

Combination of beauty and charm production cross section measurements in deep inelastic ep scattering at HERA

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H1prelim-17-071

ZEUS-prel-17-01

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## Experimental set-up

## HERA Collider

- ep collisions
- $\sqrt{s} = 300 \dots 318 \text{ GeV}$  and lower energy runs

### H1 and ZEUS:

- $4\pi$  multipurpose detectors
- $\mathcal{L} \sim 500 \text{ pb}^{-1}$ per each experiment



$$E_p = 920 \, GeV \qquad E_e = 27.5 \, GeV$$

$$\sqrt{s} = 318 \, GeV$$

## Kinematics



Any two of the variables  $(Q^2, x, y)$  define kinematics

 $Q^2 > 1 \ GeV^2$  — deep inelastic scattering (DIS)  $Q^2 < 1 \ GeV^2$  — photoproduction processes (PHP)

## Heavy flavour (HF) production in DIS



Production is directly sensitive to g PDF in the proton and to HQ masses

Overview of experimental techniques used to measure charm and beauty production at HERA

## Measurement of charm production at HERA

O. Zenaiev

## "Golden" decay channel $D^* ightarrow D^0(K\pi)\pi_s$



Dedicated H1ZEUS combination: "Combination of differential  $D^{*\pm}$  cross-section measurements in deep-inelastic ep scattering at HERA" [JHEP09 (2015) 149]



## Measurement of charm and beauty production at HERA



Recent reviews of HF production at HERA:

- O. Behnke, A. Geiser, M. Lisovyi, "Charm, Beauty and Top at HERA", Prog. Part. Nucl. Phys. 84 (2015) 1
- O.Z., "Charm Production and QCD Analysis at HERA and LHC ", Eur. Phys. J. C77 (2017) 151

## Combination of charm and beauty HERA data

## Combination procedure

- fiducial cross sections extrapolated to full phase space using consistent NLO predictions [HVQDIS], account for relevant unc.
- combined at the level of **reduced cross sections**  $\sigma_{\text{red}}^{c\bar{c}}$ ,  $\sigma_{\text{red}}^{b\bar{b}}$   $\sigma_{\text{red}}^{Q\bar{Q}} = \frac{d^2 \sigma^{Q\bar{Q}}}{dx_{\text{Bj}} dQ^2} \cdot \frac{x_{\text{Bj}}Q^4}{2\pi\alpha^2 (1+(1-\underline{y})^2)}$  (full phase space)  $(Q\bar{Q} \text{ stands either for } c\bar{c} \text{ or } b\bar{b})$
- $\bullet$  combination accounts for correlation of systematic uncertainties, as well as correlation of c and b from same measurements
- $\Rightarrow$  significant improvement in precision via cross calibration of different measurement techniques and c/b

## Combined using HERAverager program

[https://wiki-zeuthen.desy.de/HERAverager] well established combination method used in:

- previous HERA charm combination [EPJ C73 (2013) 2311]
- HERAPDF2.0 [EPJ C75 (2015) 580]
- ATLAS papers [1603.09222, 1512.02192, 1606.01736, 1612.03016]

## Input data

| Data set |  | Tagging  | $Q^2$ range |                  | $N_c$ | L                   | $\sqrt{s}$ | $N_b$ |
|----------|--|----------|-------------|------------------|-------|---------------------|------------|-------|
|          |  |          | [Ge         | V <sup>2</sup> ] |       | [pb <sup>-1</sup> ] | [GeV]      |       |
| 1        | H1 VTX [8]                               | VTX      | 5 –         | 2000             | 29    | 245                 | 318        | 12    |
| 2        | H1 D*+ HERA-I [9]                        | $D^{*+}$ | 2 -         | 100              | 17    | 47                  | 318        |       |
| 3        | H1 $D^{*+}$ HERA-II (medium $Q^2$ ) [10] | $D^{*+}$ | 5 –         | 100              | 25    | 348                 | 318        |       |
| 4        | H1 $D^{*+}$ HERA-II (high $Q^2$ ) [11]   | $D^{*+}$ | 100 -       | 1000             | 6     | 351                 | 318        |       |
| 5        | ZEUS D*+ 96-97 [12]                      | $D^{*+}$ | 1 –         | 200              | 21    | 37                  | 300        |       |
| 6        | ZEUS D*+ 98-00 [13]                      | $D^{*+}$ | 1.5 –       | 1000             | 31    | 82                  | 318        |       |
| 7        | ZEUS D <sup>0</sup> 2005 [14]            | $D^0$    | 5 –         | 1000             | 9     | 134                 | 318        |       |
| 8        | ZEUS µ 2005 [7]                          | μ        | 20 -        | 10000            | 8     | 126                 | 318        | 8     |
| 9        | ZEUS D <sup>+</sup> HERA-II [2]          | $D^+$    | 5 -         | 1000             | 14    | 354                 | 318        |       |
| 10       | ZEUS D*+ HERA-II [3]                     | $D^{*+}$ | 5 –         | 1000             | 31    | 363                 | 318        |       |
| 11       | ZEUS VTX HERA-II [4]                     | VTX      | 5 -         | 1000             | 18    | 354                 | 318        | 17    |
| 12       | ZEUS e HERA-II [5]                       | e        | 10 -        | 1000             |       | 363                 | 318        | 9     |
| 13       | ZEUS $\mu$ + jet HERA-I [6]              | μ        | 2 -         | 3000             |       | 114                 | 318        | 11    |

(corresponding references can be found in backup)

- Combined data provided in kinematic range:  $2.5 \le Q^2 \le 2000 \text{ GeV}^2$ ,  $3 \times 10^{-5} \le x_{\text{Bj}} \le 5 \times 10^{-2}$
- Input 209 c, 52 b data points  $\Rightarrow$  combined 52 c, 27 b points
- Extends previous HERA charm combination with 3 new c data sets and 5 new b: first combination of HERA b data

## Combined data

**CHARM** 

BEAUTY



 $\chi^2/dof = 149/187$ : input data are consistent

## **CHARM**



Significantly improved precision compared to input measurements

## BEAUTY



Significantly improved precision compared to input measurements

## Predictions calculated with OPENQCDRAD interfaced in xFitter

www-zeuthen.desy.de/ $\sim$ alekhin/OPENQCDRAD

www.xfitter.org

- input PDFs: HERAPDF2.0FF3A, ABM11, ABMP16, or fitted
- NLO or approx. NNLO as implemented in OPENQCDRAD
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ , varied by factor 2 (dominant unc.)
- $m_c(m_c) = 1.27 \pm 0.03$  GeV,  $m_b(m_b) = 4.18 \pm 0.03$  GeV [PDG2016], or fitted

FFN scheme,  $n_f = 3$ : reliable in this kinematic range

#### CHARM HERA (prel.) O HERAPDE2 0 FE3A H1 and ZEUS appr. NNLO ABMP16 NLO ABM11 preliminary NLO fit DIS + c + b $Q^2 = 5 \text{ GeV}^2$ $Q^2 = 7 \text{ GeV}^2$ 5 Cred 0.3 0.2 0.1 $Q^2 = 18 \text{ GeV}^2$ $O^2 = 32 \text{ GeV}^2$ $Q^2 = 12 \text{ GeV}^2$ 0.4 0.2 $0.6 - Q^2 = 60 \text{ GeV}^2$ Q<sup>2</sup> = 120 GeV<sup>2</sup> Q<sup>2</sup> = 200 GeV<sup>2</sup> 0.4 0.2 $0.6 - Q^2 = 350 \text{ GeV}^2$ $\Omega^2 = 650 \text{ GeV}^2$ 0<sup>2</sup> - 2000 GeV<sup>2</sup> 0.4 0.2 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-2</sup> 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-2</sup> 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-1</sup>

Overall reasonable description, some x slope at low and medium  $Q^2$ 

14/23

# Predictions calculated with OPENQCDRAD interfaced in xFitter

www-zeuthen.desy.de/~alekhin/OPENQCDRAD

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FFN scheme,  $n_f = 3$ : reliable in this kinematic range



Overall reasonable description, some x slope at low and medium  $Q^2$ Same in previous H1ZEUS charm combination, but within larger unc.

# Predictions calculated with OPENQCDRAD interfaced in xFitter

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www.xfitter.org

- input PDFs: HERAPDF2.0FF3A, ABM11, ABMP16, or fitted
- NLO or approx. NNLO as implemented in OPENQCDRAD
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ , varied by factor 2 (dominant unc.)
- $m_c(m_c) = 1.27 \pm 0.03$  GeV,  $m_b(m_b) = 4.18 \pm 0.03$  GeV [PDG2016], or fitted

FFN scheme,  $n_f = 3$ : reliable in this kinematic range



## Overall reasonable description, some x slope at low and medium $Q^2$ Small sensitivity to PDFs, appr. NNLO do not improve description

## Predictions calculated with OPENQCDRAD interfaced in xFitter

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- input PDFs: HERAPDF2.0FF3A, ABM11, ABMP16, or fitted
- NLO or approx. NNLO as implemented in OPENQCDRAD
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ , varied by factor 2 (dominant unc.)
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FFN scheme,  $n_f = 3$ : reliable in this kinematic range

#### BEAUTY HERA (prel.) O HERAPDF2.0 FF3A H1 and ZEUS appr. NNLO ABMP16 II O ARM11 preliminary NLO fit DIS + c + b 5<sup>bb</sup> $Q^2 = 2.5 \text{ GeV}^2$ $Q^2 = 5 \text{ GeV}^2$ $Q^2 = 7 \text{ GeV}^2$ 0.01 0.005 $Q^2 = 18 \text{ GeV}^2$ $Q^2 = 32 \text{ GeV}^2$ = 12 GeV<sup>2</sup> 0.04 0.02 $Q^2 = 60 \text{ GeV}^2$ $Q^2 = 120 \text{ GeV}^2$ $Q^2 = 200 \text{ GeV}^2$ 0.04 0.02 $Q^2 = 350 \text{ GeV}^2$ $Q^2 = 650 \text{ GeV}^2$ Q<sup>2</sup> = 2000 GeV<sup>2</sup> 0.04 0.02 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-2</sup> 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-2</sup> 10<sup>-4</sup> 10<sup>-3</sup> 10<sup>-2</sup>

## Overall good description within data uncertainties

# Predictions calculated with OPENQCDRAD interfaced in xFitter

www-zeuthen.desy.de/ $\sim$ alekhin/OPENQCDRAD

www.xfitter.org

- input PDFs: HERAPDF2.0FF3A, ABM11, ABMP16, or fitted
- NLO or approx. NNLO as implemented in OPENQCDRAD
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ , varied by factor 2 (dominant unc.)
- $m_c(m_c) = 1.27 \pm 0.03$  GeV,  $m_b(m_b) = 4.18 \pm 0.03$  GeV [PDG2016], or fitted

FFN scheme,  $n_f = 3$ : reliable in this kinematic range



## Overall good description within data uncertainties Small sensitivity to PDFs and higher order corrections

| Dataset          | PDF                    | $\chi^2$ | $\chi^2$ with PDF unc. |
|------------------|------------------------|----------|------------------------|
| HERA 2012 c [1]  | HERAPDF20_NLO_FF3A_EIG | 59       | 59                     |
| 11ERA 2012 C [1] | abm11_3n_nlo           | 62       | 62                     |
| (dof = 52)       | ABMP16_3_nnlo          | 64       | 63                     |
| New combined c   | HERAPDF20_NLO_FF3A_EIG | 86       | 85                     |
| riew combined e  | abm11_3n_nlo           | 92       | 91                     |
| (dof = 52)       | ABMP16_3_nnlo          | 101      | 99                     |
| ZEUS VTX 6 [4]   | HERAPDF20_NLO_FF3A_EIG | 14       | 14                     |
| ZE03 VIX 0 [4]   | abm11_3n_nlo           | 13       | 13                     |
| (dof = 17)       | ABMP16_3_nnlo          | 14       | 14                     |
| New combined h   | HERAPDF20_NLO_FF3A_EIG | 33       | 33                     |
| new combined b   | abm11_3n_nlo           | 34       | 34                     |
| (dof = 27)       | ABMP16_3_nnlo          | 39       | 39                     |

[1] previous HERA charm combination EPJ C73 (2013) 2311

[4] ZEUS *b* lifetime tagging measurement JHEP09 (2014) 127

(most precise individual public data sets for c and b from HERA to date)

## Quantitatively confirms observed findings:

- larger tension for new charm data owing to reduced uncertainties
- appr. NNLO does not improve data description compared to NLO
- overall small sensitivity to input PDFs

# QCD analysis and determination of charm and beauty masses

## QCD analysis of combined charm and beauty data

## Similar to HERAPDF2.0 FF:

- performed using xFitter [www.xfitter.org]
- inclusive HERA data + new combined c&b data
- NLO DGLAP [QCDNUM] and matrix elements [OPENQCDRAD],  $n_f = 3$

• 
$$\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$$
 varied by factor 2 (model unc.)

• free  $m_c(m_c)$ ,  $m_b(m_b)$ 

• 
$$\alpha_s(M_Z)^{n_f=3} = 0.106 \ (\to \alpha_s(M_Z)^{n_f=5} = 0.118)$$

- HERAPDF parametrisation, 14p
- fit uncertainty using  $\Delta\chi^2=1$
- model and parametrisation uncertainties

 $m_c(m_c) = 1290^{+46}_{-41}(\text{fit})^{+62}_{-14}(\text{mod})^{+7}_{-31}(\text{par}) \text{ MeV}$  $m_b(m_b) = 4049^{+104}_{-109}(\text{fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par}) \text{ MeV}$  $\Rightarrow$  determined precise HQ masses consistent with world average

PDG2016:  $m_c(m_c) = 1270 \pm 30$  MeV,  $m_b(m_b) = 4180^{+40}_{-30}$  MeV

## Discussion of HQ mass extraction

$$m_c(m_c) = 1290^{+46}_{-41}(\text{fit})^{+62}_{-14}(\text{mod})^{+7}_{-31}(\text{par}) \text{ MeV}$$

 $m_b(m_b) = 4049^{+104}_{-109} (\text{fit})^{+90}_{-32} (\text{mod})^{+1}_{-31} (\text{par}) \text{ MeV}$ 

Results have sizable model and parametrisation uncertainty:

- model uncertainties dominated by scale variations
- *parametrisation* uncertainties dominated by reduced 13p form: closely related to inclusive HERA data in the fit

Using inclusive HERA data only:

$$m_c(m_c) = 1798^{+144}_{-134}$$
(fit) MeV

$$m_b(m_b) = 8450^{+2280}_{-1810}$$
(fit) MeV

No full uncertainty evaluation, but observed large sensitivity to PDF parametrisation ( $\rightarrow$  13p):

$$m_c(m_c) = 1798 \rightarrow 1450 \text{ MeV},$$
  
 $m_b(m_b) = 8450 \rightarrow 3995 \text{ MeV}$ 

$$\begin{split} & xg(x) = A_g x^{Bg} (1-x)^{Cg} - A'_g x^{B'g} (1-x)^{C'g} \\ & xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1+E_{u_v}) \\ & xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}} \\ & x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1+D_{\bar{U}}x) \\ & x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}} \end{split}$$

13p:  $E_{u_v} = 0$ 

 $\Rightarrow$  inclusive HERA data alone cannot constrain HQ masses

 $\Rightarrow$  interplay of PDFs and HQ masses needs carefull treatment

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## Summary

## New preliminary combined HERA HQ data:

- 11 c + 5 b input data sets  $\Rightarrow$  c + b sets with full cross correlations
- input data are consistent
- improvement in precision w.r.t previous HERA results on charm
- first combined HERA results on beauty

## Comparison to theory and QCD analysis:

- $\bullet\,$  overall reasonable description, some tension in x slope for charm
- precise extraction of  $m_c(m_c)$ ,  $m_b(m_b)$

H1prelim-17-071, ZEUS-prel-17-01

https://www.desy.de/h1zeus/combined\_results/index.php?do=heavy\_flavours



## BACKUP. Input data

- [2] H. Abramowicz et al. [ZEUS Collaboration], "Measurement of D<sup>±</sup> Production in Deep Inelastic ep Scattering with the ZEUS detector at HERA", JHEP 05, (2013) 023 [arXiv:1302.5058].
- [3] H. Abramowicz et al. [ZEUS Collaboration], "Measurement of D<sup>e±</sup> Production in Deep Inelastic Scattering at HERA", JHEP 05, (2013) 097 [arXiv:1303.6578]. Erratum-ibid JHEP 02, (2014) 106.
- [4] H. Abramowicz et al. [ZEUS Collaboration], "Measurement of beauty and charm production in deep inelastic scattering at HERA and measurement of the beauty-quark mass", JHEP 09, (2014) 127 [arXiv:1405.6915].
- [5] H. Abramowicz et al. [ZEUS Collaboration], "Measurement of beauty production in deep inelastic scattering at HERA using decays into electrons", Eur. Phys. J. C71, (2011) 1573 [arXiv:1101.3692].
- [6] H. Abramowicz et al. [ZEUS Collaboration], "Measurement of beauty production in DIS and F2bb extraction at ZEUS", Eur. Phys. J. C69, (2010) 347 [arXiv:1005.3396].
- [7] S. Chekanov et al. [ZEUS Collaboration], "Measurement of charm and beauty production in deep inelastic ep scattering from decays into muons at HERA", Eur. Phys. J. C65, (2010) 65 [arXiv:0904.3487].
- [8] F. D. Aaron et al. [H1 Collaboration], "Measurement of the Charm and Beauty Structure Functions using the H1 Vertex Detector at HERA", Eur. Phys. J. C65, (2010) 89 [arXiv:0907.2643].
- [9] A. Aktas et al. [H1 Collaboration], "Production of D\*+- Mesons with Dijets in Deep-Inelastic Scattering at HERA", Eur. Phys. J. C51, (2007) 271 [hep-ex/0701023].
- [10] F. D. Aaron *et al.* [H1 Collaboration], "Measurement of D\*<sup>±</sup> Meson Production and Determination of F<sub>2</sub><sup>±km</sup> at low Q2 in Deep-Inelastic" Eur. Phys. J. C71, (2011) 1769 [arXiv:106.1028].
- [11] F. D. Aaron et al. [H1 Collaboration], "Measurement of the D\*+- Meson Production Cross Section and F(2)\*\*(c c-bar), at High Q\*\*2, in ep Scattering at HERA", Phys. Lett. B686, (2010) 91 [arXiv:0911.3989].
- [12] J. Breitweg et al. [ZEUS Collaboration], "Measurement of D\*+- production and the charm contribution to F2 in deep inelastic scattering at HERA", Eur. Phys. J. C12, (2000) 35 [hep-ex/9908012].
- [13] S. Chekanov et al. [ZEUS Collaboration], "Measurement of D\*+- production in deep inelastic e+- p scattering at HERA", Phys. Rev. D69, (2004) 012004 [hep-ex/0308068].
- [14] S. Chekanov et al. [ZEUS Collaboration], "Measurement of D+- and D0 production in deep inelastic scattering using a lifetime tag at HERA", Eur. Phys. J. C63, (2009) 171 [arXiv:0812.3775].

## BACKUP. Combination procedure

- Take measured visible x-section  $\sigma_{\rm vis}$  and extrapolate to full phase space  $\sigma_{\rm red}$  using consistent NLO setup:  $\sigma_{\rm red} = \sigma_{\rm vis} \frac{\sigma_{\rm red}^{\rm NLO}}{\sigma_{\rm NLO}^{\rm NLO}}$  [HVQDIS]
- Combine  $\sigma_{\rm red}$  accounting for bin-to-bin correlations [HERAverager]

### NLO setup for extrapolation as in [DESY-12-172]

- pole masses  $m_c = 1.5 \pm 0.15$  GeV,  $m_b = 4.5 \pm 0.25$  GeV consistent with extracted from data:  $m_c = 1.43 \pm 0.04$  GeV,  $m_b = 4.35 \pm 0.11$  GeV and consistent with PDG:  $m_c = 1.67 \pm 0.07$  GeV,  $m_b = 4.78 \pm 0.06$  GeV
- $\mu_R = \mu_F = \sqrt{Q^2 + 4m_Q^2}$ , varied simultaneously by factor 2
- $\alpha_s^{n_f=3}(M_Z) = 0.105 \pm 0.002 \ [\alpha_s^{n_f=5}(M_Z) = 0.116 \pm 0.002]$
- HERAPDF1.0 FFNS, n<sub>f</sub> = 3, assign 2% uncor. unc. (checked vs HERAPDF2.0: see backup)
- c fragmentation: Kartvelishvili frag. function parametrised as step function with  $k_T$  kink (H1, ZEUS meas. [DESY-08-080, DESY-08-209])
- *b* fragmentation: Peterson  $\epsilon_b = 0.0035 \pm 0.0020$  [NP B565 (2000) 245]
- charm fragmentation fractions [EPJ C76 (2016) 397]
- branching ratios PDG2016
- hadronisation uncertainties for data with jets in the final state

## BACKUP. Data combination

$$\chi^{2}(\mathbf{m}, \mathbf{b}) = \sum_{e=1}^{N_{e}} \sum_{i=1}^{N_{m}} \frac{\left(m_{i} - \sum_{j=1}^{N_{s}} \Gamma_{i}^{e, j} b^{e, j} - \mu_{i}^{e}\right)^{2}}{\sigma_{i}^{e^{2}}} + \sum_{j=1}^{N_{s}} b^{e, j^{2}}$$

Minimised in iterative procedure

Similar to HERAPDF2.0 FF, using running HQ mass definition:

- xFitter-1.2.0
- Input data:
  - HERA  $e^\pm p$  inclusive data,  $Q^2_{\rm min} > 3.5~{\rm GeV}^2~{\rm [1506.06042]}$
  - $\bullet\,$  new HERA c and b combined
- FFNS  $n_f = 3$  ('FF ABM RUNM'),  $(\alpha_s(F_L) = \alpha_s(F_2))$

• 
$$\alpha_s^{n_f=3}(M_Z) = 0.106$$

• free  $m_c(m_c)$ ,  $m_b(m_b)$ , or PDG  $m_c(m_c) = 1.27$  GeV,  $m_c(m_c) = 4.18$  GeV

- DGLAP NLO [QCDNUM]
- PDF parametrisation: 14p HERAPDF at  $\mu_{f0}^2 = 1.9 \text{ GeV}^2$ ,  $f_s = 0.4$ :

$$\begin{split} & xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g} \\ & xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1+E_{u_v} x^2) \\ & xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}} \\ & x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1+D_{\bar{U}} x) \\ & x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}} \\ \end{split}$$
  $\begin{aligned} & \text{Additional constrains:} \\ & A_{\bar{U}} = A_{\bar{D}} (1-f_s), \ B_{\bar{U}} = B_{\bar{D}}, \ C'_g = 25 \\ & \int_0^1 [\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)] x dx = 1 \\ & \int_0^1 [u(x) - \bar{u}(x)] dx = 2, \\ & \int_0^1 [d(x) - \bar{d}(x)] dx = 1 \end{aligned}$ 

• fit ( $\Delta\chi^2 = 1$ ), model (scales,  $\alpha_s$ ,  $f_s$ ,  $Q^2_{\min}$ ) and par. ( $\mu_{f0}$ ,  $E_{u_v} = 0$ ) unc.

## BACKUP. pQCD approximation of heavy flavour production

## Fixed Flavour Number Scheme (FFNS)

- $\bullet\,$  c,b-quarks are massive  $\Rightarrow\,$  not a part of the proton, produced perturbatively in hard scattering
- $\bullet$  valid for  $Q^2 \sim m_{c,b}^2$

### Zero Mass Variable Flavour Number Scheme (ZMVFNS)

- $\bullet\,$  c,b-quarks are massless  $\Rightarrow$  a part of the proton
- valid for  $Q^2 >> m_{c,b}^2$

## General Mass Variable Flavour Number Scheme (GMVFNS)

- equivalent to FFNS at low  $Q^2\,$
- equivalent to ZMVFNS at high  $Q^2$
- not unique (RT, ACOT, ...)

## BACKUP. $m_c(m_c)$ extraction in FFNS and VFNS

| IHEP 1608 (2016) 050  | variation                                | FONLL-C         | FFN             |
|-----------------------|--|-----------------|-----------------|
| 51121 1000 (2010) 050 | central                                  | $1.335\pm0.043$ | $1.318\pm0.054$ |
|                       | $Q_0^2 = 1.5$                            | 1.354 [+0.019]  | 1.329 [+0.011]  |
| 0                     | $D_{uv}$ non-zero                        | 1.340 [+0.005]  | 1.308 [-0.010]  |
| <b></b>               | $f_s = 0.3$                              | 1.338 [+0.003]  | 1.320 [+0.002]  |
|                       | $f_{s} = 0.5$                            | 1.332 [-0.003]  | 1.315 [-0.003]  |
| vEittor               | $\overline{m_b(m_b)} = 3.93 \text{ GeV}$ | 1.330 [-0.005]  | 1.312 [-0.006]  |
| XFRIEI                | $m_b(m_b) = 4.43 \text{ GeV}$            | 1.343 [+0.008]  | 1.324 [+0.006]  |
|                       | $\alpha_s(M_Z) = 0.1165$                 | 1.342 [+0.007]  | 1.332 [+0.014]  |
| 0                     | $\alpha_s(M_Z) = 0.1195$                 | 1.329 [-0.006]  | 1.300 [-0.018]  |
| ADCEI                 | $\mu_F^2 = \mu_R^2 = 2 \cdot Q^2$        | 1.347 [+0.012]  | 1.314 [-0.004]  |
| ACTLL                 | $\mu_F^2 = \mu_R^2 = Q^2/2$              | 1.361 [+0.026]  | 1.363 [+0.045]  |
|                       | FONLL Damping power $= 1$                | 1.352 [+0.017]  | -               |
|                       | FONLL Damping power $= 4$                | 1.327 [-0.008]  | -               |

## A determination of $m_c(m_c)$ from HERA data using a matched heavy-flavor scheme

- consistent results obtained in FFNS and FONLL, with somewhat different decomposition of uncertainties
- $\Rightarrow$  VFNS can be used for  $\overline{\rm MS}$  mass extraction, if all uncertainties from extra parameters are considered