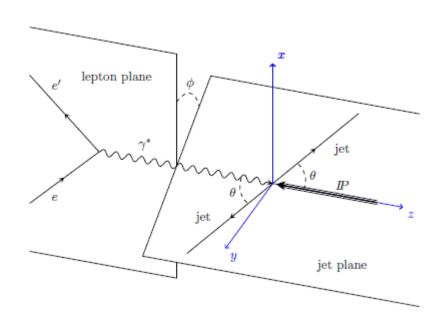


Dijets in Diffractive DIS are well descried by NLO with DPDF

Exclusive Dijets in Diffractive DIS

arXiv: 1505.05783





 ϕ - the angle between lepton plane and jet plane

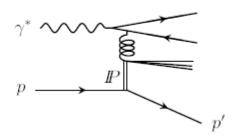
Only dijet, scattered electron and proton in the final state

 ϕ distribution $\propto 1 + A \cdot \cos 2\phi$

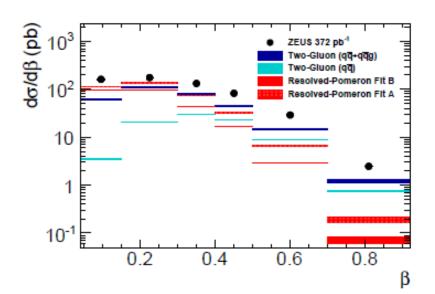
Parameter A sensitive to the nature of the object exchanged between the virtual photon and the proton

Exclusive Dijets in Diffractive DIS

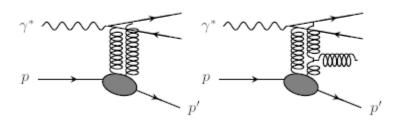
Resolved-Pomeron model



Prediction based on diffractive gluon density obtained from fits (H1 2006 fits A and B) to H1 data



Two-Gluon-Exchange model

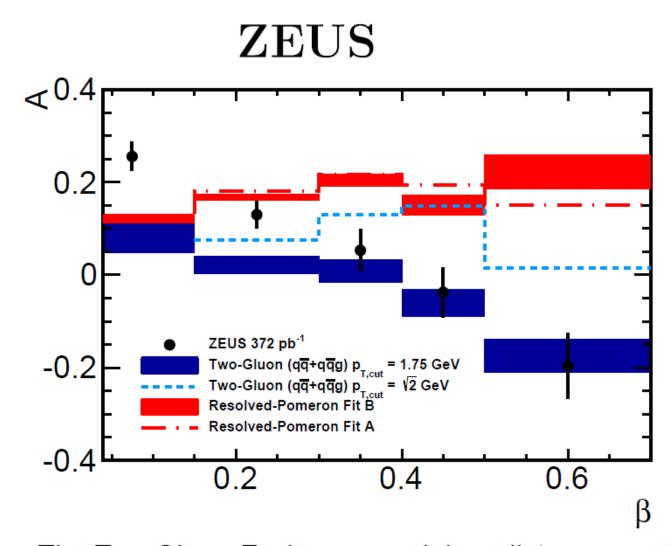


Prediction based on GRV parameterisation of the gluon density

The $q\bar{q}g$ final state is sensitive to the parton-level cut $p_{T,cut}$

 $\beta = x/x_{IP}$ - fraction of Pomeron momentum "seen" by the photon

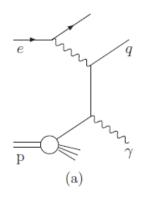
Exclusive Dijets in Diffractive DIS



The Two-Gluon-Exchange model predicts reasonably well the measured value of ${\it A}$ as a function of ${\it \beta}$

Prompt photons in photoproduction

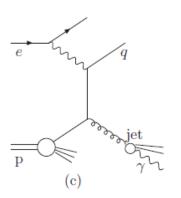
direct

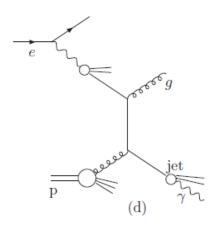


 $\frac{1}{e}$ $\frac{1}{p}$ $\frac{1}{p}$

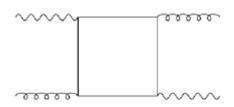
resolved

direct, fragmentation





resolved, fragmentation



box diagram

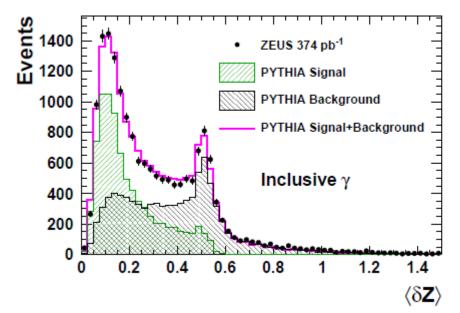
Extraction of Photon Signals

PLB 730 (2014) 293

arXiv:1312.1539

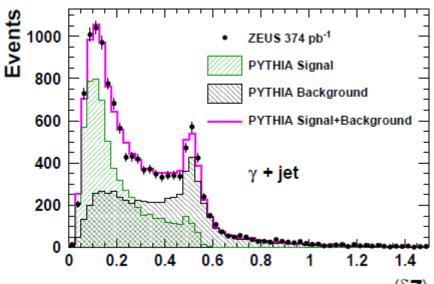
JHEP 08 (2014) 023

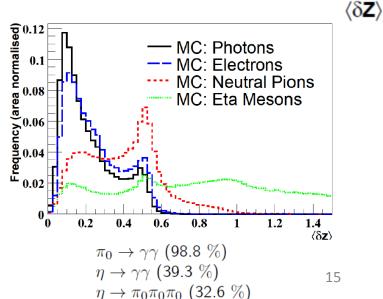
arXiv:1405.7127



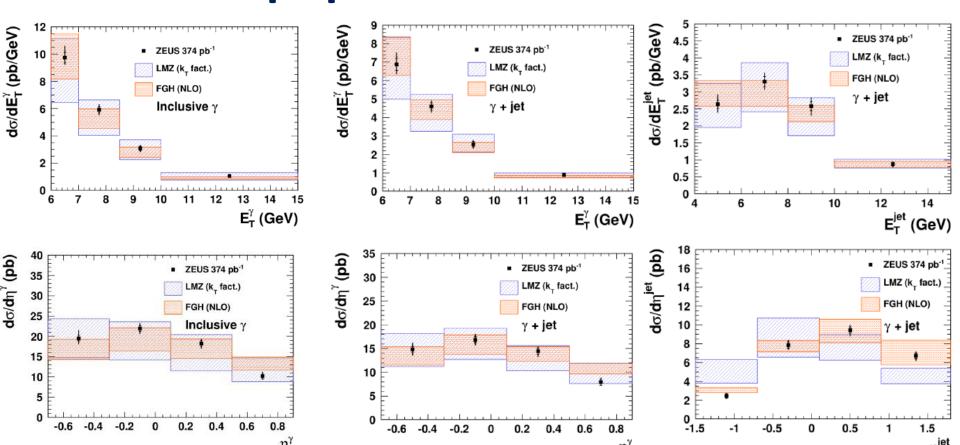
$$<\delta Z> = \frac{\sum_{i} E_{i} |Z_{i} - Z_{cluster}|}{w_{cell} \sum_{i} E_{i}}$$

 E_T -weighted mean of $|Z_{CELL} - Z_{Mean}|$





Prompt photon Cross Sections



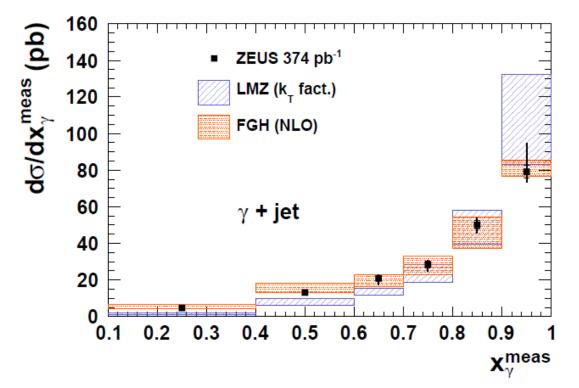
fixed order calculations

- by M. Fontannaz, J.Ph. Guillet and G. Heinrich Eur. Phys. J. C 21 (2001) 303,
 Eur. Phys. J. C 34 (2004) 191 (FGH)
- components:

k_T -factorisation approach

- calculated by A.V.Lipatov, M.A. Malyshev, N.P.Zotov, Phys. Rev. D 72 (2005) 054002, Phys. Rev. D 81 (2010) 094027, Phys. Rev. D 88 (2013) 074001 (LMZ):
- investigation of the photoproduction of the isolated photon at HERA in the framework of kt-factorisation QCD approach
- both direct and resolved processes are considered
- the box contribution was included
- fragmentation contribution is neglected

Direct/Resolved Contributions to γ + jet



Direct LO process final state:

- jet
- photon
- scattered electron (escape undetected)
- proton remnant (escape undetected)

$$\Rightarrow x_{\gamma}^{\text{meas}} = 1 \text{ (LO direct)}$$

Resolved LO process final state:

- all mentioned above
- + resolved photon remnant

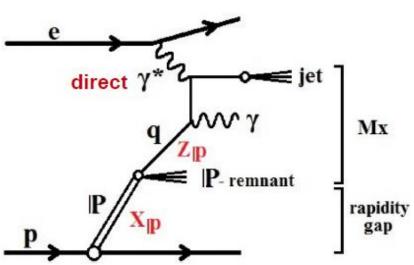
$$\Rightarrow \ \mathrm{x}_{\gamma}^{\mathrm{meas}} < 1$$

$$x_{\gamma}^{\text{meas}} = \frac{E^{\gamma} - p_Z^{\gamma} + E^{\text{jet}} - p_Z^{\text{jet}}}{E^{\text{all}} - p_Z^{\text{all}}}$$

very good description of the $x_{\gamma}^{\rm meas}$ by FGH reasonable description by LMZ (typically theory within 1-2 sigma from data)

LMZ is somewhat too "direct"

Prompt photons in diffractive photoproduction

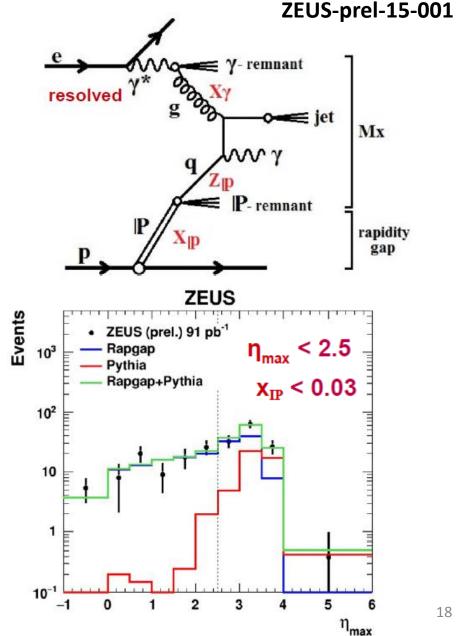


$$x_{IP} = \Sigma(E + p_z)_{all EFOs} / 2 E_p$$

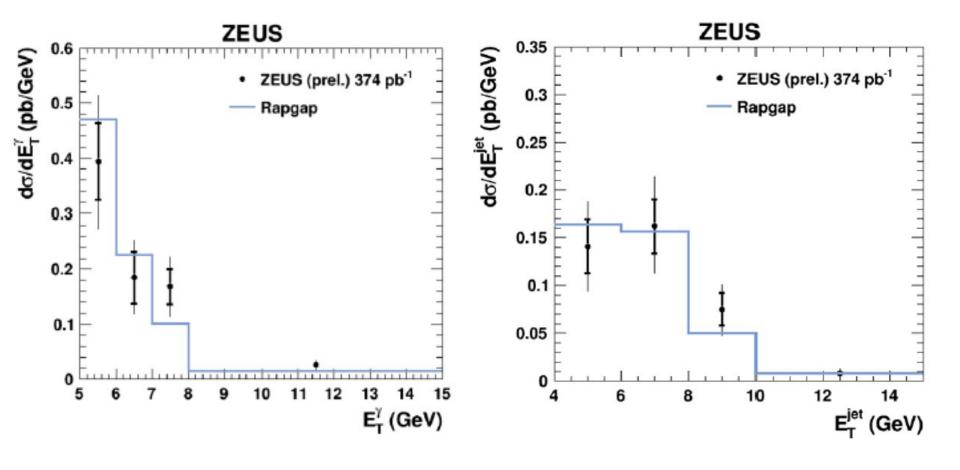
x_{IP} = fraction of proton energy taken by pomeron.

Z_{IP} = fraction of pomeron energy taken in scatter.

η_{max} = maximum value of
pseudorapidity of outgoing
particles in scatter
(Ignore forward proton.)

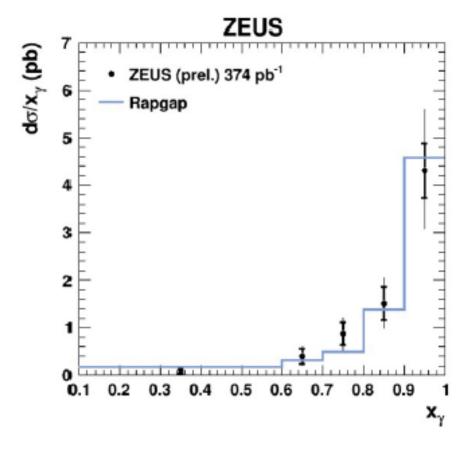


Prompt photons in diffractive photoproduction

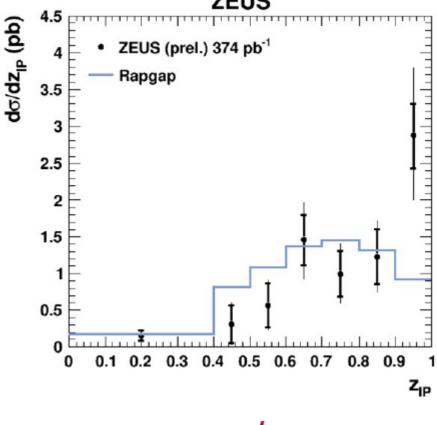


RAPGAP describes the shapes of most of the kinematic variables reasonably well.

Prompt photons in diffractive photoproduction



$$x_{y} = \Sigma_{y+jet}(E - p_{z}) / \Sigma_{all EFOs}(E - p_{z})$$

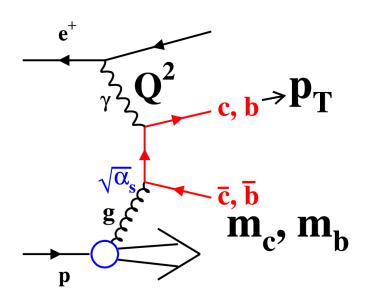


$$z_{IP} = \Sigma_{\gamma + jet}(E + p_z) / \Sigma_{all EFOs}(E + p_z)$$

dominated by the direct photoproduction

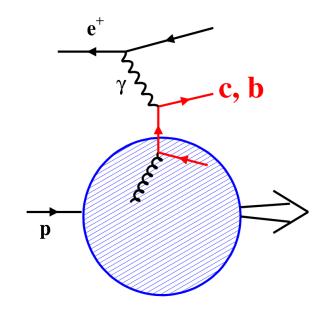
z_{IP} shows a peak at high values

Heavy Quark Production





Expected to be valid at scales ~ m_{b,c}
Programs exist to calculate fully differential cross sections (HVQDIS, FMNR)



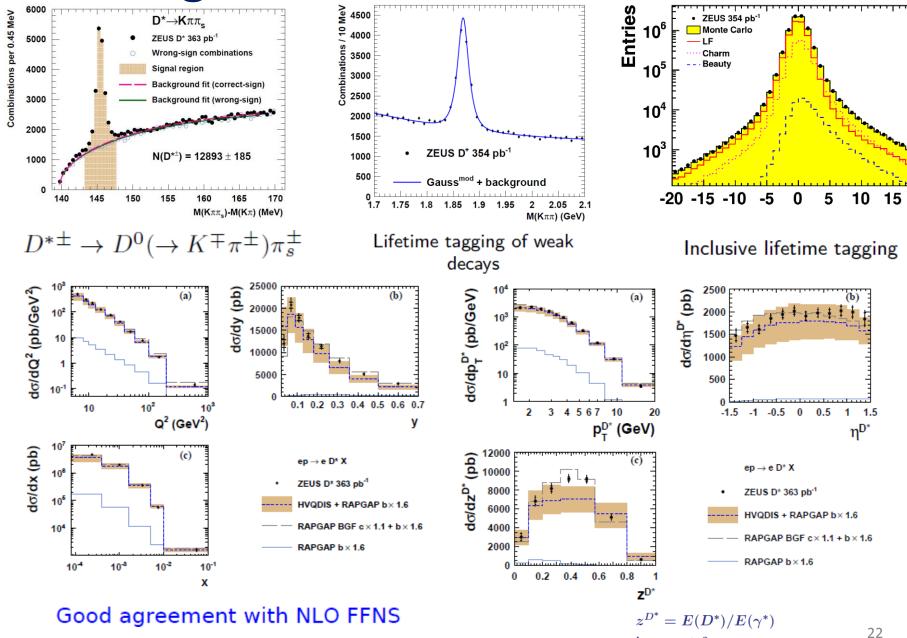
Massless scheme (ZM-VFNS)

Allows resummation of terms proportional to log(Q²/m_{b,c}²) Expected to be valid at scales >>m_{b,c}

Mixed schemes (GM-VFNS)

Employ both FFNS and ZM-VFNS Interpolation is ambiguous → various approaches (RT, ACOT etc.) exist

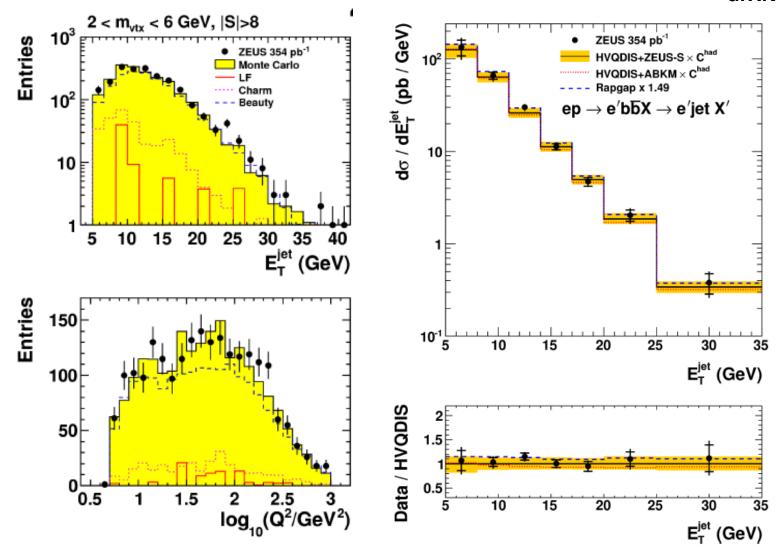
HQ signals and Cross Sections in DIS



in p rest frame

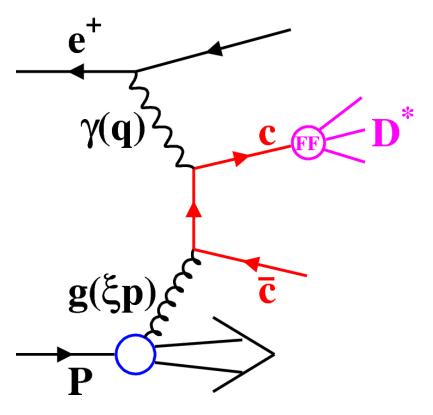
Beauty signals and x-sections in DIS

JHEP 09 (2014) 127 arXiv:1405.6915



H1+ZEUS D*[±] Cross Sections in DIS

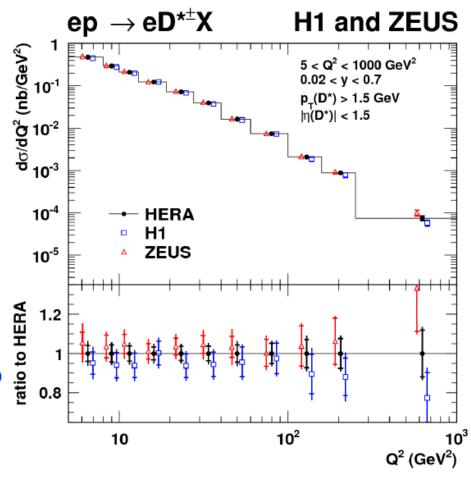
arXive: 1505.05783



- → Consistent data
- → Combined data reach precision ~5%

Combine D* visible cross sections

EPJ C71 (2011) 1769 H1 medium Q² PL B686 (2010) 91 H1 high Q² JHEP 05 (2013) 097 ZEUS all Q²



NLO QCD predictions: HVQDIS

Massive scheme \rightarrow only light flavours in pdf: u,d,s,g; NLO = o(α_s^2)

HVQDIS setup for ep \rightarrow cc X \rightarrow D*X (uncertainties):

- $\mu_r = \mu_f = \sqrt{Q^2 + 4m_c^2}$ vary independently by factor 0.5 and 2
- $m_c^{pole} = 1.50 \pm 0.15 \text{ GeV}$
- $\alpha_s^{nf=3}(m_Z) = 0.105 \pm 0.002$ (corresponds to $\alpha_s^{nf=5}(m_Z) = 0.116 \pm 0.002$)
- HERAPDF1.0 FFNS
- Fragmentation:
 - Longitudinal: Karvelishvili FF with α_K(D*)

\hat{s} range	$\alpha_K(D^*)$
$\hat{s} \leq \hat{s}_1$	6.1 ± 0.9
$\hat{s}_1 < \hat{s} \le \hat{s}_2$	3.3 ± 0.4
$\hat{s} > \hat{s}_2$	2.67 ± 0.31

$$\hat{s}_1 = 70 \pm 40 \text{ GeV}^2$$

 $\hat{s}_2 = 324 \text{ GeV}^2$

- Transverse: $f(k_T) = k_T \exp(-\frac{2k_T}{\langle k_T \rangle})$; $\langle k_T \rangle = 0.35 \pm 0.15 \text{ GeV}$
- $f(c \rightarrow D^*) = 0.2287 \pm 0.0056$

Use HVQDIS also to predict small additional component: ep \rightarrow bb X \rightarrow D* X

Customised NLO QCD predictions: HVQDIS

Try to find parameters such that calculation describes normalisation & shapes of all differential cross sections presented in the following

•
$$\mu_r = \sqrt{Q^2 + 4m_c^2} \quad \Longrightarrow \quad$$

$$0.5\sqrt{Q^2+4m_c^2}$$

→ Increase cross section

•
$$m_c^{pole} = 1.50 \text{ GeV} \longrightarrow$$

1.40 GeV

→ Increase cross section

- · Fragmentation:
 - Longitudinal: Karvelishvili FF with α_K(D*)

\hat{s} range	$\alpha_K(D^*)$
$\hat{s} \leq \hat{s}_1$	6.1 ± 0.9
$\hat{s}_1 < \hat{s} \le \hat{s}_2$	3.3 ± 0.4
$\hat{s} > \hat{s}_2$	2.67 ± 0.31

•
$$\hat{s}_1 = 70 \text{ GeV}^2$$



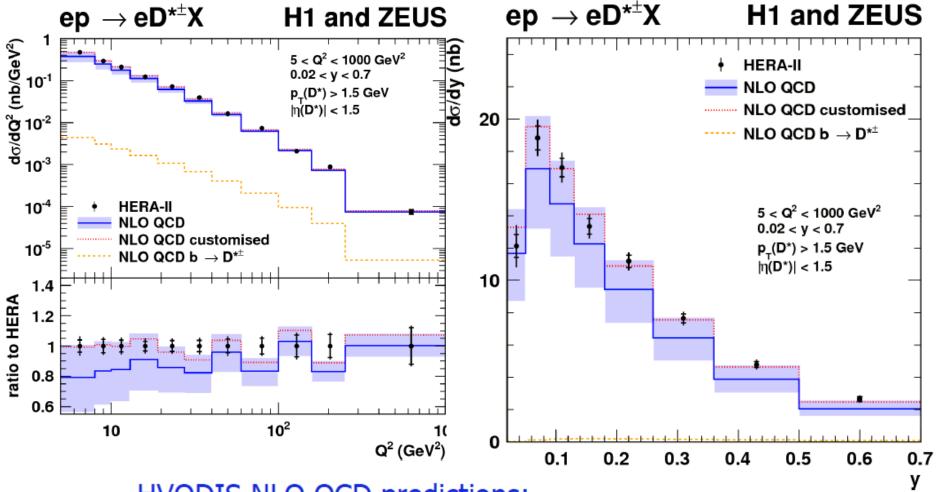
 30 GeV^2

→ Soften fragmentation

Leave all other parameters at their default values

This is no prediction → but may give hints in which direction to develop theory

H1+ZEUS D** Cross Sections in DIS

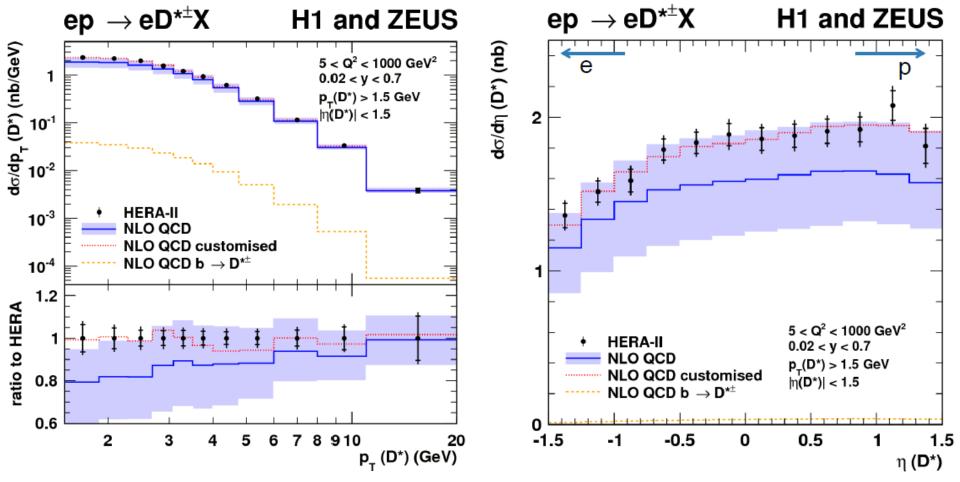


HVQDIS NLO QCD predictions:

- Describe data reasonably
- Large uncertainties 10-30% → need NNLO and improved fragmentation models

H1+ZEUS D** Cross Sections in DIS

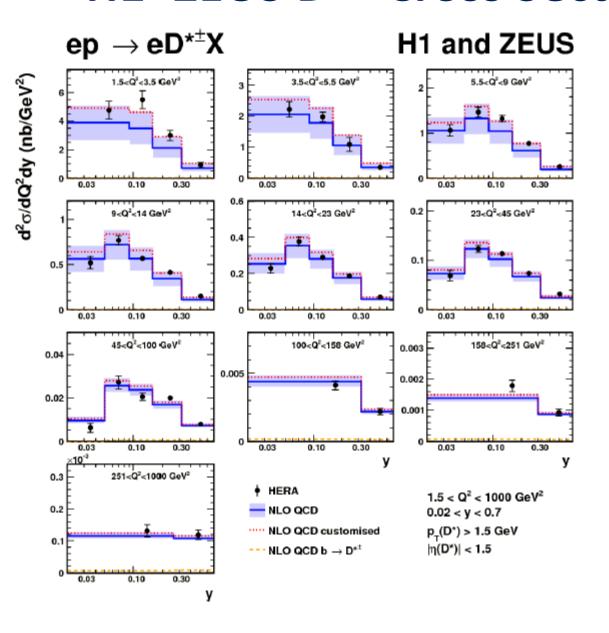
DESY 15-037 March 2015



HVQDIS NLO QCD predictions:

- Describe data reasonably
- Large uncertainties 10-30% → need NNLO and improved fragmentation models

H1+ZEUS D** Cross Sections in DIS



Input data: as before

EPJ C71 (2011) 1769 H1 PL B686 (2010) 91 H1 JHEP 05 (2013) 097 ZEUS

plus ZEUS HERA I data

PR D69 (2004) 012004 ZEUS

Summary & Prospects

No new data but a lot of new results



Extensive results on jet production Precise α_s measurements



New results on diffractive dijets in DIS and photoproduction Conformation of diffraction suppression in photoproduction



New precise measurements of prompt photon production Verification of collinear and k, predictions



Extensive results on charm and beauty production Precise combined D** Cross Sections in DIS



More final and combined H1+ZEUS results

Back-up Slides

HERAPDF 1.0, 1.5, 2.0

HERAPDF1.0

HERAPDF1.5

HERAPDF2.0 (prel.)

H1prelim-14-041(2) ZEUS-prel-14-005(7)

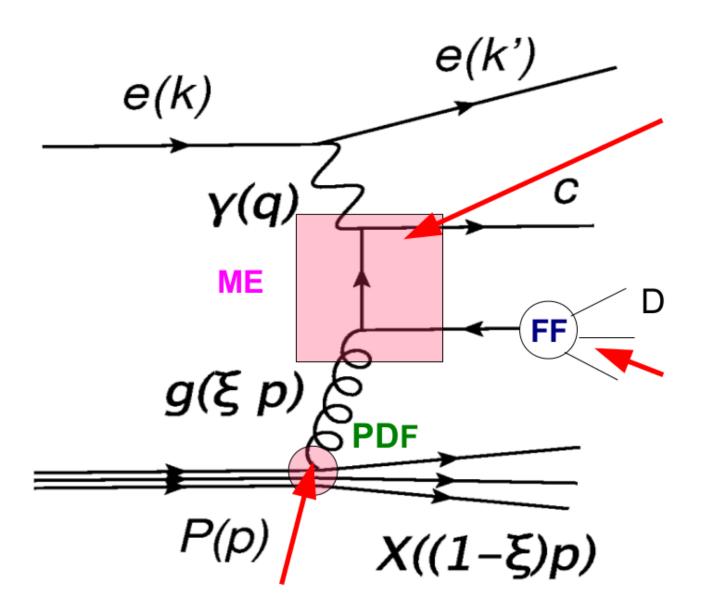
	Data Set		x G	rid	Q^2/Ge	V ² Grid	L	e^{+}/e^{-}	\sqrt{s}
			from	to	from	to	pb ⁻¹		GeV
. [HERA I $E_p = 820$ GeV and $E_p = 920$ GeV data sets								
	H1 svx-mb	95-00	0.000005	0.02	0.2	12	2.1	e^+p	301, 319
П	H1 low Q^2	96-00	0.0002	0.1	12	150	22	e^+p	301, 319
П	H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p	301
П	H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p	301
П	H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p	319
П	H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p	319
П	H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p	319
П	H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p	319
П	H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p	319
	ZEUS BPC	95	0.000002	0.00006	0.11	0.65	1.65	e^+p	300
П	ZEUS BPT	97	0.0000006	0.001	0.045	0.65	3.9	e^+p	300
П	ZEUS SVX	95	0.000012	0.0019	0.6	17	0.2	e^+p	300
П	ZEUS NC	96-97	0.00006	0.65	2.7	30000	30.0	e^+p	300
	ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p	300
	ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p	318
	ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p	318
	ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p	318
П	ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p	318

	HERA II $E_p = 920 \text{ GeV}$ data sets								
	H1 NC	03-07	0.0008	0.65	60	30000	182	e^+p	319
	H1 CC	03-07	0.008	0.40	300	15000	182	e^+p	319
	H1 NC	03-07	0.0008	0.65	60	50000	151.7	e^-p	319
	H1 CC	03-07	0.008	0.40	300	30000	151.7	e^-p	319
	H1 NC med Q^2	03-07	0.0000986	0.005	8.5	90	97.6	e^+p	319
	H1 NC low Q^2	03-07	0.000029	0.00032	2.5	12	5.9	e^+p	319
ı	ZEUS NC	06-07	0.005	0.65	200	30000	135.5	e^+p	318
	ZEUS CC	06-07	0.0078	0.42	280	30000	132	e^+p	318
	ZEUS NC	05-06	0.005	0.65	200	30000	169.9	e^-p	318
	ZEUS CC	04-06	0.015	0.65	280	30000	175	e^-p	318
	ZEUS NC nominal	06-07	0.000092	0.008343	7	110	44.5	e^+p	318
	ZEUS NC satellite	06-07	0.000071	0.008343	5	110	44.5	e^+p	318
	HERA II $E_p = 575 \text{Ge}$	V data sets							
	H1 NC high Q ²	07	0.00065	0.65	35	800	5.4	e^+p	252
	H1 NC low Q^2	07	0.0000279	0.0148	1.5	90	5.9	e^+p	252
	ZEUS NC nominal	07	0.000147	0.013349	7	110	7.1	e^+p	251
	ZEUS NC satellite	07	0.000125	0.013349	5	110	7.1	e^+p	251
	HERA II $E_p = 460 \text{Ge}$	V data sets							
	H1 NC high Q2	07	0.00081	0.65	35	800	11.8	e^+p	225
	H1 NC low Q^2	07	0.0000348	0.0148	1.5	90	12.2	e^+p	225
	ZEUS NC nominal	07	0.000184	0.016686	7	110	13.9	e^+p	225
	ZEUS NC satellite	07	0.000143	0.016686	5	110	13.9	e^+p	225



All results are final and published!

Heavy Quark Production



Secondary vertex method

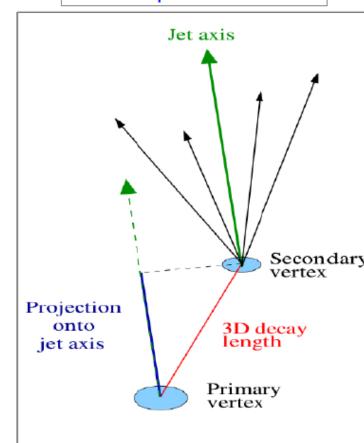
Phase space of the measurement:

Tag: jet + secondary vertex

- Employs long lifetime of ground state hadrons containing charm or beauty quarks

- Select tracks belonging to a jet
 - > p_⊤(track)>500 MeV
- Fit a secondary vertex
- Project decay length onto a jet axis
- Calculate decay length significance

 $5 < Q^{2} < 1000 \text{ GeV}^{2}$ 0.02 < y < 0.7 $E_{T}^{\text{jet}} > 5(4.2) \text{ GeV}$ $-1.6 < \eta^{\text{jet}} < 2.2$



Quark mass definitions

Pole quark mass

- Based on (unphysical) concept of quark being a free parton
- Pole mass is ambiguous up to corrections of $O(\lambda_{QCD})$

Running quark mass (\bar{MS})

- \overline{MS} (minimal substraction scheme) mass definition m(μ_R) realizes running mass (scale dependence)
- renormalization group equation (mass anomalous dimenstion γ)

$$\left(\mu_R^2 \frac{\delta}{\delta \mu_R^2} + \beta(\alpha_s) \frac{\delta}{\delta \alpha_s}\right) m(\mu_R) = \gamma(\alpha_s) m(\mu_R)$$

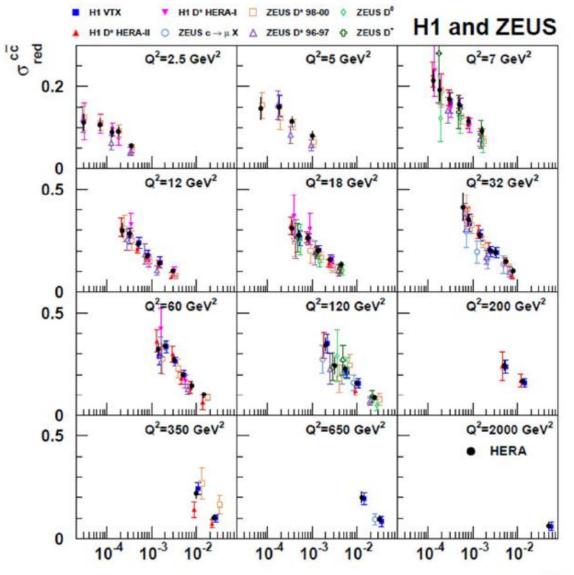
Charm DIS Data Samples

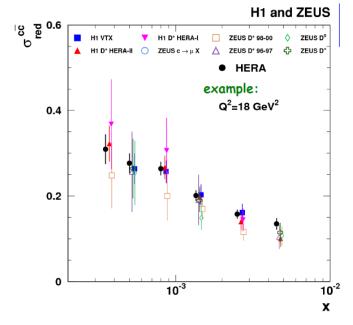
Data set		Tagging method	Q^2 range		N	\mathcal{L}	
			[GeV ²]			$ [pb^{-1}] $	
1	H1 VTX [14]	Inclusive track lifetime	5	_	2000	29	245
2	H1 D^* HERA-I [10]	D^{*+}	2	_	100	17	47
3	H1 <i>D</i> * HERA-II [18]	D^{*+}	5	_	100	25	348
4	H1 <i>D</i> * HERA-II [15]	D^{*+}	100	_	1000	6	351
5	ZEUS D* (96-97) [4]	D^{*+}	1	_	200	21	37
6	ZEUS D* (98-00) [6]	D^{*+}	1.5	_	1000	31	82
7	ZEUS D^0 [12]	$D^{0,\mathrm{no}D^{st+}}$	5	_	1000	9	134
8	ZEUS D^{+} [12]	D^+	5	_	1000	9	134
9	ZEUS μ [13]	μ	20	_	10000	8	126

- two independent experiments
- a large variety of tagging techniques: inclusive methods using the large lifetime of charmed hadrons, inclusive track lifetime, complete reconstruction of charmed mesons, D*+, charm semileptonic decay, μ
- a large number of measurements, $\sum N = 155$ data points, in a common grid spanning the x Q² plane (except for [14] where scaling factors, always smaller than 18 %, have been applied to migrate the original measurements to the closest point of the common grid)
- developped a combination method taking into account properly correlated and uncorrelated uncertainties (155 data points in 52 bins)

key observable:
$$\sigma^{c\bar{c}}_{\rm red} \ = \ \frac{{\rm d}^2\sigma^{c\bar{c}}}{{\rm d}x{\rm d}Q^2} \cdot \frac{xQ^4}{2\pi\alpha^2(Q^2)\left(1+(1-y)^2\right)}$$

Combined Charm Reduced x-sections

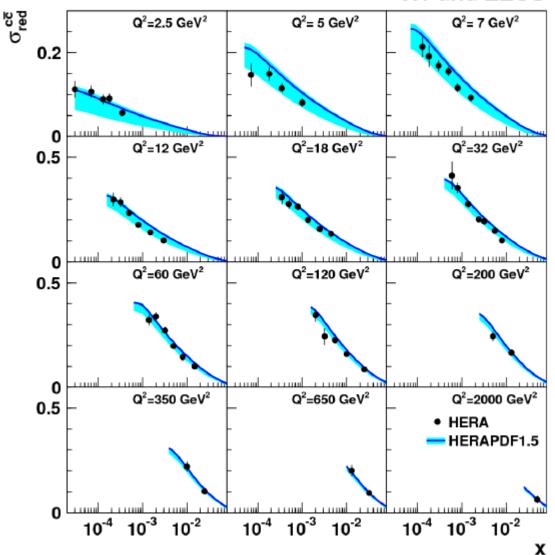




- ✓ good consistency of data among the several possible tests χ^2 / ndf = 62 / 103
- √ good complementarity of data
- ✓ 10 % uncertainty on average,
 6 % at small x and medium Q²

Sensitivity to m_c



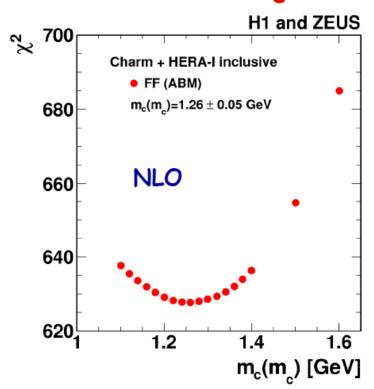


well described using HERAPDF1.5 (fitted from inclusive DIS only)

strong charm mass dependence (blue band: 1.35->1.6 GeV)

Can be used to constraint m_c

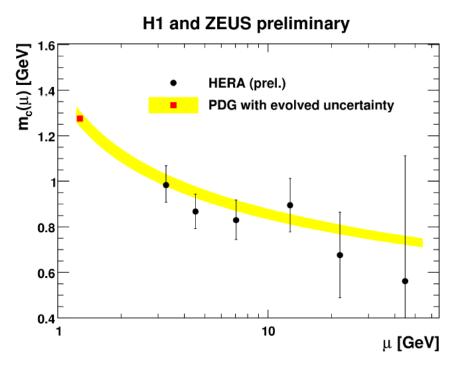
m_c value and running



$$m_c(m_c) = 1.26 \pm 0.05_{exp} \pm 0.03_{mod} \ \pm 0.02_{param} \pm 0.02_{\alpha_c} \ GeV$$

Errors are experimental, model, parametrisation and α_s

Consistent with PDG: $m_c(m_c) = 1.275 \pm 0.025 \text{ GeV}$

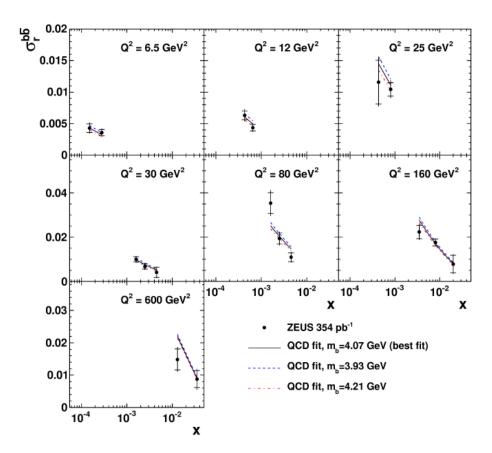


$$\mu = \sqrt{Q^2 + 4m_c^2}$$

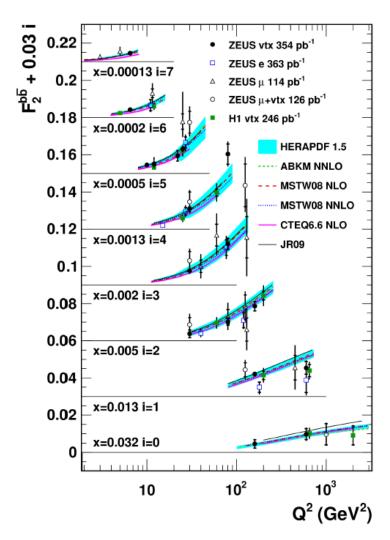
$$m_c(\mu) = m_c(m_c) \frac{\left(\frac{\alpha_s(\mu)}{\pi}\right)^{\frac{1}{\beta_0}}}{\left(\frac{\alpha_s(m_c)}{\pi}\right)^{\frac{1}{\beta_0}}}$$

$$\beta_0 \text{ for } N_f = 3 \text{ is } \frac{9}{4}$$

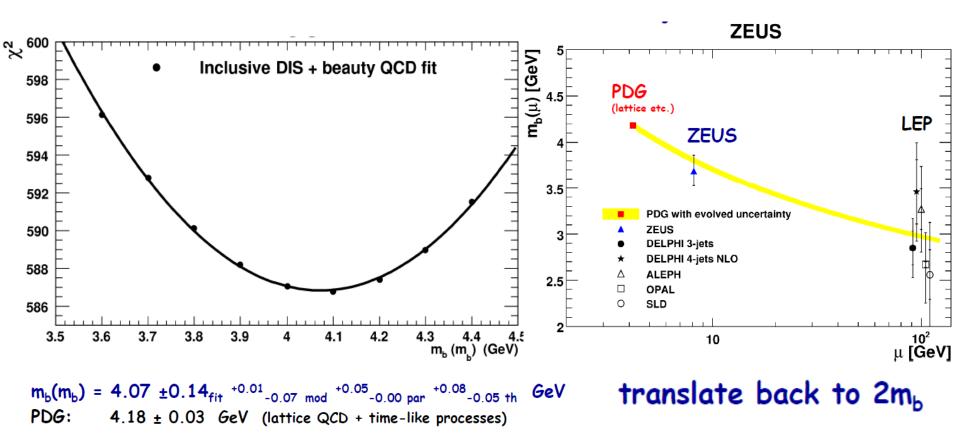
Beauty Reduced x-sections and F₂



Sensitivity to m_b comes mostly from low Q²

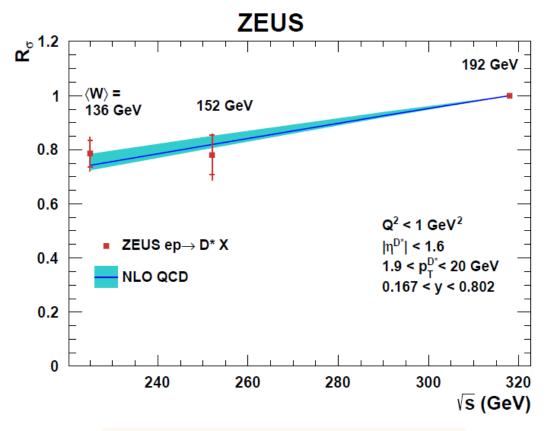


m_b value and running



mass running consistent with QCD

Ratio of D** PhP x-sections at different Vs



DESY 14 082 May 2014

Parameters for NLO QCD calculation:

Fixed-flavor-number scheme (FFNS):

Strong coupling constant : $\alpha_s(Mz) = 0.118$,

mass of c quarks: m =1.50 GeV

Fragmentation fraction f(c->D*)=0.237

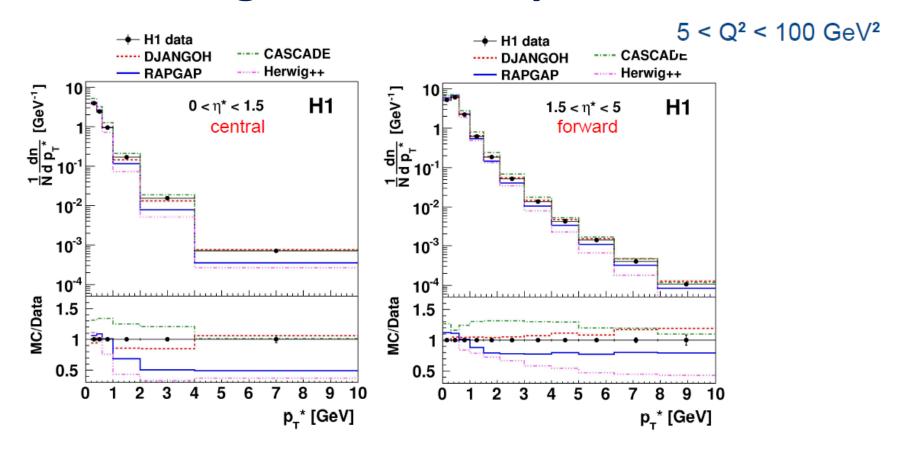
PDFs: proton - ZEUS-S FFNS

photon - GRV-G HO

Peterson fragmentation function: = 0.079

Scales were set to $\mu = \sqrt{m_c^2 + p_T^2}$

Charged Particle Spectra in DIS



DJANGOH does best, RAPGAP is also satisfactory at low p_{τ} but not at high p_{τ} .

CASCADE (based on CCFM) is the least successful model.