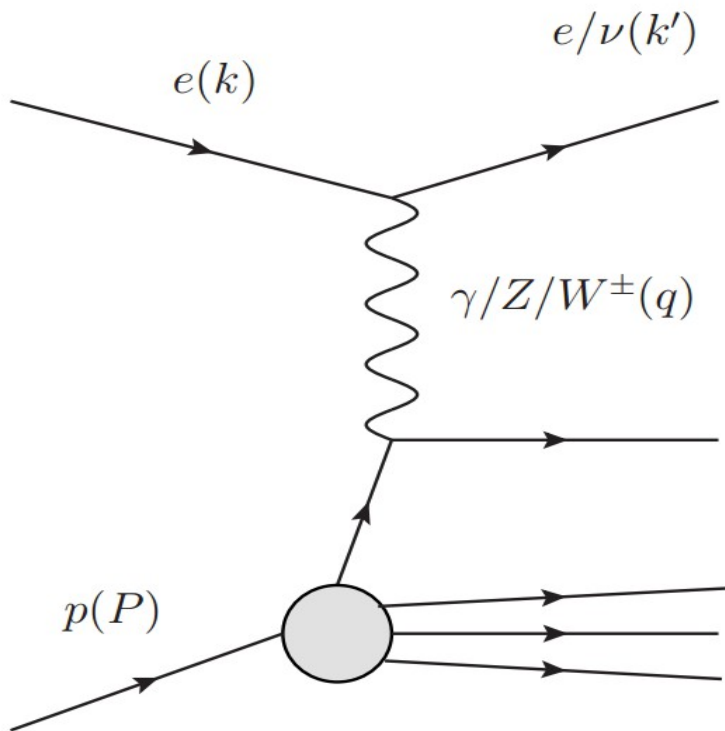


# Simultaneous QCD & Electroweak fits to HERA inclusive DIS data

K. Wichmann on behalf of H1 & ZEUS collaborations

# Deep Inelastic Scattering at HERA



- Lepton beams polarised for HERAII  
 → crucial for the EW measurements

$$X(P') \quad E_p = 920 (820, 460, 575) \text{ GeV}$$

$$E_e = 27.5 \text{ GeV}$$

$$\sqrt{s} = 318 (300, 225, 252) \text{ GeV}$$

$$Q^2 = -q^2 = -(k - k')^2$$

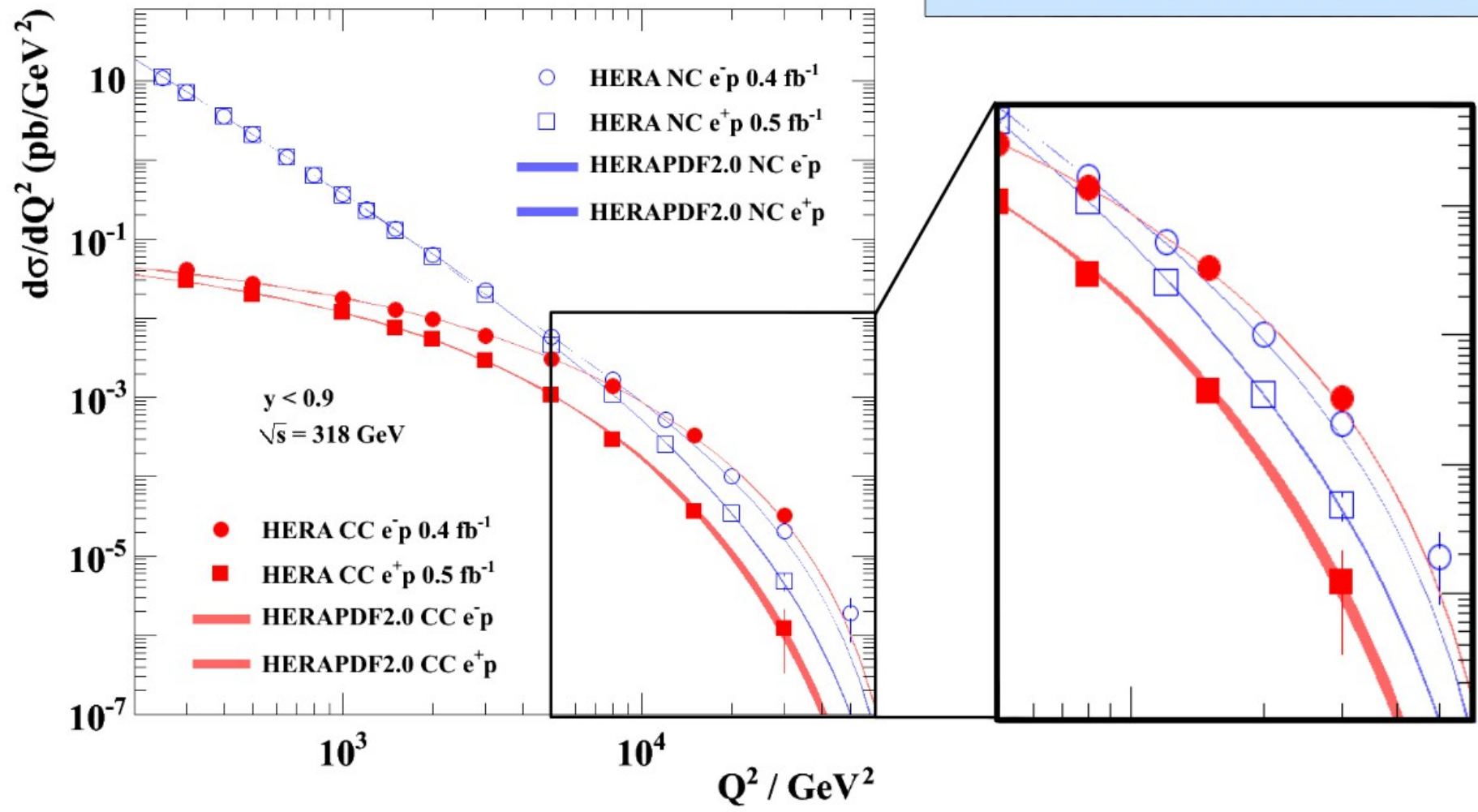
$$x_{Bj} = \frac{Q^2}{2pq} \quad y = \frac{pq}{pk}$$

$$s = (p + k)^2 \quad Q^2 = xys$$

Experimental luminosity (H1 & ZEUS):  
 ~ 0.5 fb<sup>-1</sup> data from each experiment

Fantastic precision of HERA inclusive final data

# H1 and ZEUS



- NC and CC at tree level

$$Y_{\pm} = 1 \pm (1-y)^2$$

$$\frac{d\sigma_{NC}^{\pm}}{dQ^2 dx} = \frac{2\pi\alpha^2}{x} \left[ \frac{1}{Q^2} \right]^2 (Y_+ F_2 + Y_- x F_3 + y^2 F_L)$$

$$\frac{d\sigma_{CC}^{\pm}}{dQ^2 dx} = \frac{1 \pm P}{2} \frac{G_F^2}{4\pi x} \left[ \frac{m_W^2}{m_W^2 + Q^2} \right]^2 (Y_+ W_2^{\pm} \pm Y_- x W_3^{\pm} - y^2 W_L^{\pm})$$



# Polarised DIS

- Generalised structure functions depend on e-beam polarisation

$$P_e = \frac{N_R - N_L}{N_R + N_L}$$

$$\begin{aligned} \tilde{F}_2^\pm &= F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z, \\ x\tilde{F}_3^\pm &= -(a_e \pm P_e v_e) \chi_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \chi_Z^2 x F_3^Z \end{aligned}$$

- Structure functions in QP model

**NC**

$$[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] x(q + \bar{q}),$$

$$[xF_3^{\gamma Z}, xF_3^Z] = \sum_q [e_q a_q, v_q a_q] 2x(q - \bar{q}),$$

**CC**

$$\frac{d^2\sigma_{CC}(e^+p)}{dx_{Bj}dQ^2} = (1 + P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(\bar{u} + \bar{c}) + (1 - y)^2 (d + s + b)]$$

$$\frac{d^2\sigma_{CC}(e^-p)}{dx_{Bj}dQ^2} = (1 - P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(u + c) + (1 - y)^2 (\bar{d} + \bar{s} + \bar{b})]$$

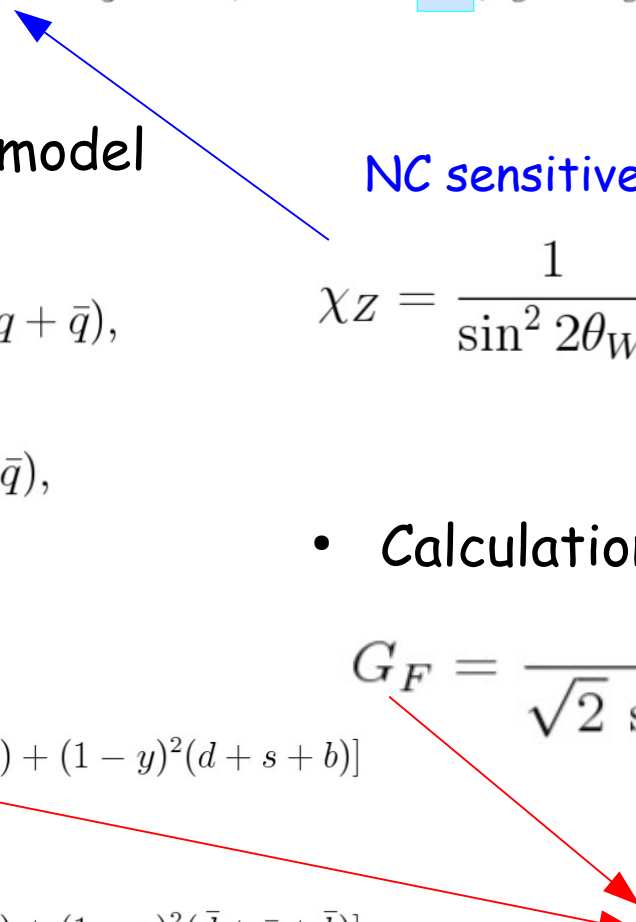
NC sensitive to  $\sin^2\theta_W$  via

$$\chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2} \frac{1}{1 - \Delta R}$$

- Calculation in on-shell scheme

$$G_F = \frac{\pi\alpha_0}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

CC sensitive to  $\sin^2\theta_W$

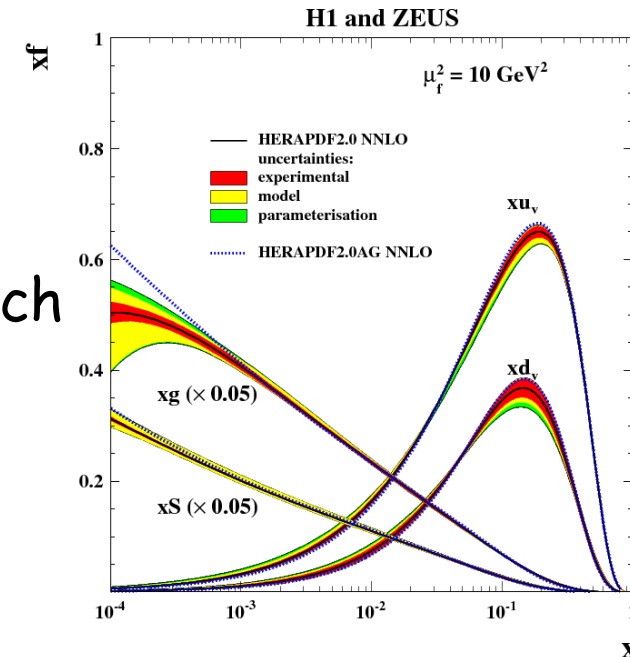


# Global QCD fits

- Data: NC & CC,  $e^+p$  and  $e^-p$  scattering
- Global PDF fits closely follow HERAPDF2.0 approach
- DGLAP evolution using QCDNUM
- 13 parameter fit (HERAPDF2.0 - DUBAR)

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

$$xg(x), xu_v(x), xd_v(x), x\bar{U}(x), x\bar{D}(x)$$



- Starting scale  $Q^2_0 = 1.9 \text{ GeV}^2$
- Model and parameterisation uncertainties  $\rightarrow$  HERAPDF2.0
- Corrections calculated using EPRC code:  $\Delta R$

[desy.de/~hspiesb/eprc.html](http://desy.de/~hspiesb/eprc.html)

- No ISR/FSR corrections

# Vector and axial-vector couplings



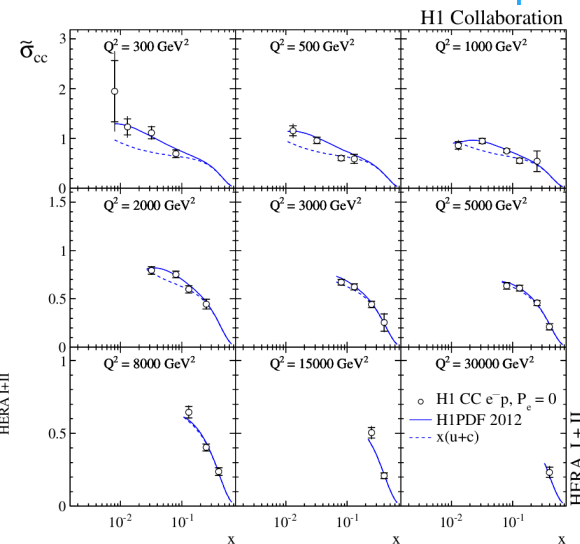
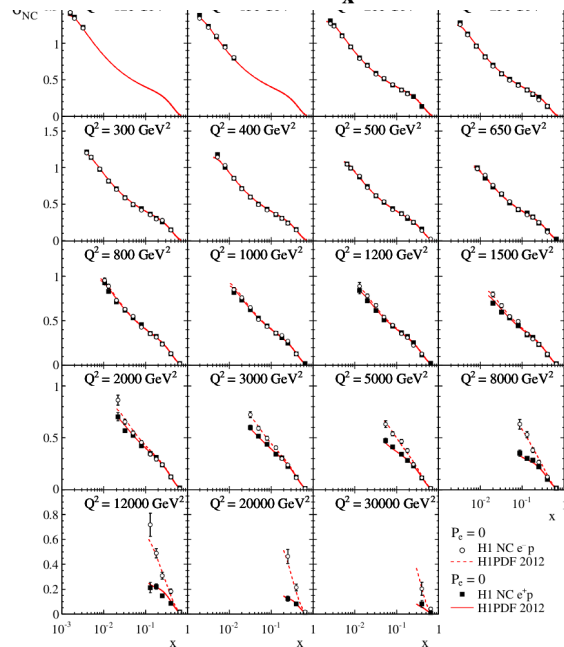
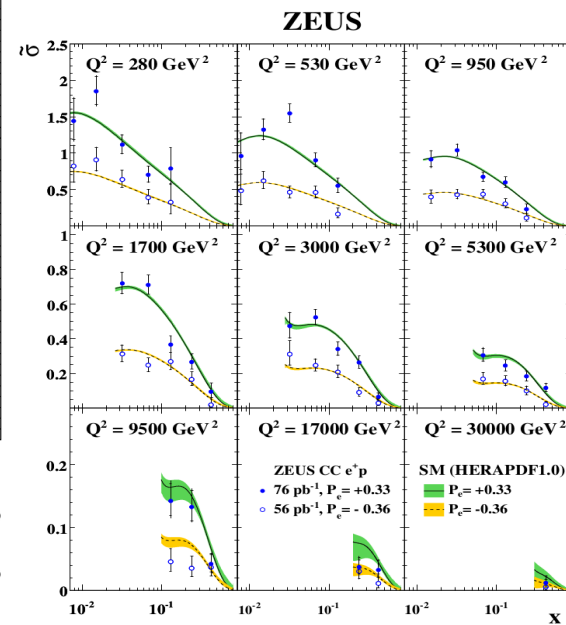
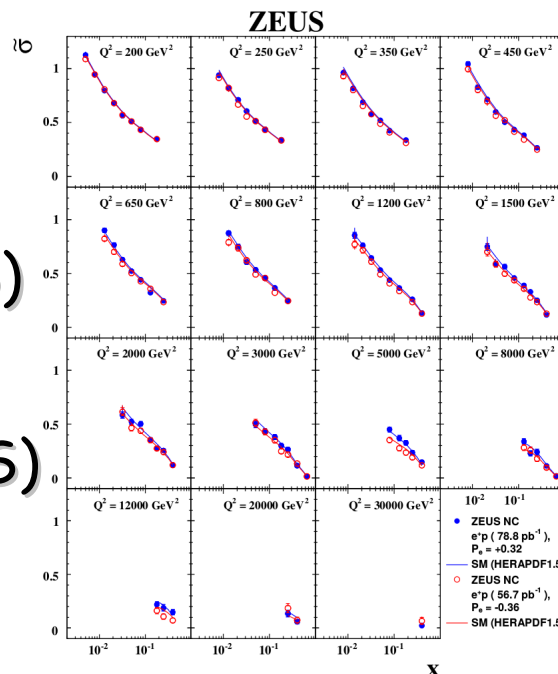
# Data used



## Uncombined data sets

- All HERAI data (H1 & ZEUS)
  - unpolarised
- Reduced  $E_p$  data (H1 & ZEUS)
- HERAII
  - H1 unpolarised data
  - **ZEUS polarised data**

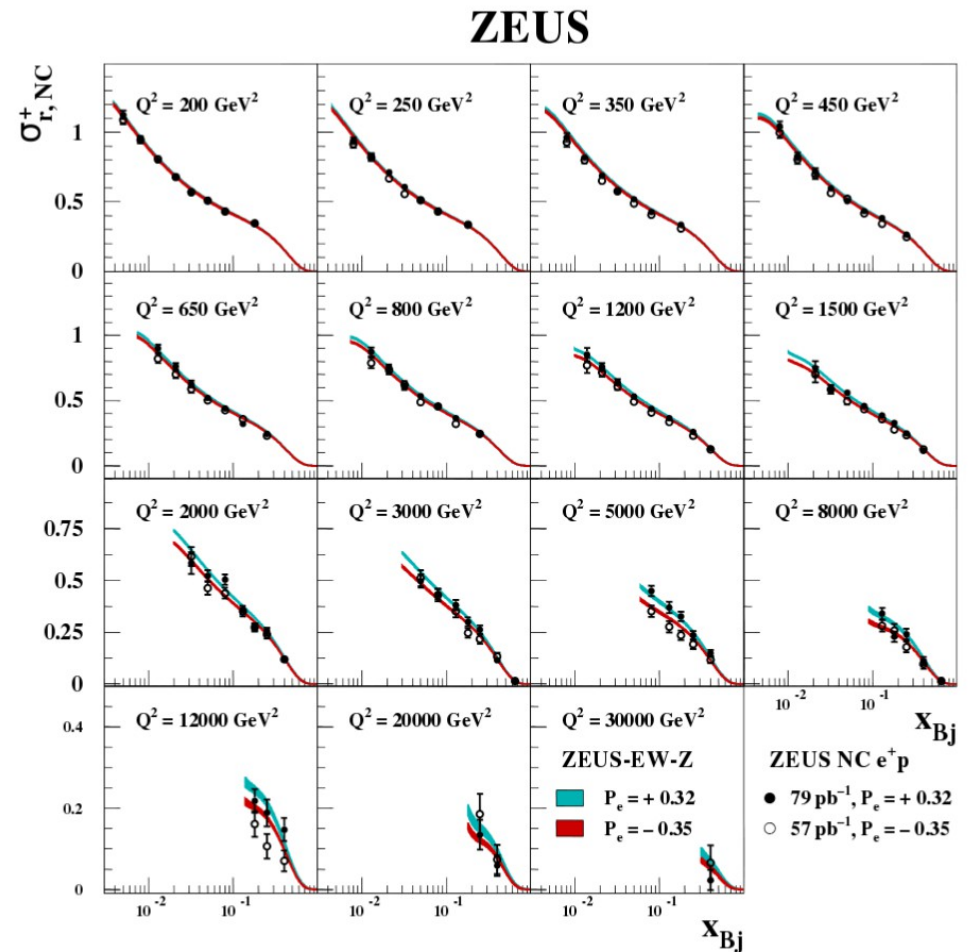
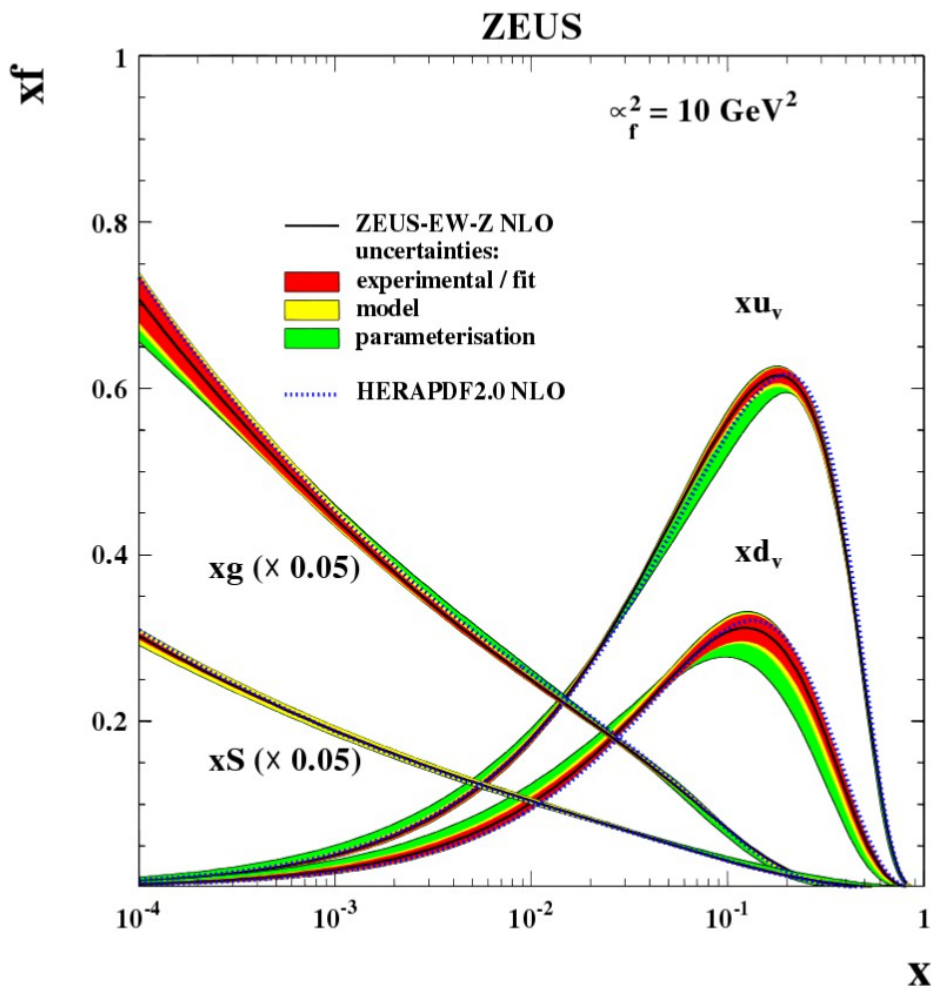
Data from  $Q^2 = 3.5 \text{ GeV}^2$



- DGLAP evolution @ NLO
- HF scheme - GN VFNS NLO (RT OPT)

# Fit results

- Simultaneous QCD and EW fit:
  - 13 QCD parameters + 4 EW couplings

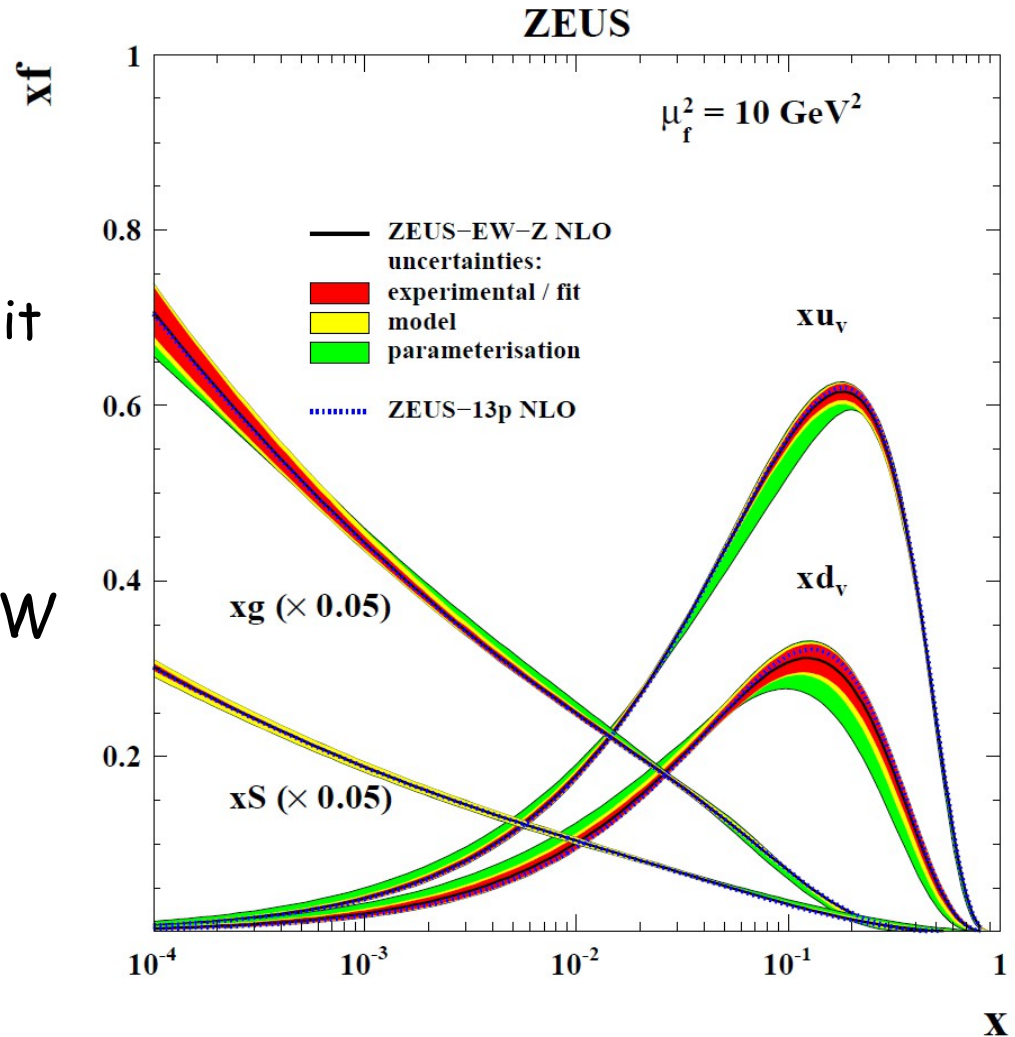






# QCD & EW parameters uncorrelated

- Reference fit ZEUS-13p:
  - QCD parameters fixed to 13p fit
  - Only 4 EW couplings fitted
- Very similar results
- Correlation between QCD and EW parameters small





# QCD & EW parameters uncorrelated

- Detailed studies performed to check stability of EW couplings with respect to various QCD parameters
  - HPDF1: QCD parameters and all constants fixed to HERAPDF2.0
  - HPDF2: QCD parameters fixed to HERAPDF2.0 + on-shell value of  $\sin^2\theta_w$
  - 13p - reference fit described before

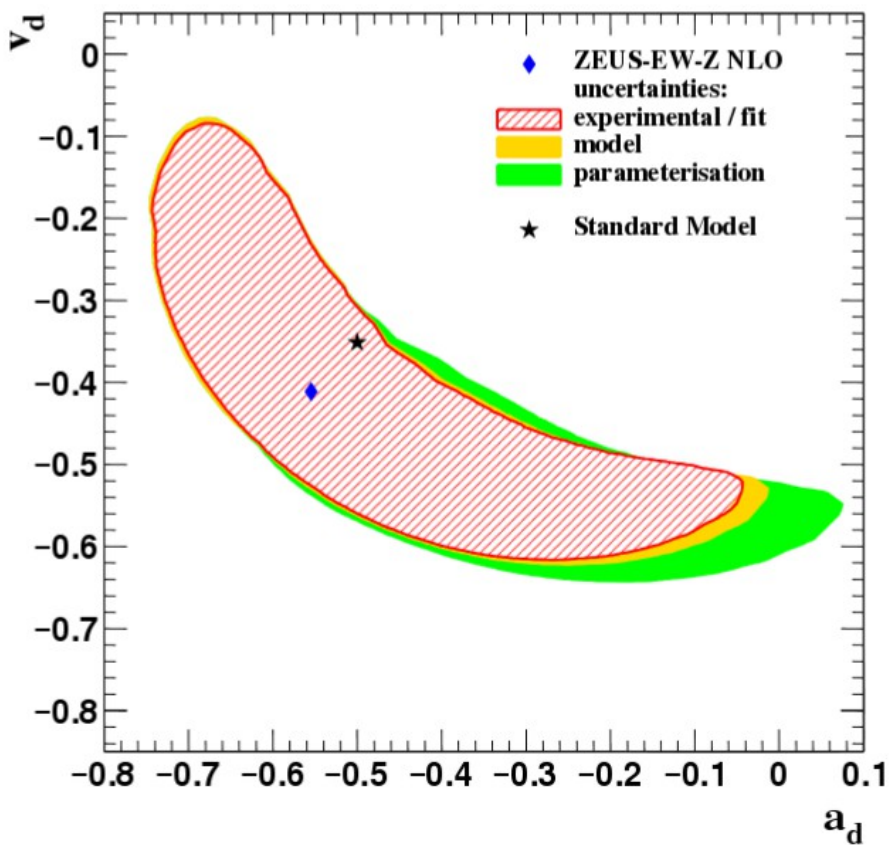
→ Results for couplings very similar

	$a_u$	exp	tot	$a_d$	exp	tot	$v_u$	exp	tot	$v_d$	exp	tot
EW-Z	+0.50	+0.09 -0.05	+0.12 -0.05	-0.56	+0.34 -0.14	+0.41 -0.15	+0.14	+0.08 -0.08	+0.09 -0.09	-0.41	+0.24 -0.16	+0.25 -0.20
13p	+0.49	+0.07 -0.04		-0.57	+0.30 -0.13		+0.15	+0.08 -0.08		-0.40	+0.22 -0.17	
HPDF1	+0.47	+0.06 -0.03		-0.62	+0.23 -0.11		+0.16	+0.08 -0.08		-0.35	+0.22 -0.19	
HPDF2	+0.49	+0.06 -0.03		-0.63	+0.24 -0.11		+0.15	+0.08 -0.08		-0.36	+0.22 -0.19	
SM	+0.50			-0.50			+0.20			-0.35		

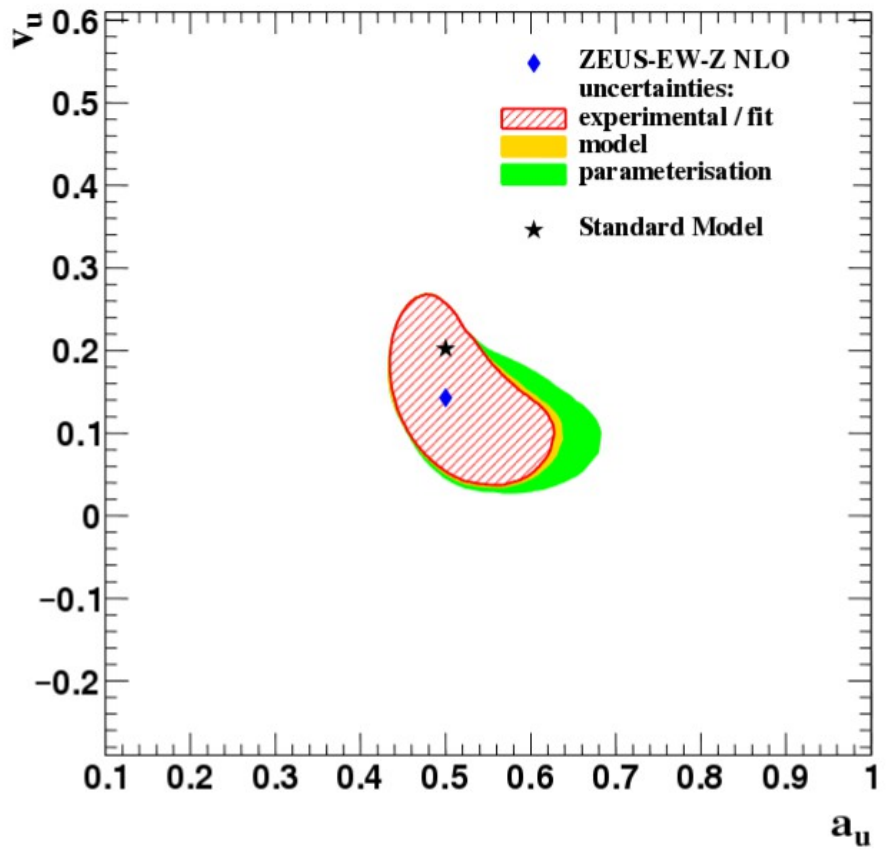
# Correlations



ZEUS



ZEUS

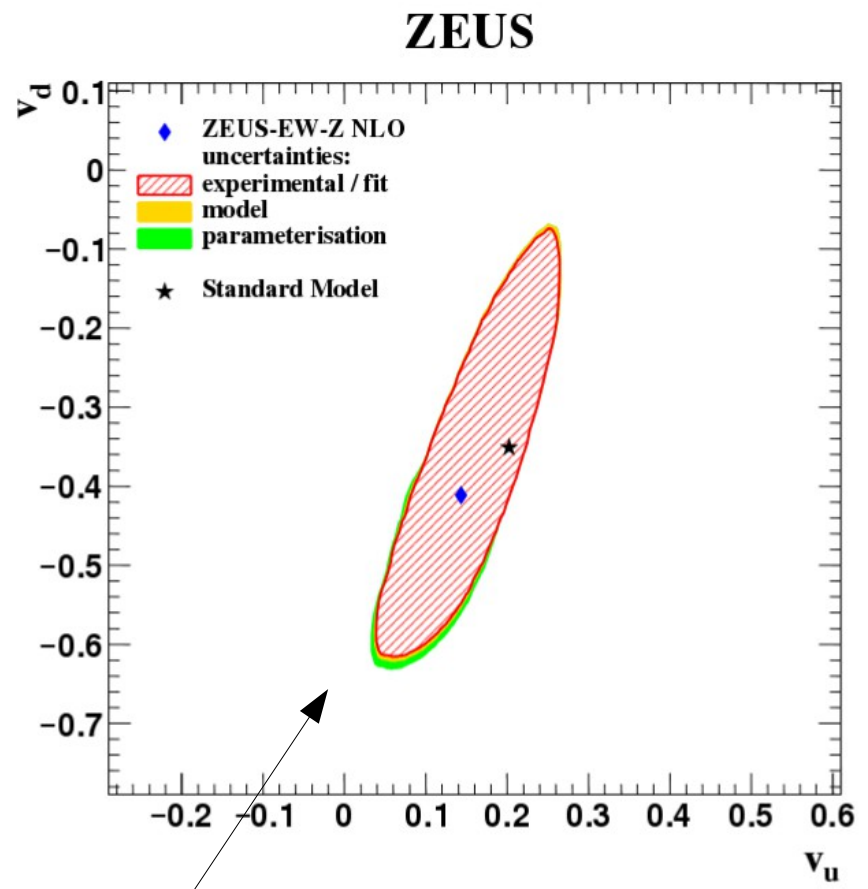
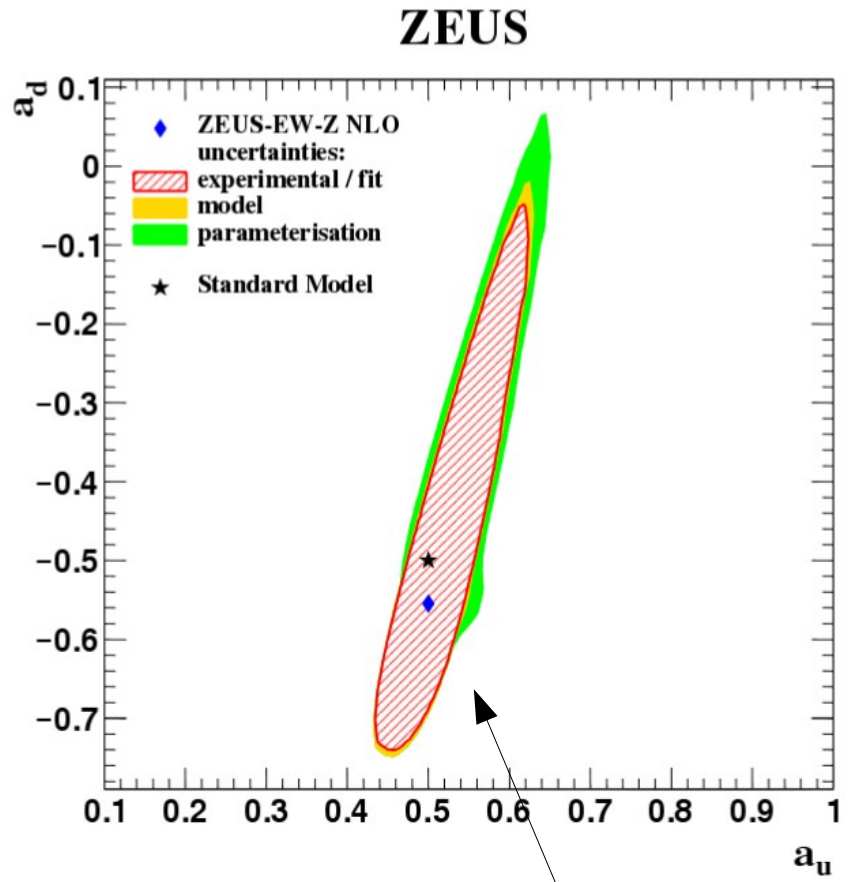


$a_u = 0.50$	$\begin{matrix} +0.09 \\ -0.05 \end{matrix}$	$\begin{matrix} +0.04 \\ -0.02 \end{matrix}$	$\begin{matrix} +0.08 \\ -0.01 \end{matrix}$	$= 0.50$	$\begin{matrix} +0.12 \\ -0.05 \end{matrix}$	$0.5$	Standard Model
$a_d = -0.56$	$\begin{matrix} +0.34 \\ -0.14 \end{matrix}$	$\begin{matrix} +0.11 \\ -0.05 \end{matrix}$	$\begin{matrix} +0.20 \\ -0.00 \end{matrix}$	$= -0.56$	$\begin{matrix} +0.41 \\ -0.15 \end{matrix}$	$-0.5$	
$v_u = 0.14$	$\begin{matrix} +0.08 \\ -0.08 \end{matrix}$	$\begin{matrix} +0.01 \\ -0.00 \end{matrix}$	$\begin{matrix} +0.03 \\ -0.01 \end{matrix}$	$= 0.14$	$\begin{matrix} +0.09 \\ -0.09 \end{matrix}$	$0.202$	
$v_d = -0.41$	$\begin{matrix} +0.24 \\ -0.16 \end{matrix}$	$\begin{matrix} +0.04 \\ -0.07 \end{matrix}$	$\begin{matrix} +0.00 \\ -0.08 \end{matrix}$	$= -0.41$	$\begin{matrix} +0.25 \\ -0.20 \end{matrix}$	$-0.351$	

# Correlations



- Vector and axial-vector couplings in the fit show high correlation



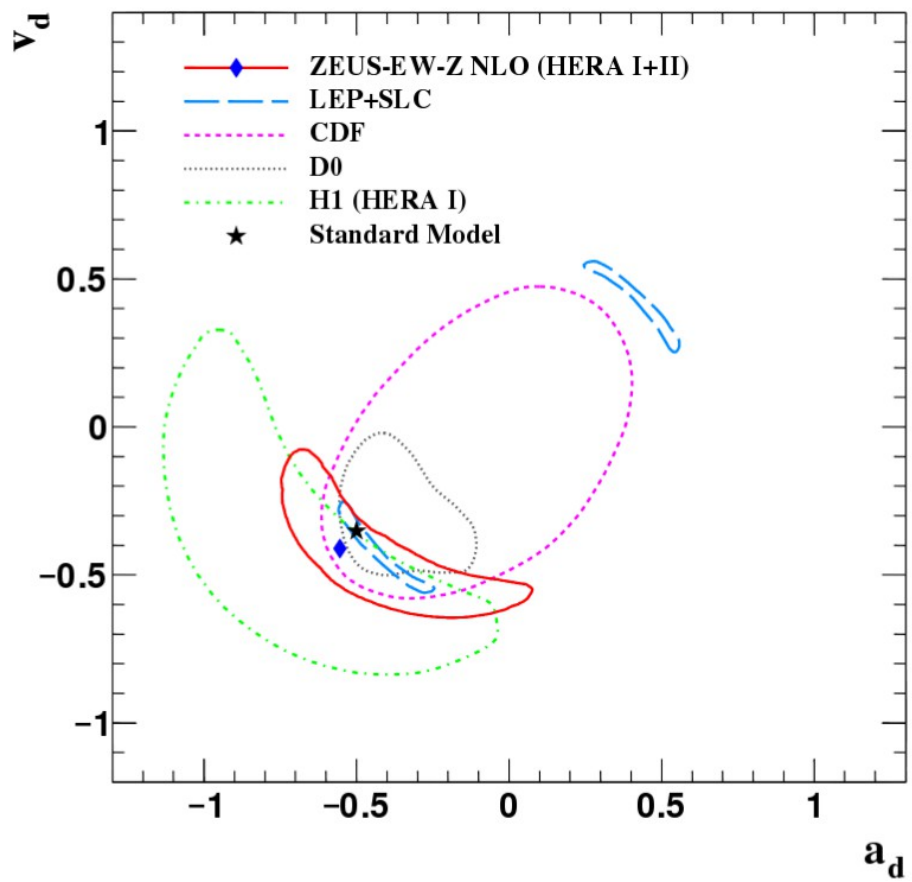
	$a_u$	$a_d$	$v_u$	$v_d$
$a_u$	1.000	0.861	-0.555	-0.729
$a_d$	0.861	1.000	-0.636	-0.880
$v_u$	-0.555	-0.636	1.000	0.851
$v_d$	-0.729	-0.880	0.851	1.000

Insignificant correlations of couplings to PDF parameters

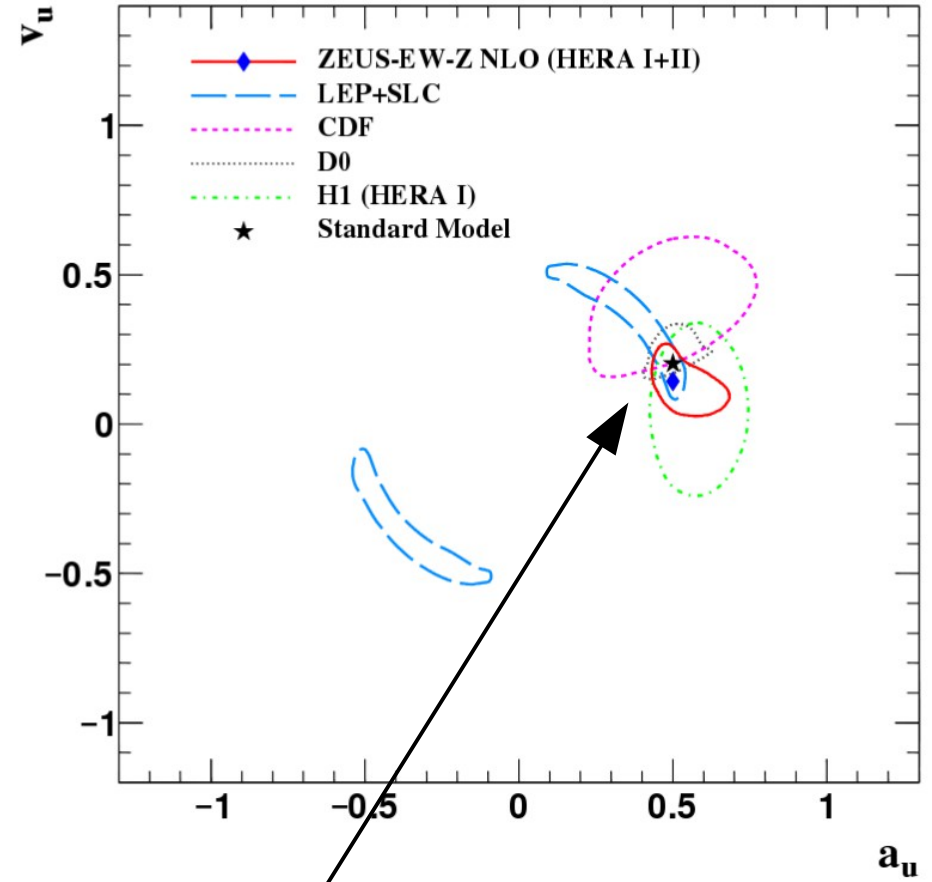
# Comparison with other measurements



## ZEUS



## ZEUS



HERA data remarkably sensitive to **u-type** quark couplings

# H1 fit methodology

H1prelim-16-041



- Lots of basics same as in ZEUS measurement
  - Differences/different approaches pointed out
- Calculations performed strictly in on-shell scheme
  - Parameters are:  $\alpha$ ,  $m_W$ ,  $m_Z$ ,  $(m_t, m_H, \dots)$
- Polarisation measurements considered as independent measurements in fits
- New C++ code for PDF and more general fits developed: Alpos
- DGLAP evolution @ NNLO

## $\chi^2$ Definition

- Uncertainties on cross sections are assumed to be 'log-normal' distributed (relative uncertainties)
- Uncertainties on polarisation measurements are assumed to be 'normal' distributed
- Correlations of syst. uncertainties between different datasets are considered

$$\chi^2 = (\log(d) - \log(t))^T V_R^{-1} (\log(d) - \log(t)) + (d - t)^T V_A^{-1} (d - t)$$

## Fit parameters

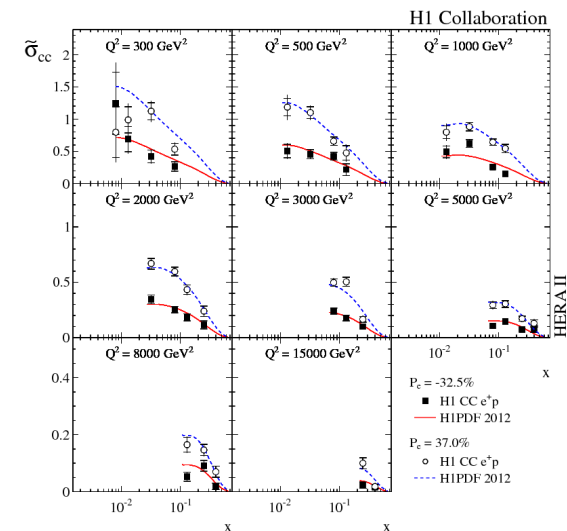
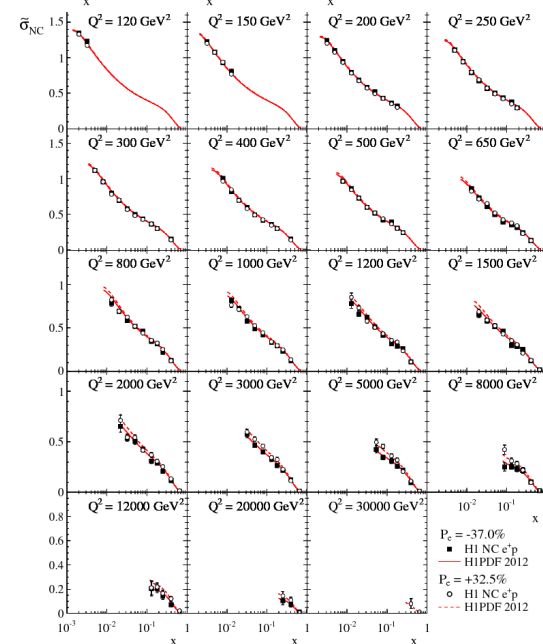
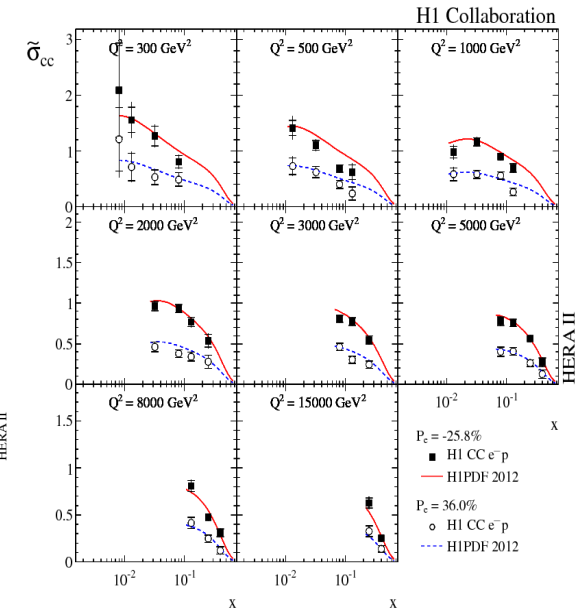
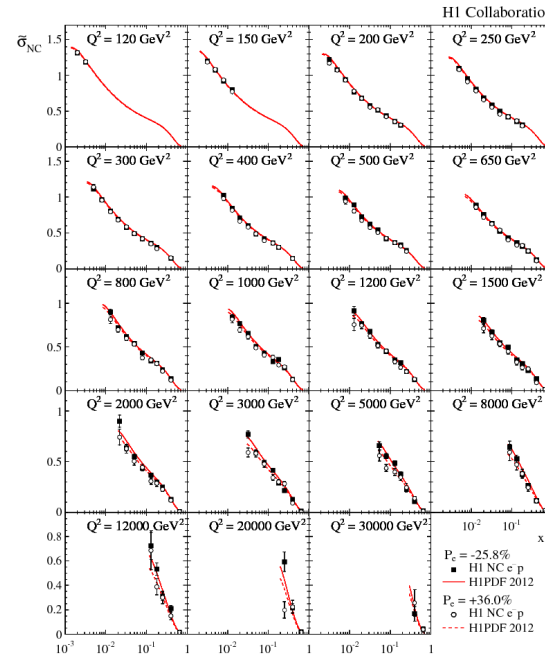
- 13 PDF parameters
- 4 polarisation values
- 4 Light-quark couplings (or other SM parameters)
- More general also 'nuisance parameters' of syst. uncertainties

# Data used

## Uncombined data sets

- H1 HERAI data
  - unpolarised
- Reduced  $E_p$  H1 data
- HERA II
  - H1 polarised data

Data from  $Q^2 = 12 \text{ GeV}^2$



- HF scheme: ZM-VFNS as implemented in QCDNUM

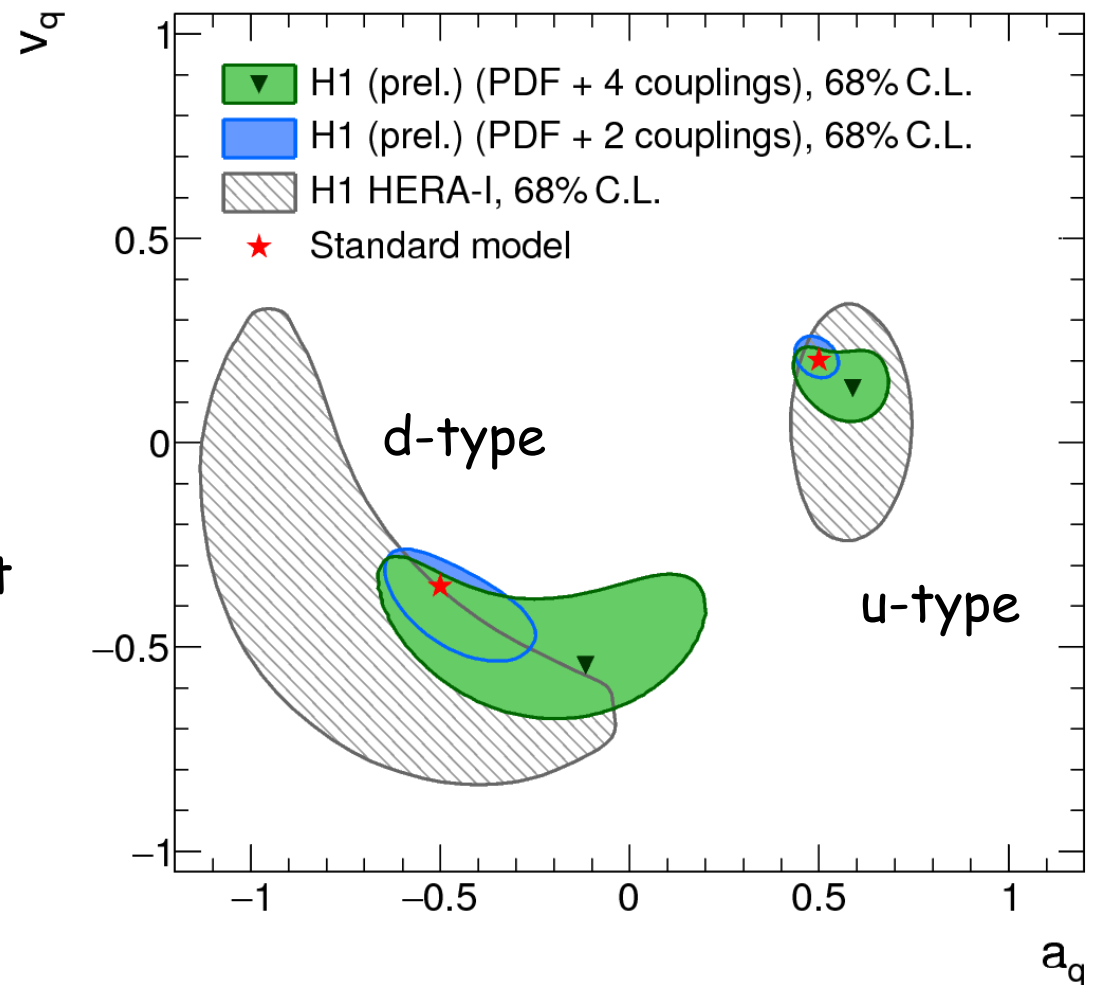
# Light quark couplings @ H1

## Fit: PDF + 4 couplings

- $\chi^2 / \text{ndf} = 1370.5 / (1388 - 21)$
- u-type couplings constrained better than d-type
- sensitivity from valence quarks
- Results compatible with SM
- PDF uncertainties small
- Considerably improved sensitivity using final H1 HERA-II data
- Polarisation in HERA-II important for vector couplings

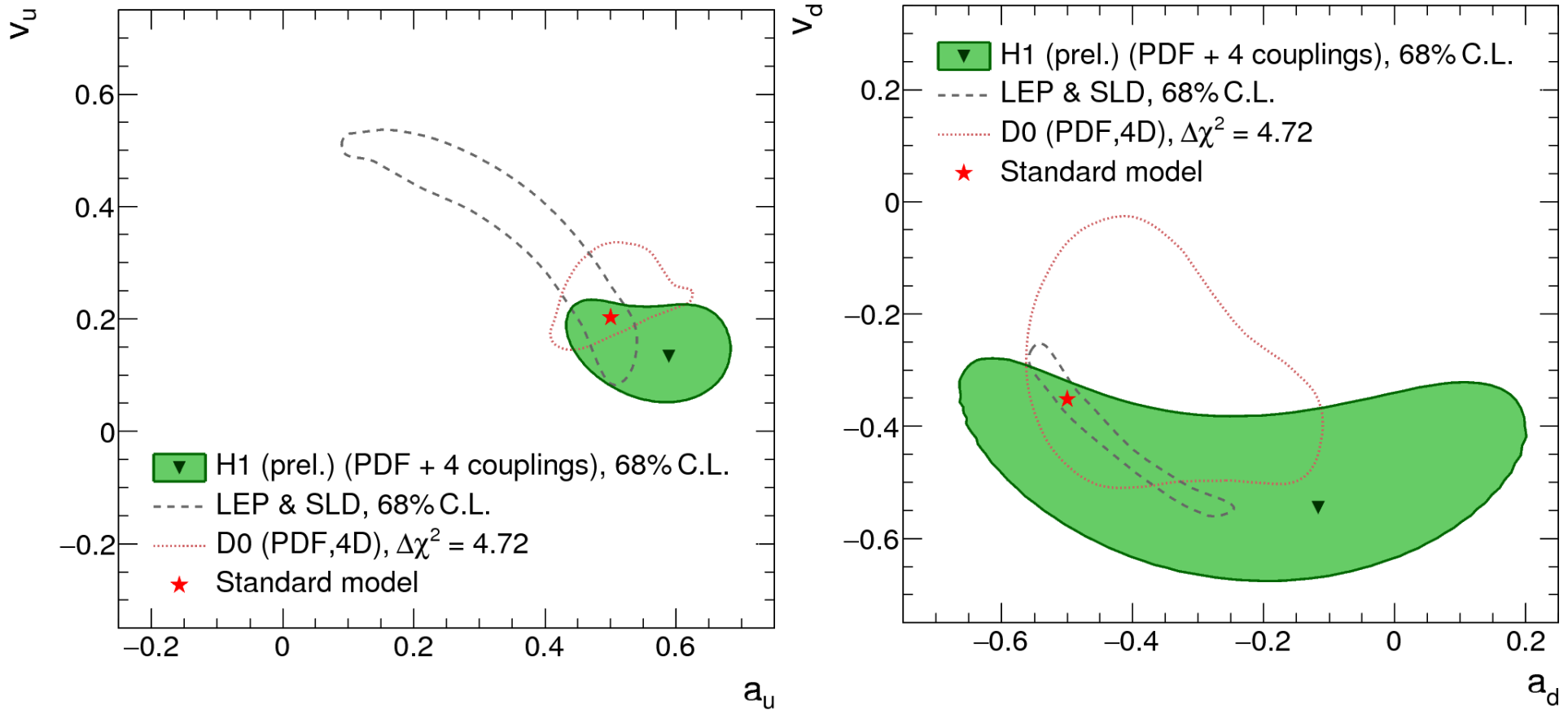
## Fit: PDF + 2 couplings

- Reduced correlations and uncertainties
- Correlations between  $a_u - a_d$  and  $v_u - v_d$  large



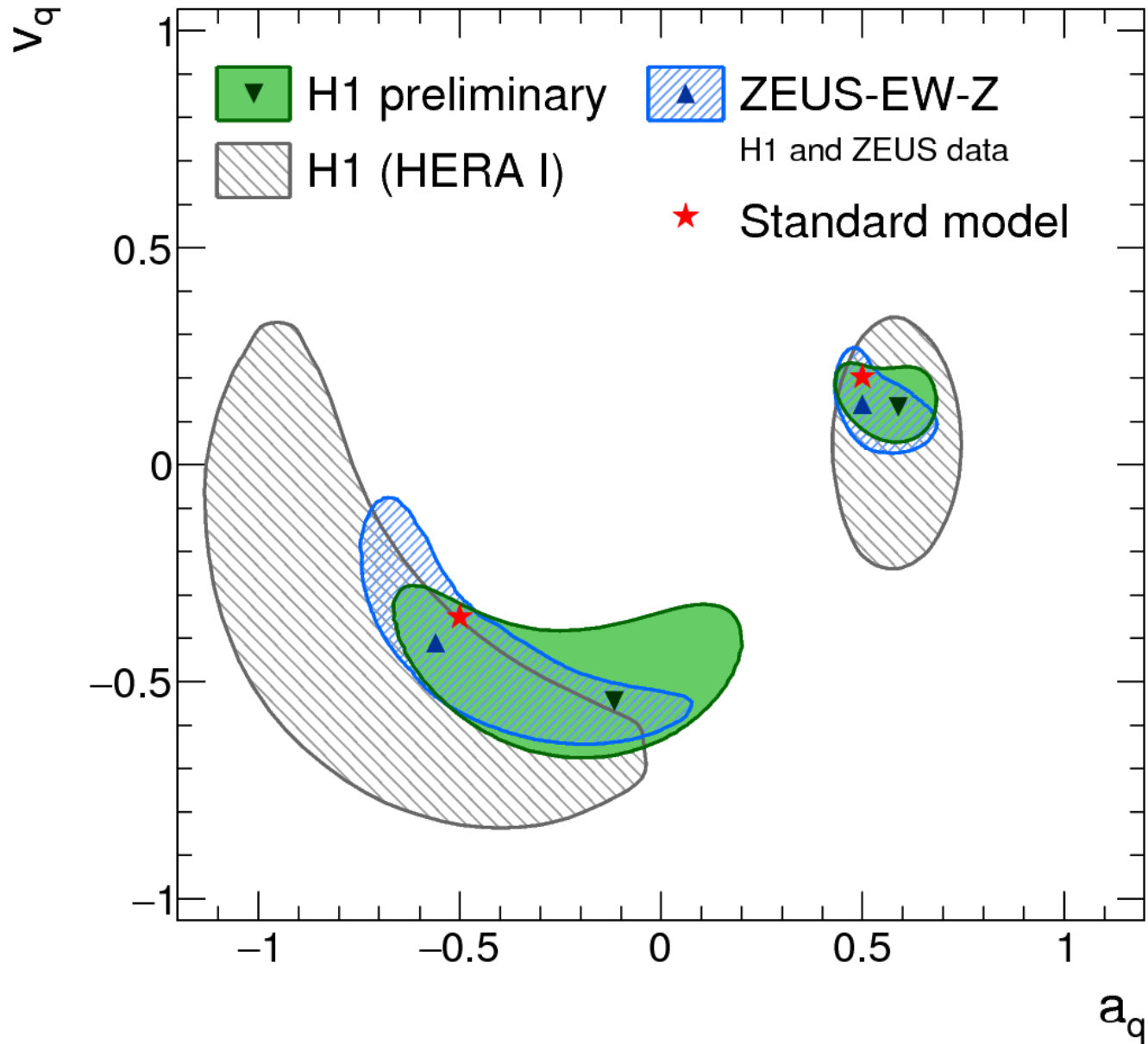


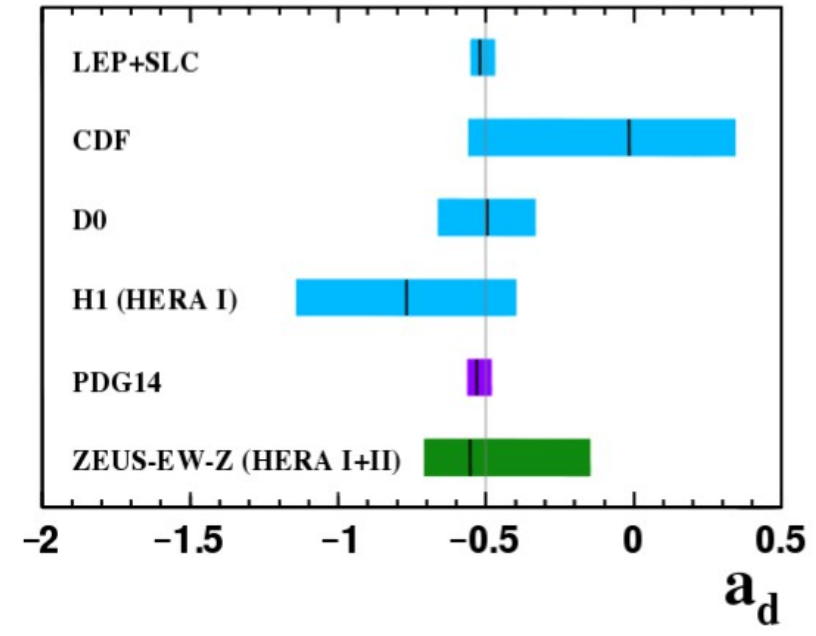
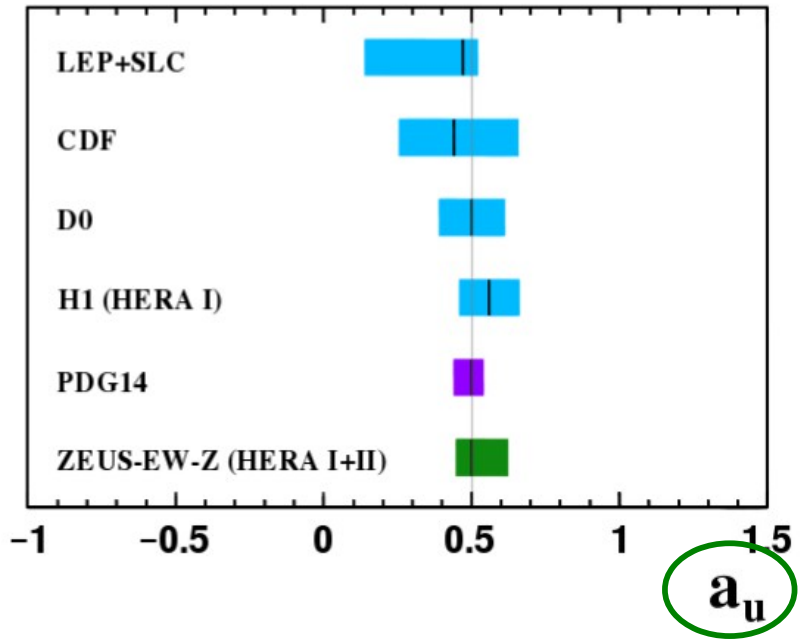
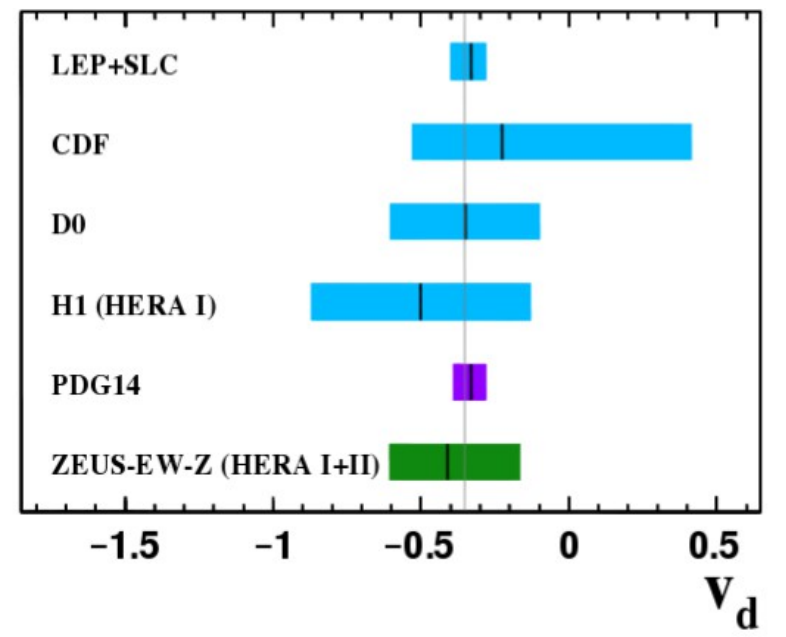
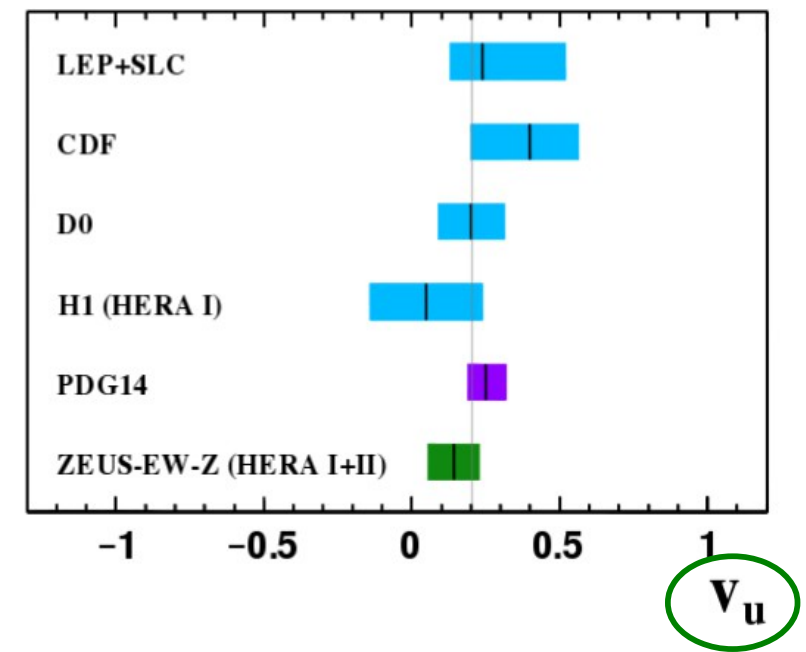
# Comparison with other measurements



- Comparable precision of complementary processes

# H1 and ZEUS Results Combined





High sensitivity of HERA data to u-type quark couplings

# Probing Standard Model

## Standard Model is now overconstrained

- Important to study consistency in many complementary processes
- HERA: Space-like momentum transfers
- Only purely virtual exchange of bosons



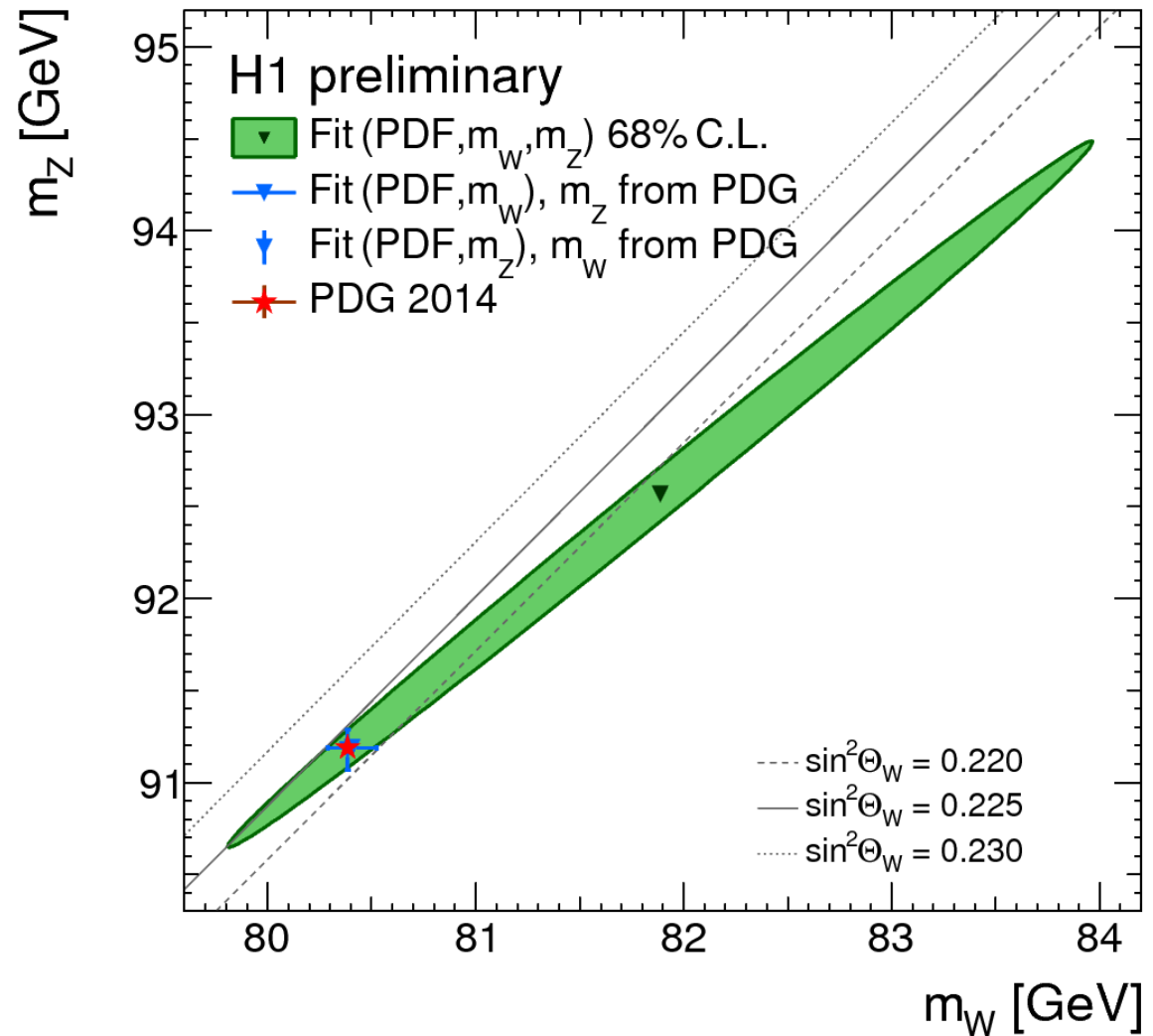
# Boson masses

## $(m_W - m_Z) + \text{PDF fits}$

- Assume  $\alpha$  is known
- on-shell masses  $m_W$  and  $m_Z$  are only free EW parameters
- Agreement with SM
- Large correlation between  $m_W$  and  $m_Z$

## Mass of W boson

- Take other masses ( $m_Z$ ) as external input to calculations



$$m_W = 80.407 \pm 0.118 \text{ (exp, pdf-fit)} \pm 0.005 \text{ (} m_Z, m_t, m_H \text{)} \text{ GeV}$$

$$M_W^{PDG 14} = 80.385 \pm 0.015 \text{ GeV}$$

# Study of Standard Model Parameters

## Different view on SM parameters

- Fermi coupling constant  $G_F$

$$G_F = \frac{\pi \alpha}{\sqrt{2} m_W^2 \sin^2 \theta_W} (1 + \Delta r)$$

- Weak mixing angle

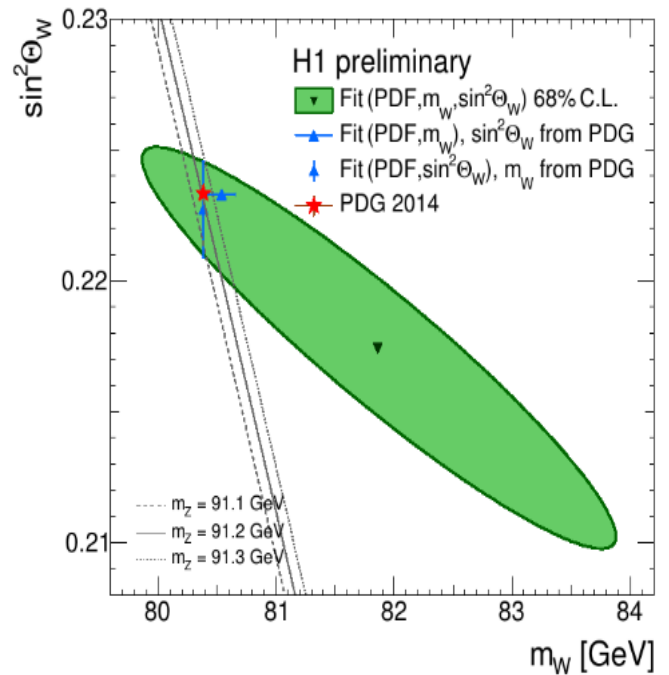
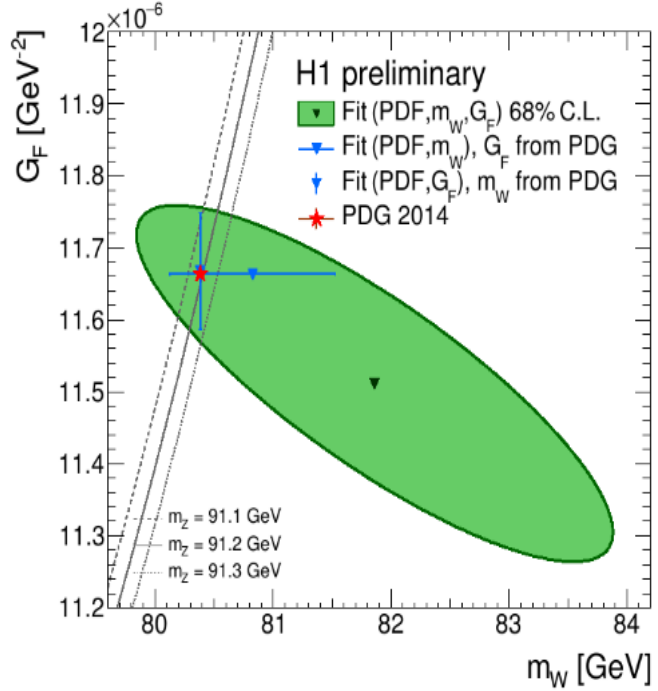
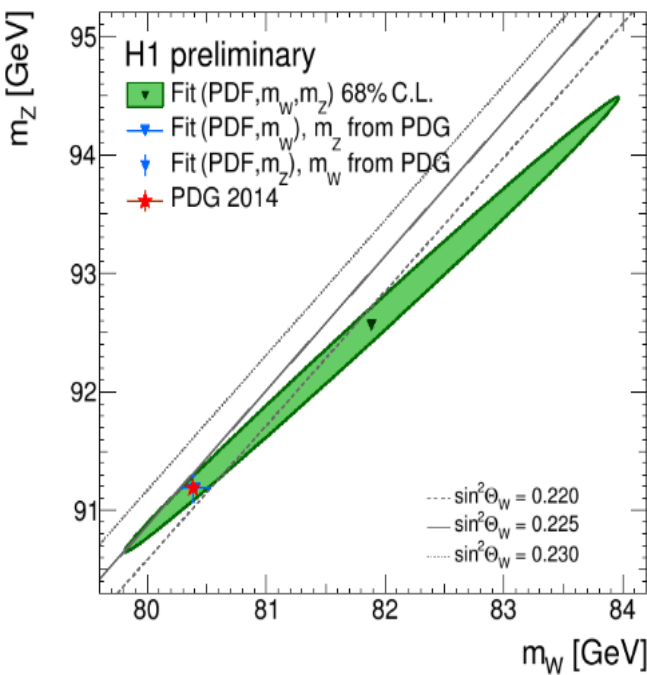
$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

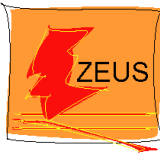
## Perform calculations consistently in on-shell scheme ( $\alpha, m_Z, m_W$ )

- Calculate  $m_Z$  (iteratively) from  $G_F$  or  $\sin^2 \theta_W$

## Results from fits together with PDF and $m_W$

- H1 values consistent with precise values from PDG
- Correlation to  $m_W$  are different for  $m_Z$ ,  $\sin^2 \theta_W$  and  $G_F$





# Simultaneous extraction of $\sin^2\theta_W$ and $M_W$

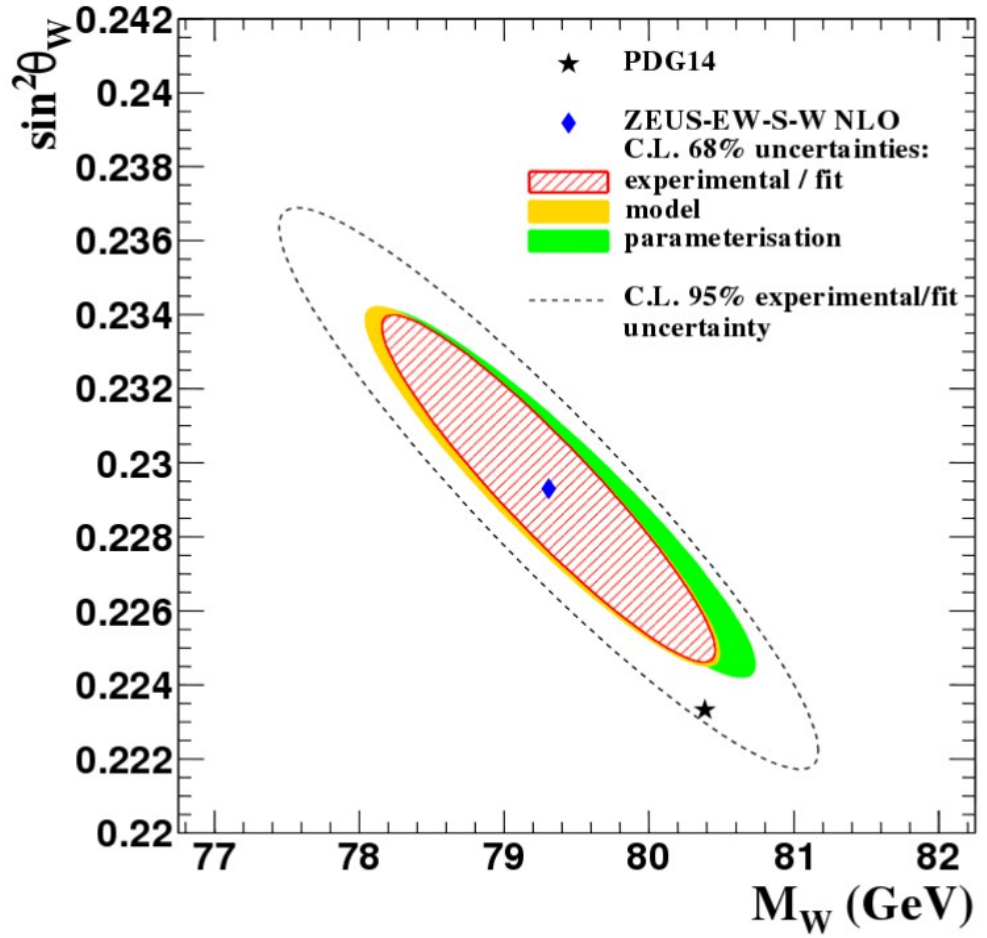
- Similar measurement by ZEUS

$$M_W = 79.30 \pm 0.76_{(expl/fit)} \begin{matrix} +0.38 \\ -0.08(mod) \end{matrix} \begin{matrix} +0.48 \\ -0.10(par) \end{matrix} GeV = 79.30^{+0.98}_{-0.77(tot)} GeV$$

$$\sin^2\theta_W = 0.2293 \pm 0.0031_{(expl/fit)} \begin{matrix} +0.0005 \\ -0.0001(mod) \end{matrix} \begin{matrix} +0.0003 \\ -0.0001(par) \end{matrix} = 0.2293^{+0.0032}_{-0.0031(tot)}$$

- All extracted quantities agree with world average values

## ZEUS



$$M_W^{PDG14} = 80.385 \pm 0.015 GeV$$

$$\sin^2\theta_W^{PDG14 On-shell} = 0.22333 \pm 0.00011$$

$$corr(M_W, \sin^2\theta_W) = -0.930$$



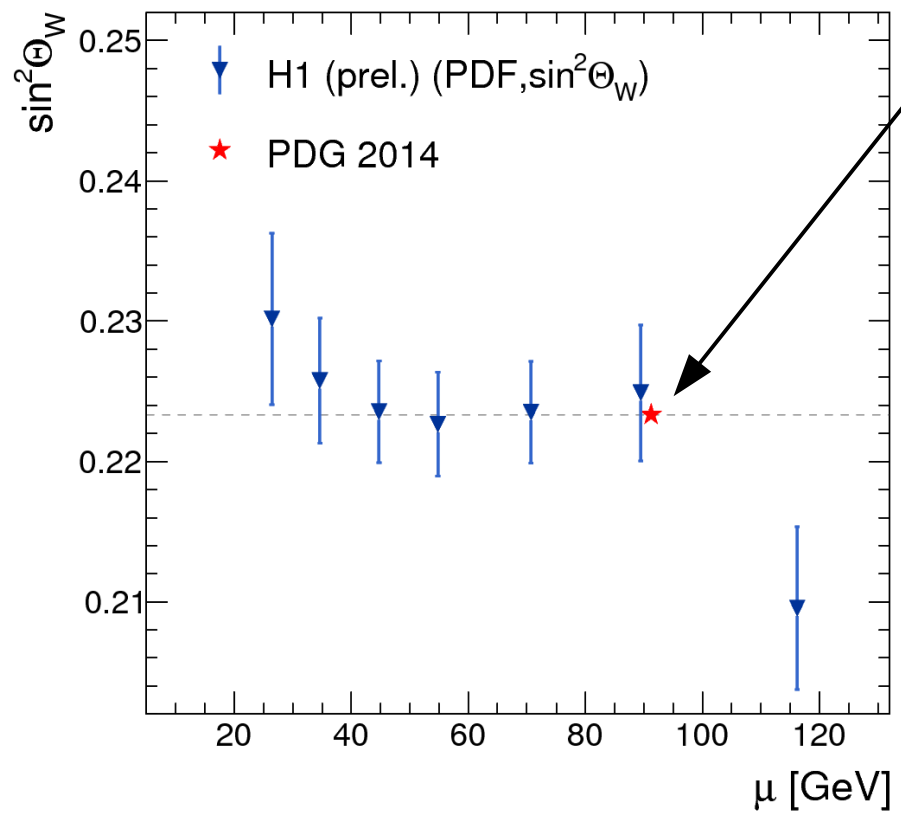
# On-shell $\sin^2\theta_W$

- $\sin^2\theta_W$  determined simultaneously with PDF parameters (ZEUS-EW-S)

$$\sin^2\theta_W = 0.2252 \pm 0.0011_{(exp/fit)} \begin{matrix} +0.0003 \\ -0.0001(mod) \end{matrix} \begin{matrix} 0.0007 \\ -0.0001(par) \end{matrix} = \mathbf{0.2252^{+0.0013}_{-0.0011}(tot)}$$

- Consistent with PDG14

$$\sin^2\theta_W^{PDG14\ On-shell} = 0.22333 \pm 0.00011$$

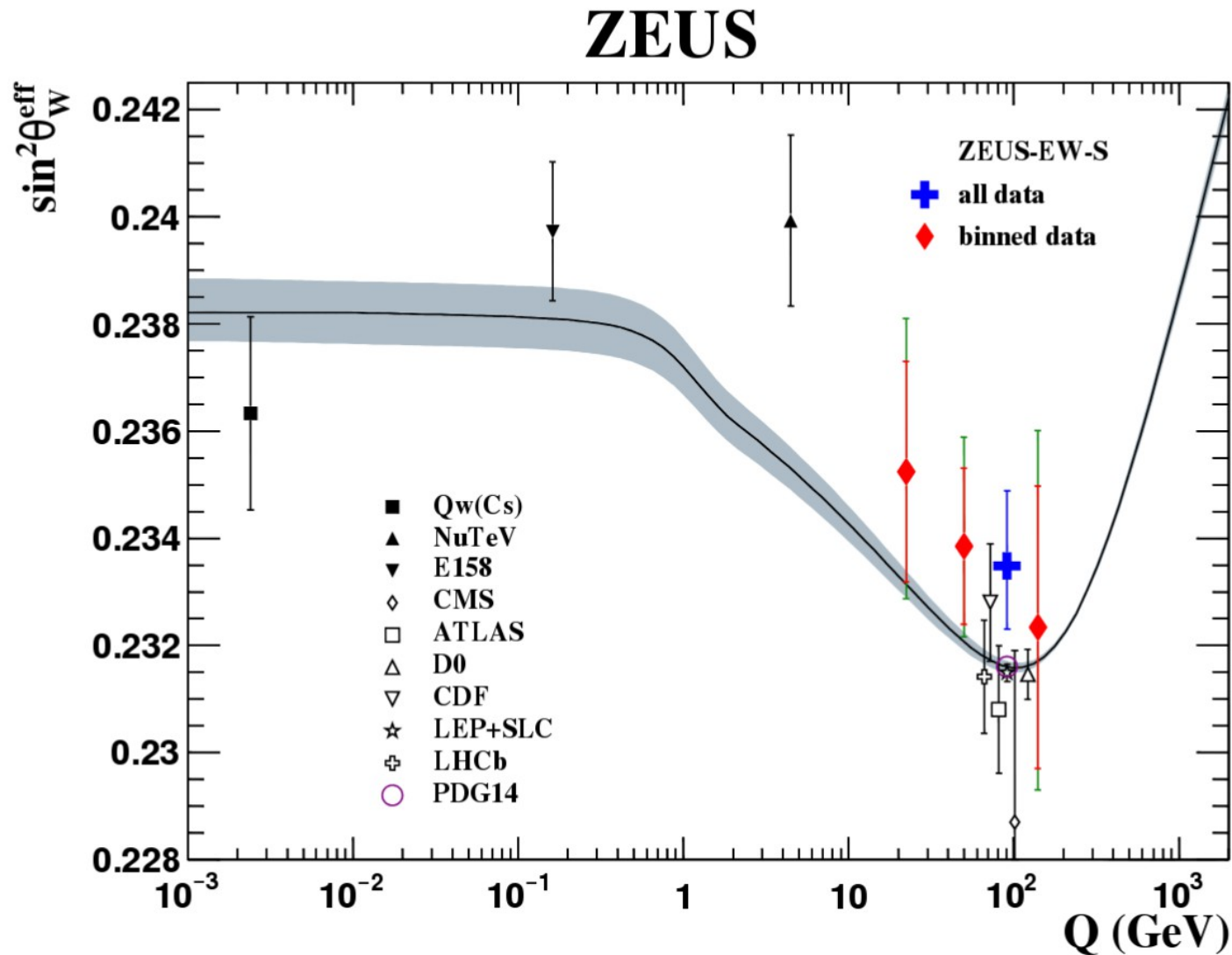


- $\sin^2\theta_W$  determined in  $Q^2$  bins
- Unique measurement of weak mixing angle at different scales
- Agreement with PDG14
- Can be translated to  $\overline{MS}$  scheme



# Effective $\sin^2\theta_W$

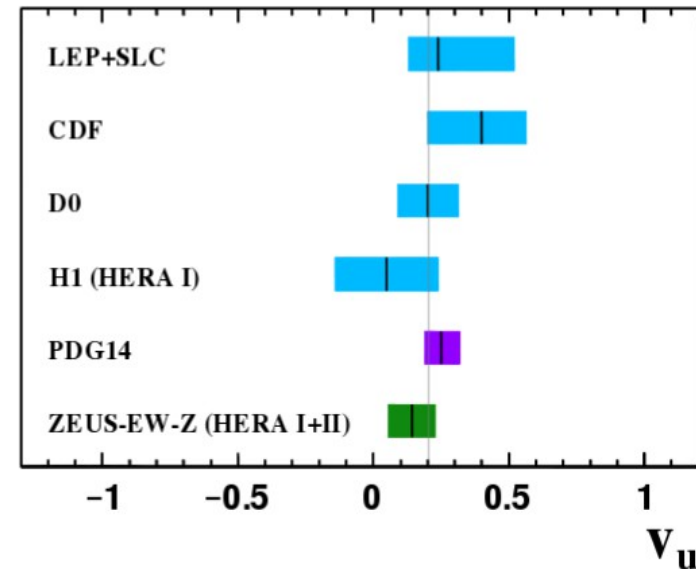
- On-shell measurements were translated to  $\sin^2\theta_W^{eff}$
- First observation of effective  $\sin^2\theta_W$  running from single machine





# Summary

- HERA polarised inclusive data sensitive to electroweak parameters
  - Simultaneous PDF and EW fits
- Axial and vector-axial couplings to quarks agree with world average
- Measurements of **u-type** quark couplings among the most accurate

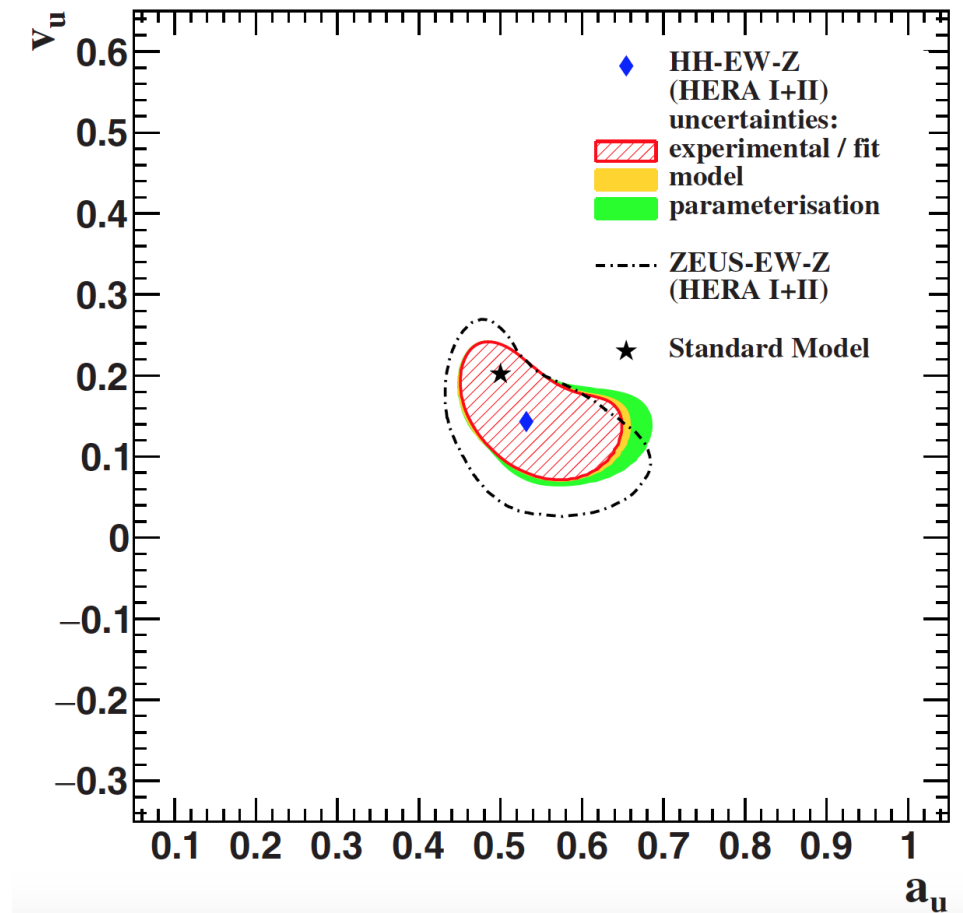
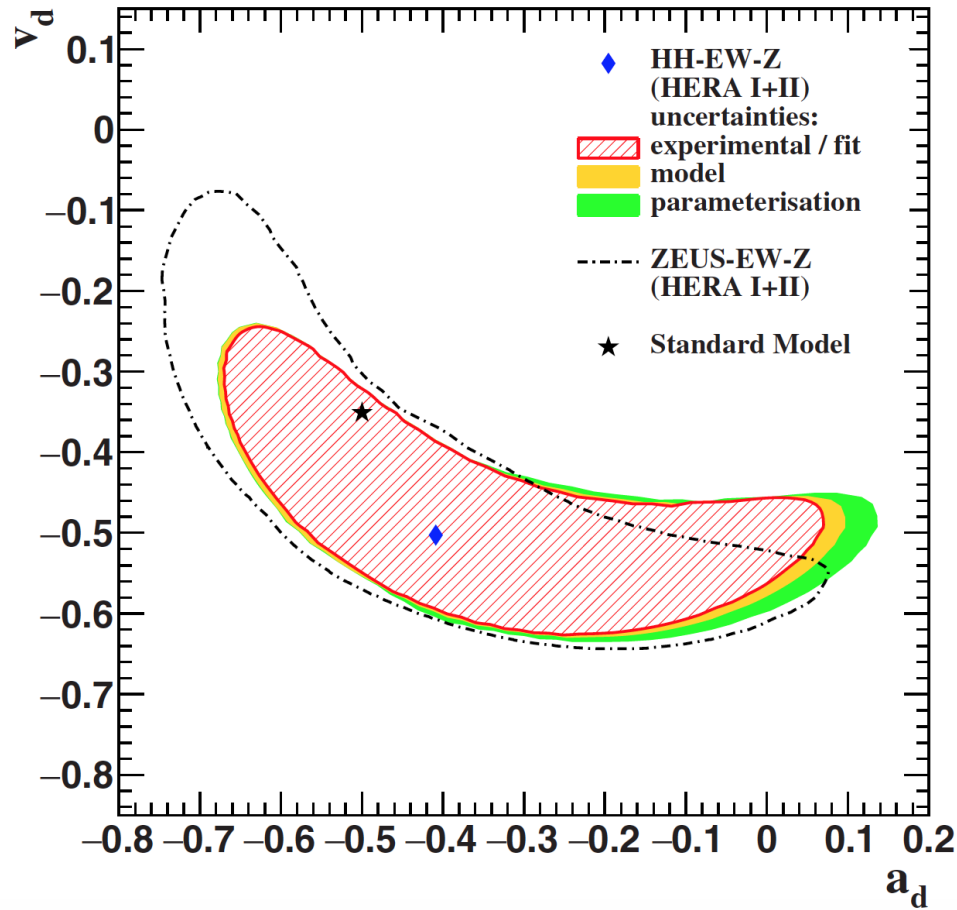


- Standard Model tests performed
  - Good consistency for  $M_Z$ ,  $M_W$ ,  $G_F$  and weak mixing angle
  - value of  $\sin^2\theta_W$  competitive with measurements from neutrino sector
- $\sin^2\theta_W$  on-shell and effective determined for different scales
- Mass of  $W$  boson was determined at space-like momentum transfer

# Back-up slides

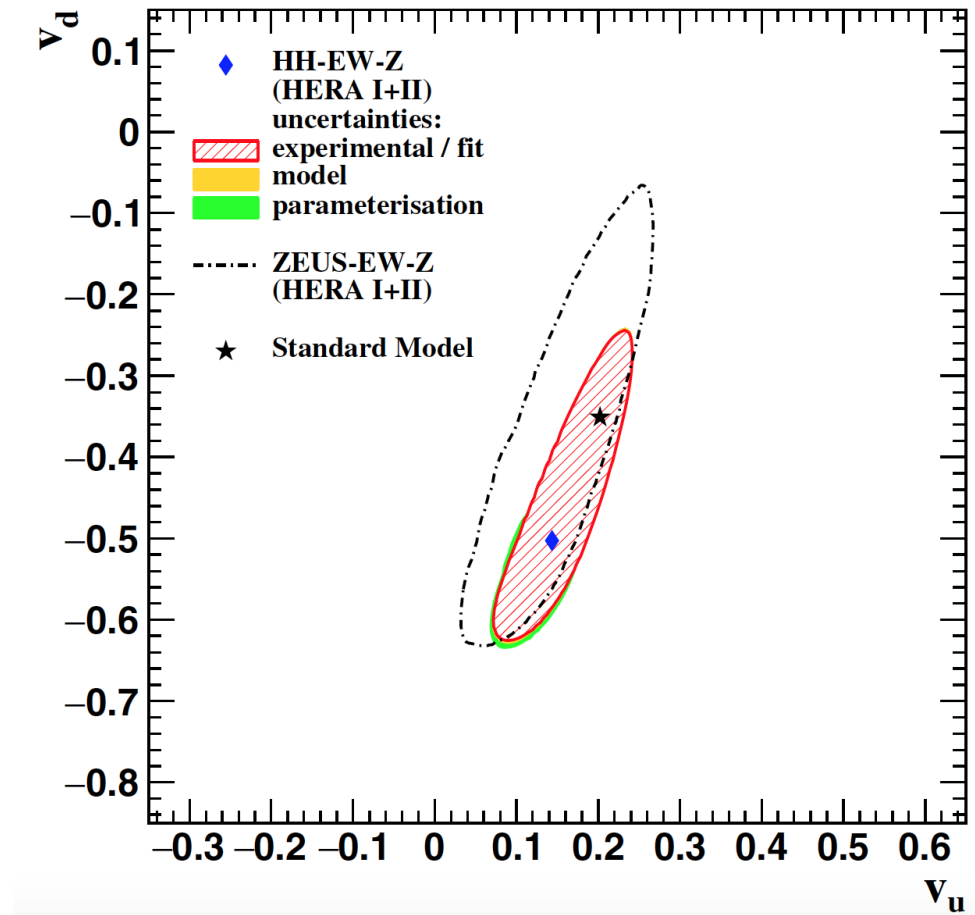
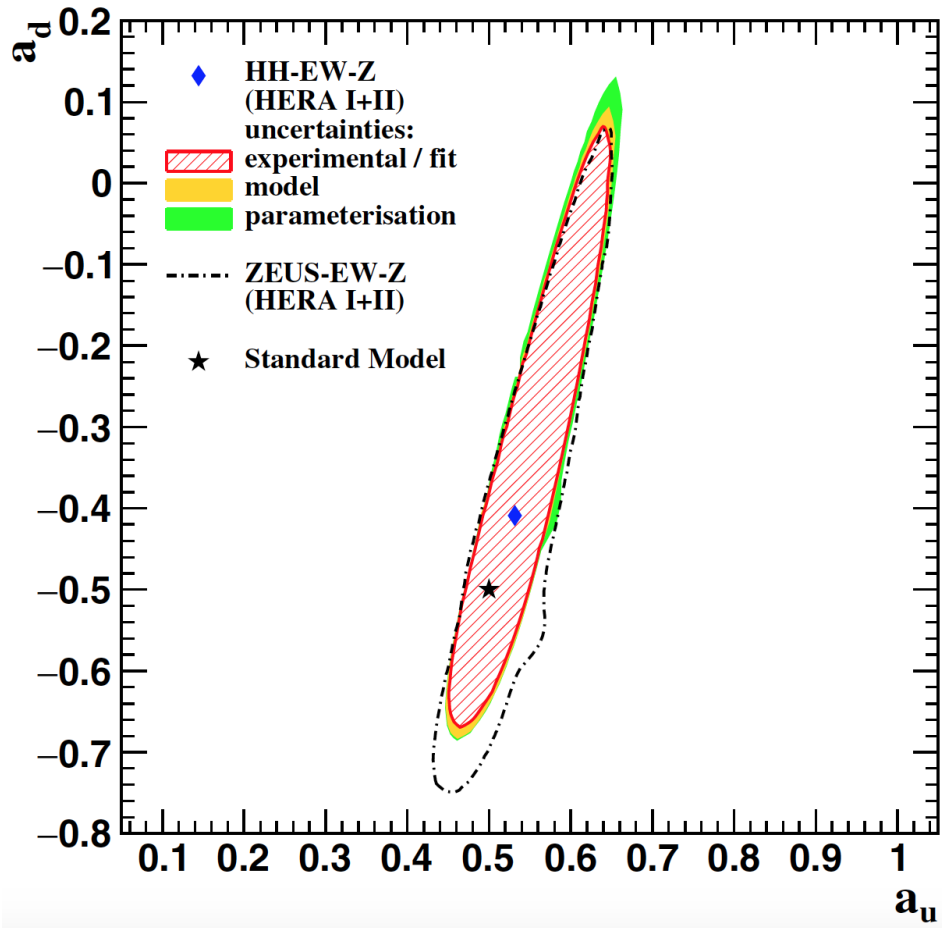
# Using all polarised HERA data in QCD+EW fit

DESY-16-063, accepted by PRD



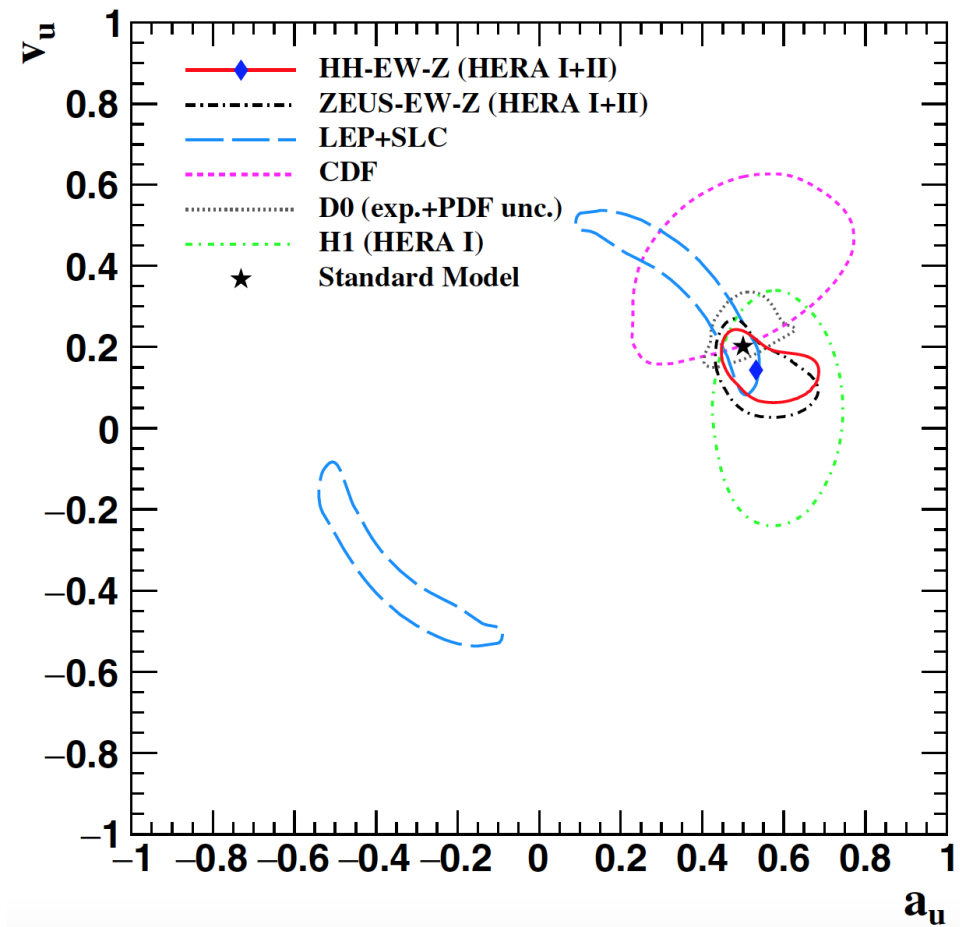
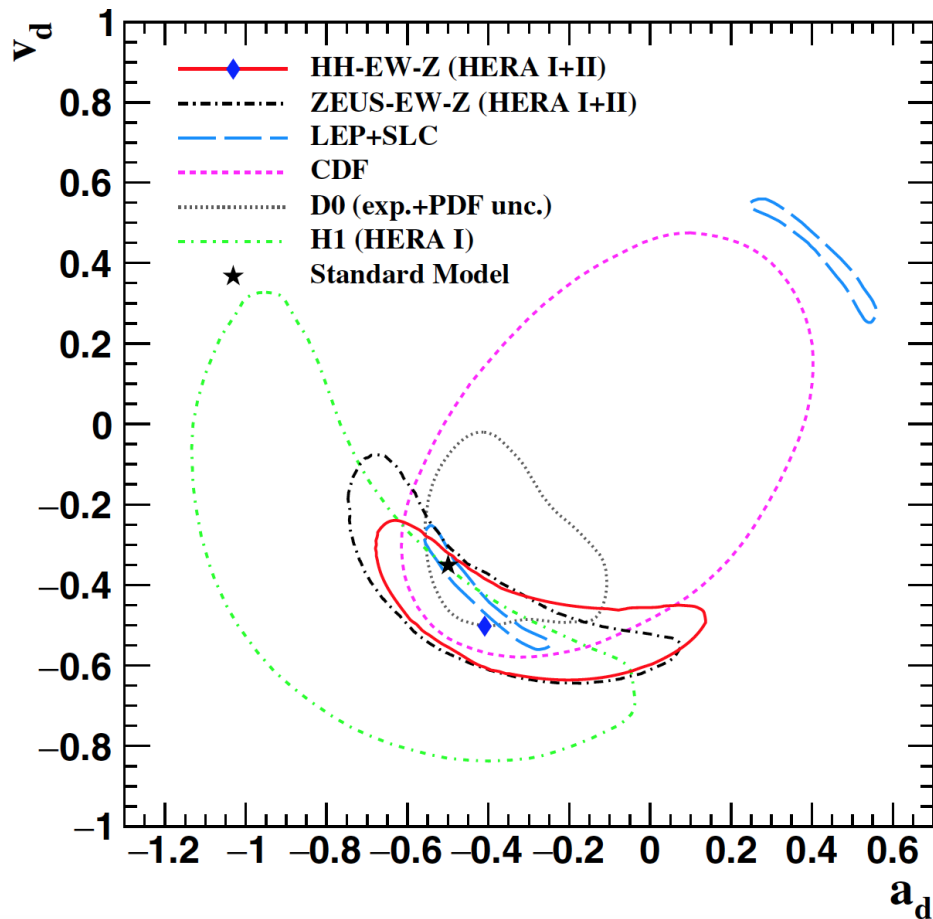
# Using all polarised HERA data in QCD+EW fit

DESY-16-063, accepted by PRD



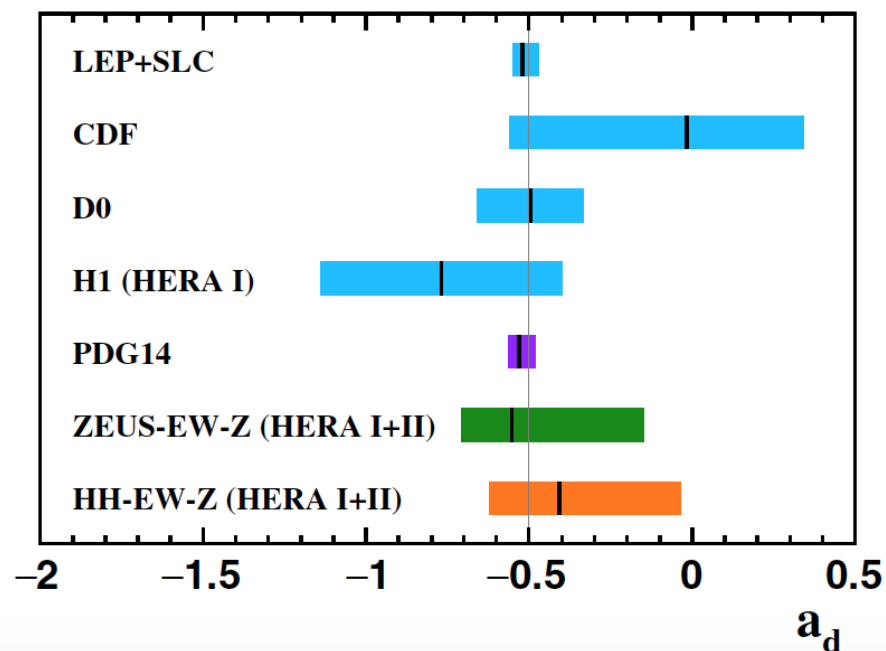
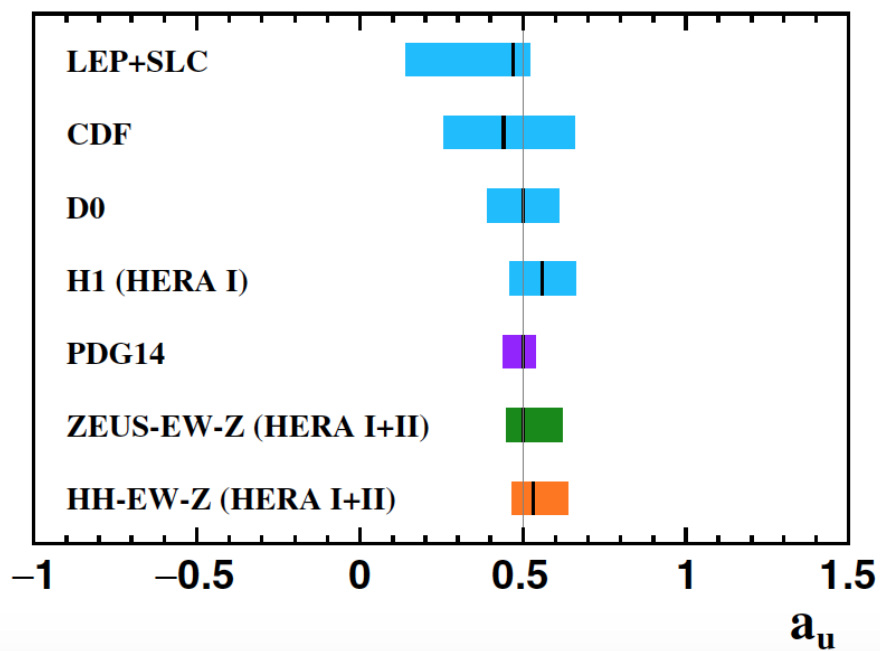
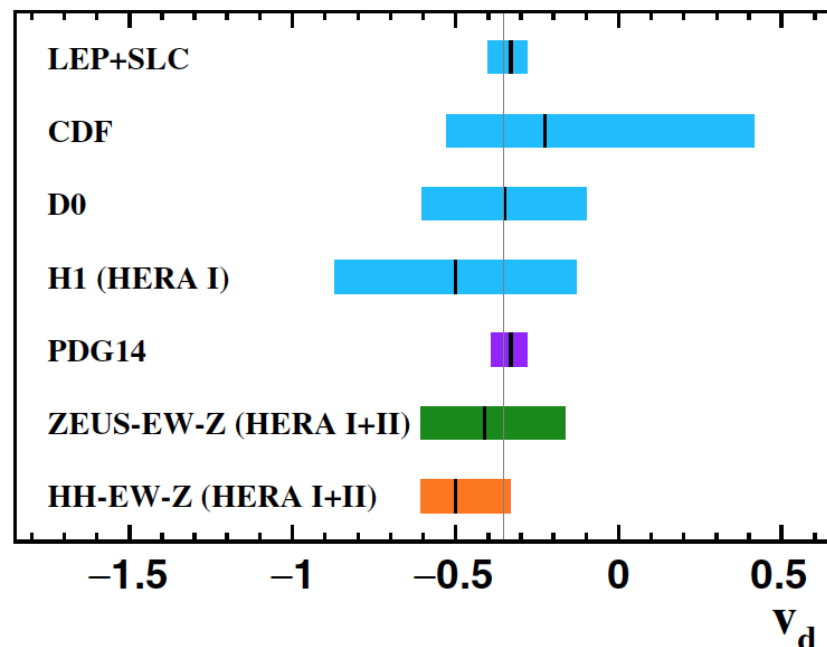
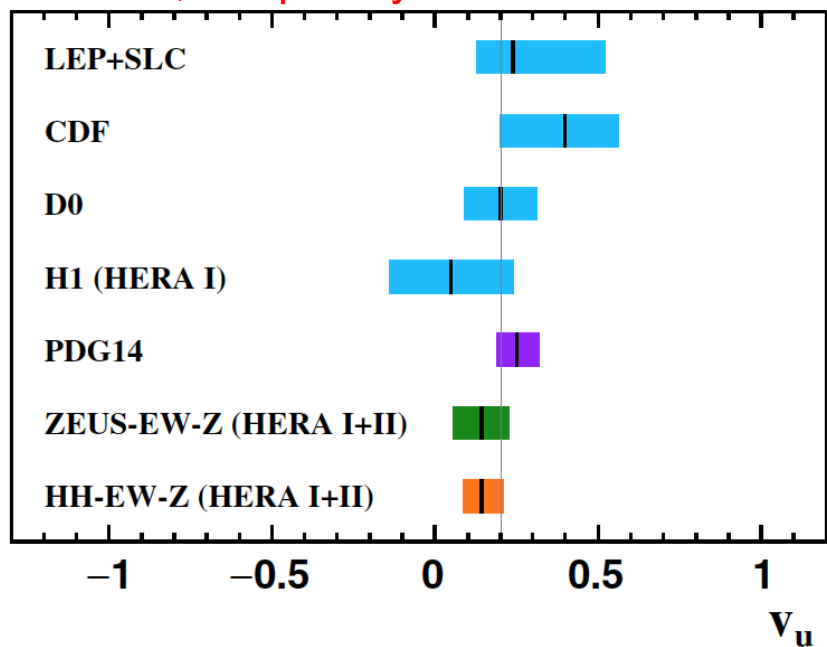
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# On $\sin^2\theta_W(+X)$ fits to DIS data

DIS inclusive cross sections depend on  $\sin^2\theta_W$  through:

- **Z propagator** in NC cross sections;
- **Vector couplings** of Z to quarks;

$$\tilde{F}_2^\pm = F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z$$

$$x\tilde{F}_3^\pm = -(a_e \pm P_e v_e) \chi_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \chi_Z^2 x F_3^Z$$

$$\chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2} \frac{1}{1 - \Delta R}$$

- **W propagator** ( $G_F$ );

$$\frac{d^2\sigma_{CC}(e^+p)}{dx_{Bj}dQ^2} = (1 + P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x[(\bar{u} + \bar{c}) + (1 - y)^2(d + s + b)]$$

$$\frac{d^2\sigma_{CC}(e^-p)}{dx_{Bj}dQ^2} = (1 - P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x[(u + c) + (1 - y)^2(\bar{d} + \bar{s} + \bar{b})]$$

$$G_F = \frac{\pi\alpha}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

$\Delta R$  is an EW correction.

[arXiv:hep-ph/9902277](https://arxiv.org/abs/hep-ph/9902277)

Re-expressing  $G_F$  through  $\sin^2\theta_W$  and  $M_W$  allows to use both CC and NC for  $\sin^2\theta_W$  determination.

- Current analysis exploits all three dependences for  $\sin^2\theta_W$  extraction.
- $\sin^2\theta_W$  values extracted in current analysis correspond to **On-shell scheme**.



## Quark couplings to Z

Now consider fits to electroweak NC couplings as well as PDF parameters

The total cross-section :  $\sigma = \sigma^0 + P \sigma^P$

The unpolarised cross-section is given by  $\sigma^0 = Y_+ F_2^0 + Y_- xF_3^0$

$$F_2^0 = \sum_i A_i^0(Q^2) [xq_i(x, Q^2) + xq_i(\bar{x}, Q^2)]$$

$$xF_3^0 = \sum_i B_i^0(Q^2) [xq_i(x, Q^2) - xq_i(\bar{x}, Q^2)]$$

$$A_i^0(Q^2) = e_i^2 - 2 e_i \mathbf{v}_i \mathbf{v}_e P_Z + (\mathbf{v}_e^2 + \mathbf{a}_e^2)(\mathbf{v}_i^2 + \mathbf{a}_i^2) P_Z^2$$

$$B_i^0(Q^2) = -2 e_i \mathbf{a}_i \mathbf{a}_e P_Z + 4 \mathbf{a}_i \mathbf{a}_e \mathbf{v}_i \mathbf{v}_e P_Z^2$$

$$P_Z = \frac{1}{\sin^2 2\theta} \frac{Q^2}{(M_Z^2 + Q^2)}$$

The polarised cross-section is given by  $\sigma^P = Y_+ F_2^P + Y_- xF_3^P$

$$F_2^P = \sum_i A_i^P(Q^2) [xq_i(x, Q^2) + xq_i(\bar{x}, Q^2)]$$

$$xF_3^P = \sum_i B_i^P(Q^2) [xq_i(x, Q^2) - xq_i(\bar{x}, Q^2)]$$

$$A_i^P(Q^2) = 2 e_i \mathbf{v}_i \mathbf{a}_e P_Z - 2 \mathbf{v}_e \mathbf{a}_e (\mathbf{v}_i^2 + \mathbf{a}_i^2) P_Z^2$$

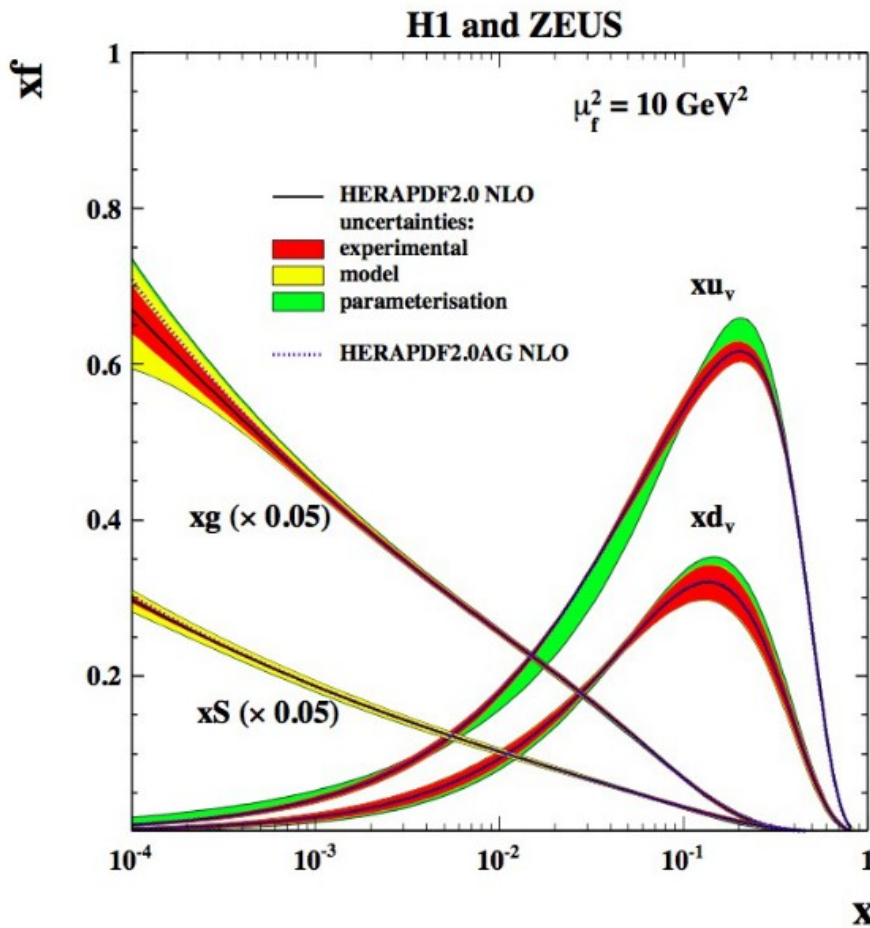
$$B_i^P(Q^2) = 2 e_i \mathbf{a}_i \mathbf{v}_e P_Z - 2 \mathbf{a}_i \mathbf{v}_i (\mathbf{v}_e^2 + \mathbf{a}_e^2) P_Z^2$$

$P_Z \gg P_Z^2$  ( $\gamma Z$  interference is dominant)  
 $\mathbf{v}_e$  is very small ( $\sim 0.04$ ).

→ unpolarized  $xF_3 \rightarrow \mathbf{a}_i$ ,  
 polarized  $F_2 \rightarrow \mathbf{v}_i$

From slides by Amanda Cooper-Sarkar

# Color decomposition of uncertainties



## Experimental uncertainties:

- Hessian method
- Conventional  $\Delta\chi^2 = 1 \Rightarrow 68\% \text{ CL}$

Variation	Standard Value	Lower Limit	Upper Limit
$Q_{\min}^2$ [GeV <sup>2</sup> ]	3.5	2.5	5.0
$Q_{\min}^2$ [GeV <sup>2</sup> ] HiQ2	10.0	7.5	12.5
$M_c$ (NLO) [GeV]	1.47	1.41	1.53
$M_c$ (NNLO) [GeV]	1.43	1.37	1.49
$M_b$ [GeV]	4.5	4.25	4.75
$f_s$	0.4	0.3	0.5
$\mu_{f_0}$ [GeV]	1.9	1.6	2.2

Adding D and E parameters to each PDF

## Parametrisation uncertainties

- largest deviation

## Model uncertainties

- all variations added in quadrature

# Fit methodology I

## ***Determine light-quark couplings***

- Use iterative minimisation procedure ('fit') of cross section predictions to data

## ***Unfortunate correlation***

- PDFs have considerable uncertainties
- These PDFs are essentially determined from H1 structure function data  
-> Large correlations
- Consider PDF uncertainty by simultaneous fit of PDFs and light quark couplings

## ***Consistency of fit-parameters in SM formalism***

- Perform calculations strictly in on-shell scheme  
Parameters are:  $\alpha$ ,  $m_Z$ ,  $m_W$ , ( $m_t$ ,  $m_H$ , ...)

## ***Polarisation measurement***

- Measurements of the beam polarisations are measurements on their own  
-> Consider these measurements as independent measurements in fit

## ***1-loop EW corrections***

- May be considered in terms of 'EW form factors'
- Are ignored in the present analysis, but will be included in the future

# Fit methodology II

## New C++-based fitting code for PDF and more general fits developed (Alpos)

- DGLAP evolution of PDFs in NNLO QCD (QCDNUM with ZMVFNS)
- PDFs are parameterised at starting scale  $Q_0^2 = 1.9\text{GeV}^2$  (similar to HERAPDF2.0)

$xg$	$xg$	$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}$	
$xu_v$	$xU = xu + xc$	$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2)$	<div style="display: flex; align-items: center; margin-bottom: 5px;"> <div style="width: 15px; height: 10px; background-color: #ccc; margin-right: 5px;"></div> <span>fixed or constrained by sum-rules</span> </div> <div style="display: flex; align-items: center;"> <div style="width: 15px; height: 10px; background-color: #add8e6; margin-right: 5px;"></div> <span>parameters set equal but free</span> </div>
$xd_v$	$xD = xd + xs$	$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$	
$x\bar{U}$	$x\bar{U} = x\bar{u} + x\bar{c}$	$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x)$	
$x\bar{D}$	$x\bar{D} = x\bar{d} + x\bar{s}$	$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$	

- Use only data with  $Q^2 \geq 12 \text{ GeV}^2$

## $\chi^2$ Definition

- Uncertainties on cross sections are assumed to be 'log-normal' distributed (relative uncertainties)
- Uncertainties on polarisation measurements are assumed to be 'normal' distributed
- Correlations of syst. uncertainties between different datasets are considered

$$\chi^2 = (\log(d) - \log(t))^T V_R^{-1} (\log(d) - \log(t)) + (d - t)^T V_A^{-1} (d - t)$$

## Fit parameters

- 13 PDF parameters
- 4 polarisation values
- 4 Light-quark couplings (or other SM parameters)
- More general also 'nuisance parameters' of syst. uncertainties

# Polarised deep-inelastic ep scattering

## Neutral and charged current at tree level

$$\frac{d\sigma_{NC}^{\pm}}{dQ^2 dx} = \frac{2\pi\alpha^2}{x} \left[ \frac{1}{Q^2} \right]^2 (Y_+ F_2 + Y_- x F_3 + y^2 F_L)$$

$$\frac{d\sigma_{CC}^{\pm}}{dQ^2 dx} = \frac{1 \pm P}{2} \frac{G_F^2}{4\pi x} \left[ \frac{m_W^2}{m_W^2 + Q^2} \right]^2 (Y_+ W_2^{\pm} \pm Y_- x W_3^{\pm} - y^2 W_L^{\pm})$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

## Calculations in on-shell scheme

$$G_F = \frac{2\pi\alpha}{2\sqrt{2}m_W^2} \left( 1 - \frac{m_W^2}{m_Z^2} \right)^{-1} (1 + \Delta r)$$

## Corrections to $G_F$

$$\Delta r = \Delta r(\alpha, m_W, m_Z, m_t, m_H, \dots)$$

## Parameters to calculations

Parameters to cross section calculation:  $\alpha, m_Z, m_W (m_t, m_H, \dots)$

More general, also couplings:  $v_e, a_e, v_u, a_u$  and  $v_d, a_d$

## Generalised structure functions

$$F_2 = F_2^y + \kappa_Z (-v_e \mp P a_e) F_2^{yZ} + \kappa_Z^2 (v_e^2 + a_e^2 \pm P v_e a_e) F_2^Z$$

$$x F_3 = +\kappa_Z (\pm a_e + P v_e) F_3^{yZ} + \kappa_Z^2 (\mp 2 v_e a_e - P (v_e^2 + a_e^2)) x F_3^Z$$

## Z<sup>0</sup>-exchange

$$\kappa_Z(Q^2) = \frac{Q^2}{Q^2 + m_Z^2} \frac{G_F m_Z^2}{2\sqrt{2}\pi\alpha}$$

## Structure functions in QPM

$$[F_2, F_2^{yZ}, F_2^Z] = x \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] \{q + \bar{q}\}$$

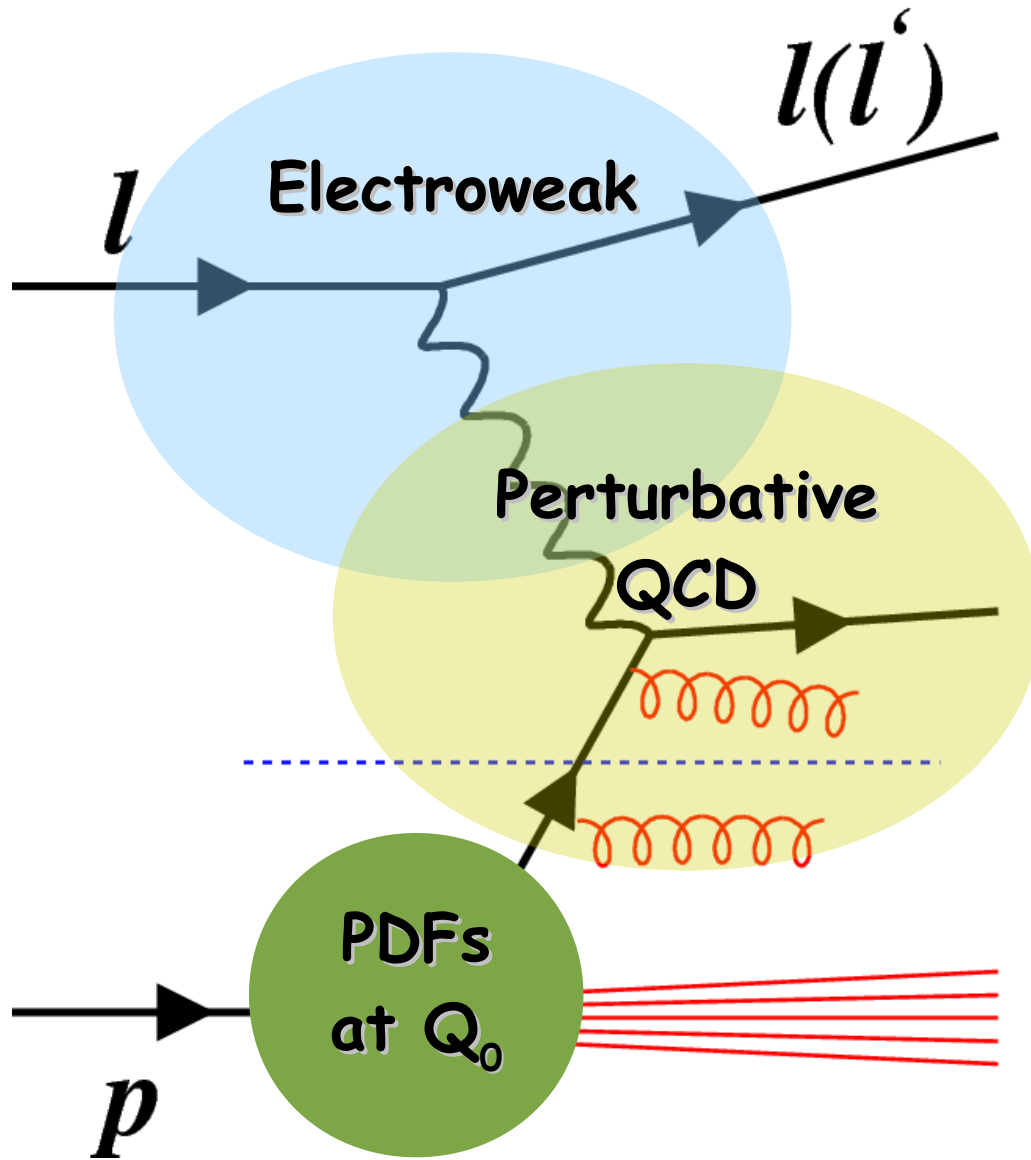
$$[x F_3^{yZ}, x F_3^Z] = x \sum_q [2e_q a_q, 2v_q a_q] \{q - \bar{q}\}$$

## Weak couplings to Z-boson

$$v_f = I_{f,L}^{(3)} - 2e_f \sin^2 \theta_W \quad (f = e, u, d, \dots)$$

$$a_f = I_{f,L}^{(3)}$$

# Deep Inelastic Scattering @ HERA



- Fix pQCD & PDFs  
! Test Electroweak
- Fix Electroweak  
! Test pQCD & PDFs

- Fix Electroweak & pQCD  
! Determine PDFs