

# Combination and QCD analysis of charm and beauty

## production cross sections in DIS at HERA



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DESY Hamburg  
for the H1 and ZEUS  
collaborations

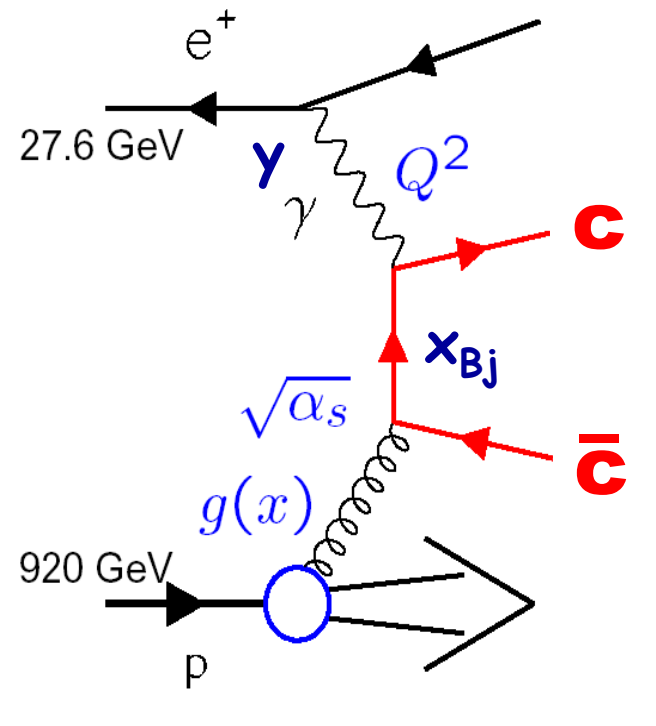
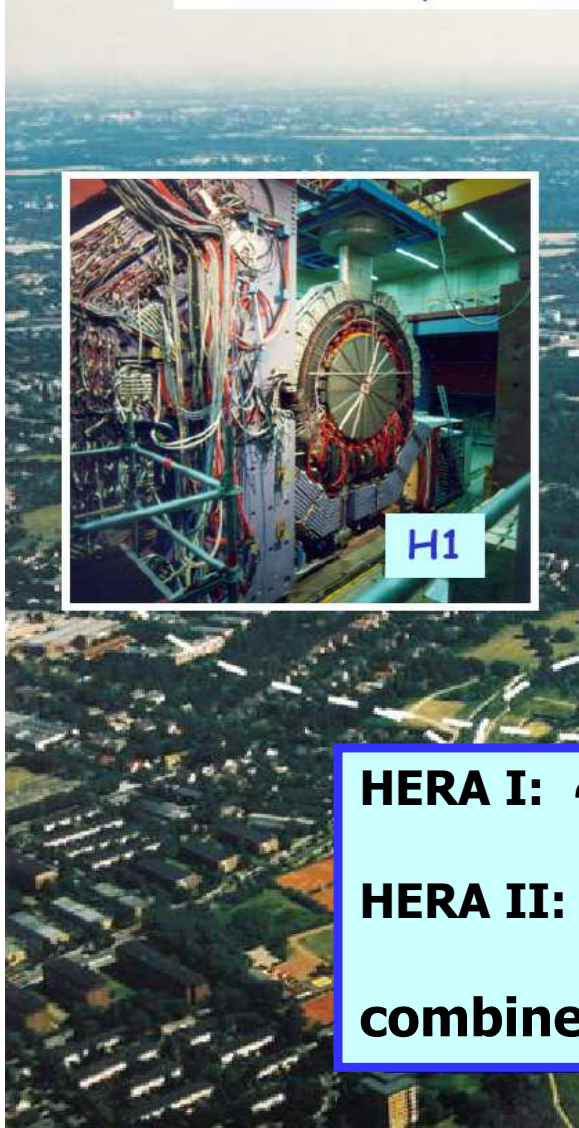


DIS 2018, Kobe, Japan, 18 April 2018

- Introduction: heavy flavours in DIS
- Combination of charm and beauty data
- Comparison with QCD predictions
- QCD fit and determination of  $m_c(m_c)$  and  $m_b(m_b)$
- Discussion

final results  
freshly released:  
arXiv:1804.01019

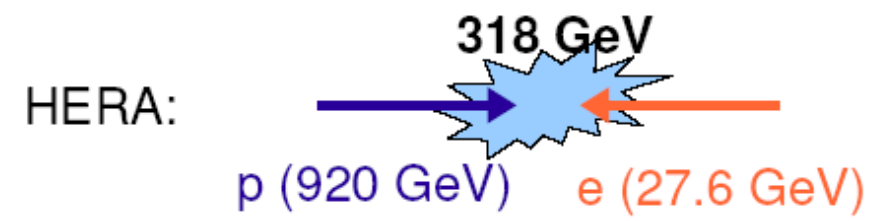
# The HERA ep collider and experiments



up to 30%  
of cross section

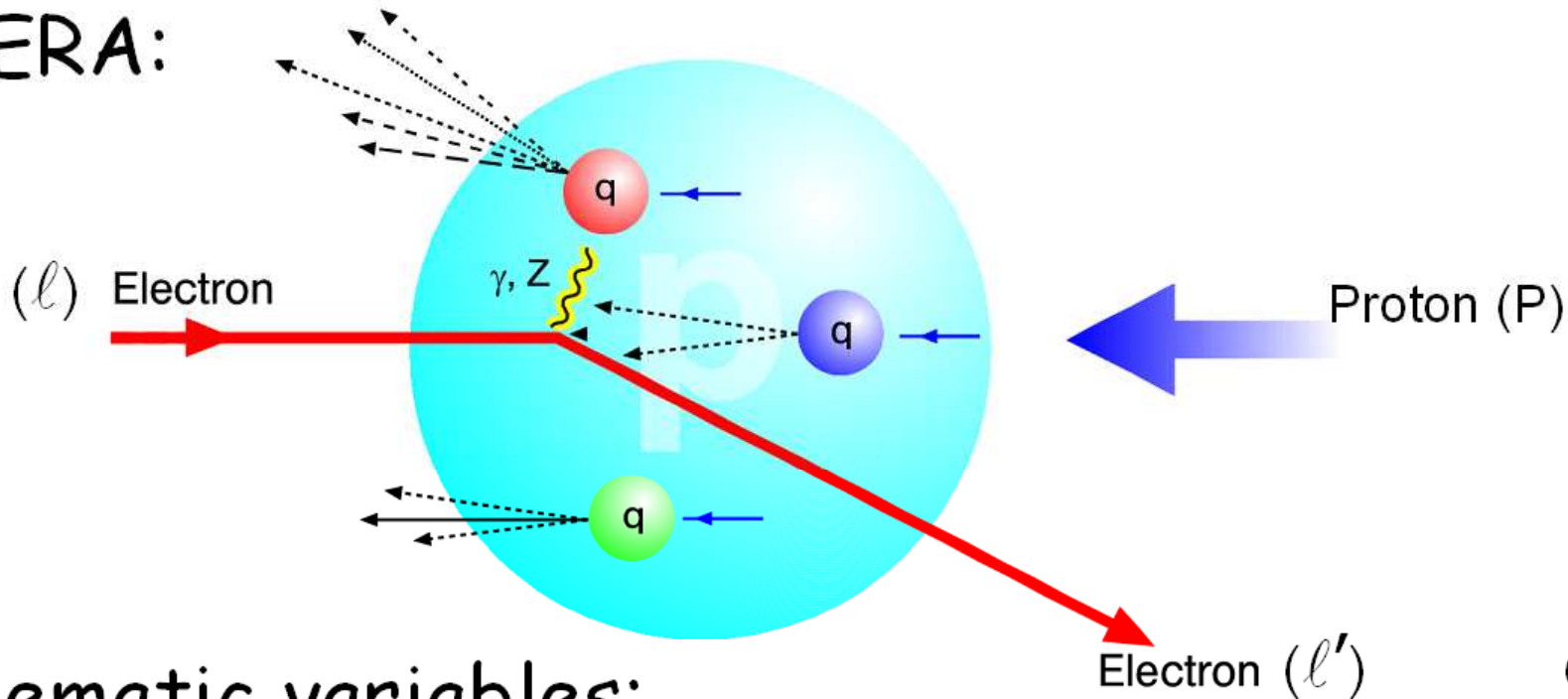


**HERA I:  $\sim 130 \text{ pb}^{-1}$  (physics)**  
**HERA II:  $\sim 380 \text{ pb}^{-1}$  (physics)**  
**combined:  $\sim 2 \times 0.5 \text{ fb}^{-1}$**



# Deep Inelastic ep Scattering at HERA

HERA:



kinematic variables:

$$q = l - l'$$

$Q^2 = -q^2$  photon (or  $Z$ ) virtuality, squared momentum transfer

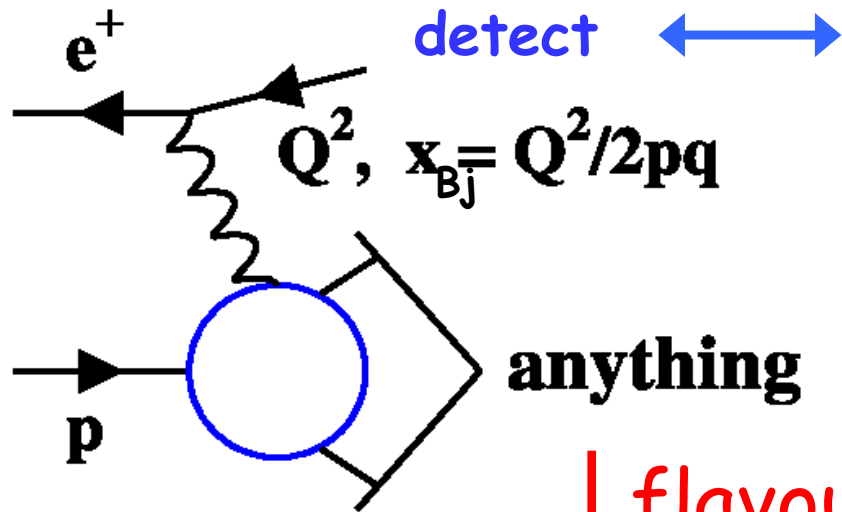
$x_{Bj} = \frac{Q^2}{2Pq}$  Bjorken scaling variable, for  $Q^2 \gg (2m_q)^2$ : momentum fraction of  $p$  constituent (equivalent in LO QPM only)

$y = \frac{qP}{lP}$  inelasticity,  $\gamma$  momentum fraction (of  $e$ )

$Q^2 \lesssim 1 \text{ GeV}^2$ :  
photoproduction

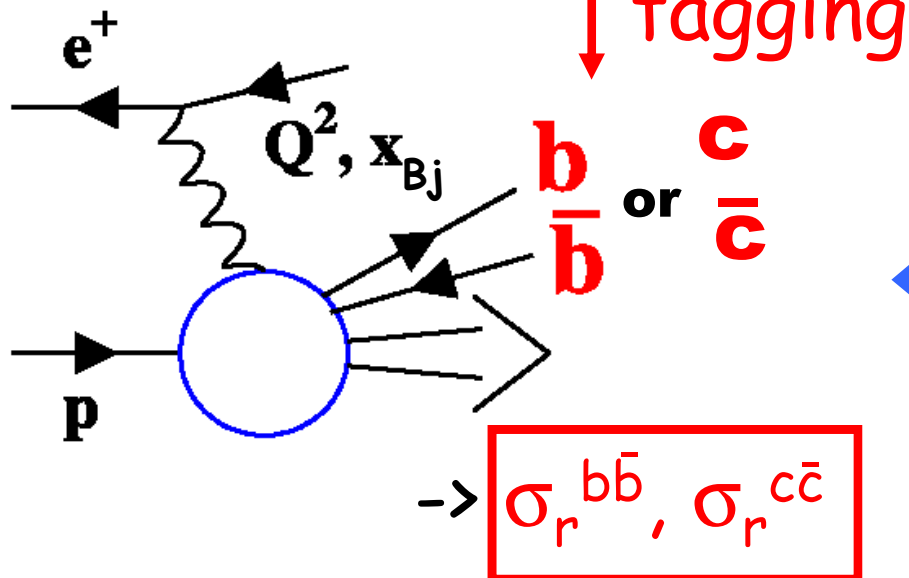
$Q^2 \gtrsim 1 \text{ GeV}^2$ :  
DIS

# Heavy flavour contributions to $\sigma_r$

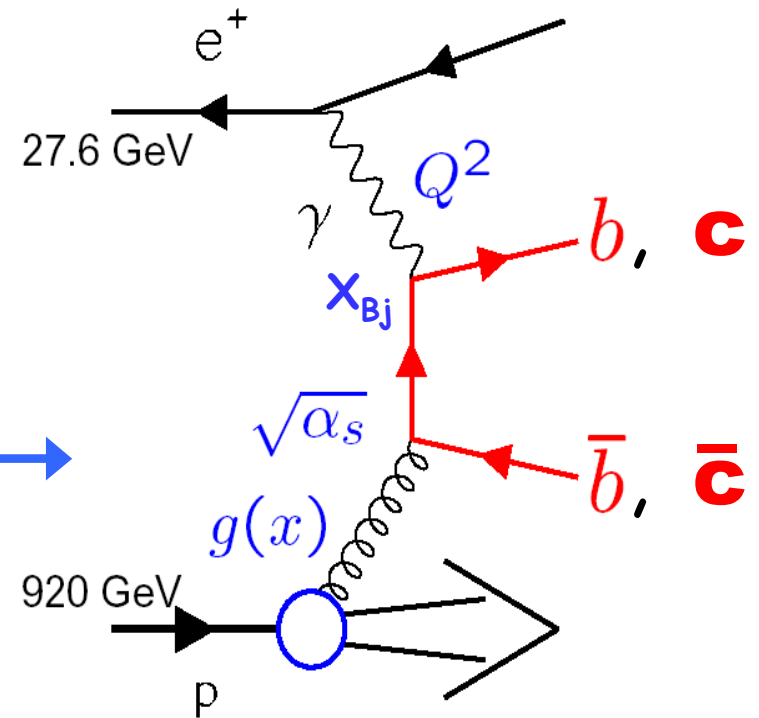


Measure cross section

$$\frac{d^2\sigma}{dx dQ^2}_{Bj} \approx \frac{2\pi\alpha^2}{Q^4 x_{Bj}} [1 + (1-y)^2] \sigma_r(x_{Bj}, Q^2)$$



QCD  $\longleftrightarrow$

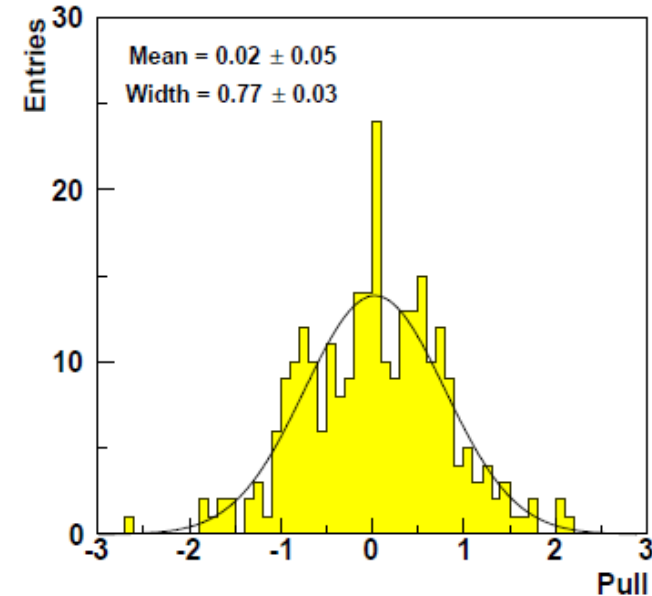


# Combination of 13 charm+beauty data sets

arXiv:1804.01019



Dataset	Tagging	$Q^2$ range [GeV <sup>2</sup> ]	$\mathcal{L}$ [pb <sup>-1</sup> ]	$\sqrt{s}$ [GeV]	$N_c$	$N_b$
1 H1 VTX [14]	VTX	5 – 2000	245	318	29	12
2 H1 $D^{*\pm}$ HERA-I [10]	$D^{*+}$	2 – 100	47	318	17	
3 H1 $D^{*\pm}$ HERA-II (medium $Q^2$ ) [20]	$D^{*+}$	5 – 100	348	318	25	
4 H1 $D^{*\pm}$ HERA-II (high $Q^2$ ) [15]	$D^{*+}$	100 – 1000	351	318	6	
5 ZEUS $D^{*+}$ 96-97 [4]	$D^{*+}$	1 – 200	37	300	21	
6 ZEUS $D^{*+}$ 98-00 [6]	$D^{*+}$	1.5 – 1000	82	318	31	
7 ZEUS $D^0$ 2005 [12]	$D^0$	5 – 1000	134	318	9	
8 ZEUS $\mu$ 2005 [13]	$\mu$	20 – 10000	126	318	8	8
9 ZEUS $D^+$ HERA-II [21]	$D^+$	5 – 1000	354	318	14	
10 ZEUS $D^{*+}$ HERA-II [22]	$D^{*+}$	5 – 1000	363	318	31	
11 ZEUS VTX HERA-II [23]	VTX	5 – 1000	354	318	18	17
12 ZEUS $e$ HERA-II [19]	$e$	10 – 1000	363	318	9	
13 ZEUS $\mu$ + jet HERA-I [16]	$\mu$	2 – 3000	114	318		11



good data consistency:  
 $\chi^2 = 149/187$  dof

**3 additional charm datasets** w.r.t. EPJ C73 (2013) 2311

**beauty** combined for the **first time**

account for **all systematic correlations** between data points, data sets, and between charm and beauty

# Charm combination

arXiv:1804.01019

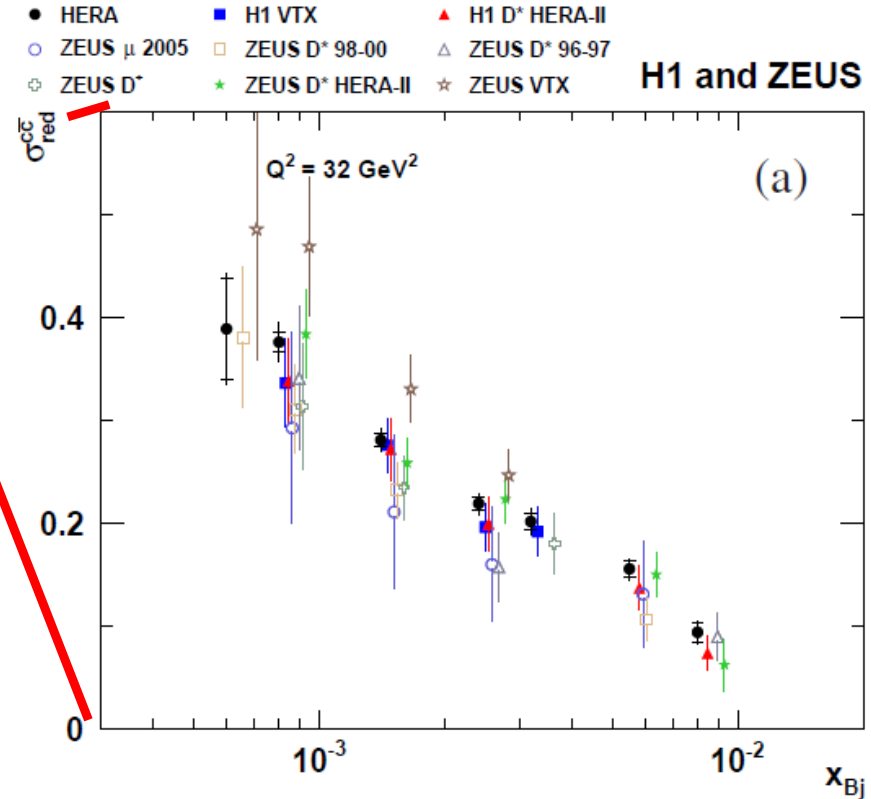
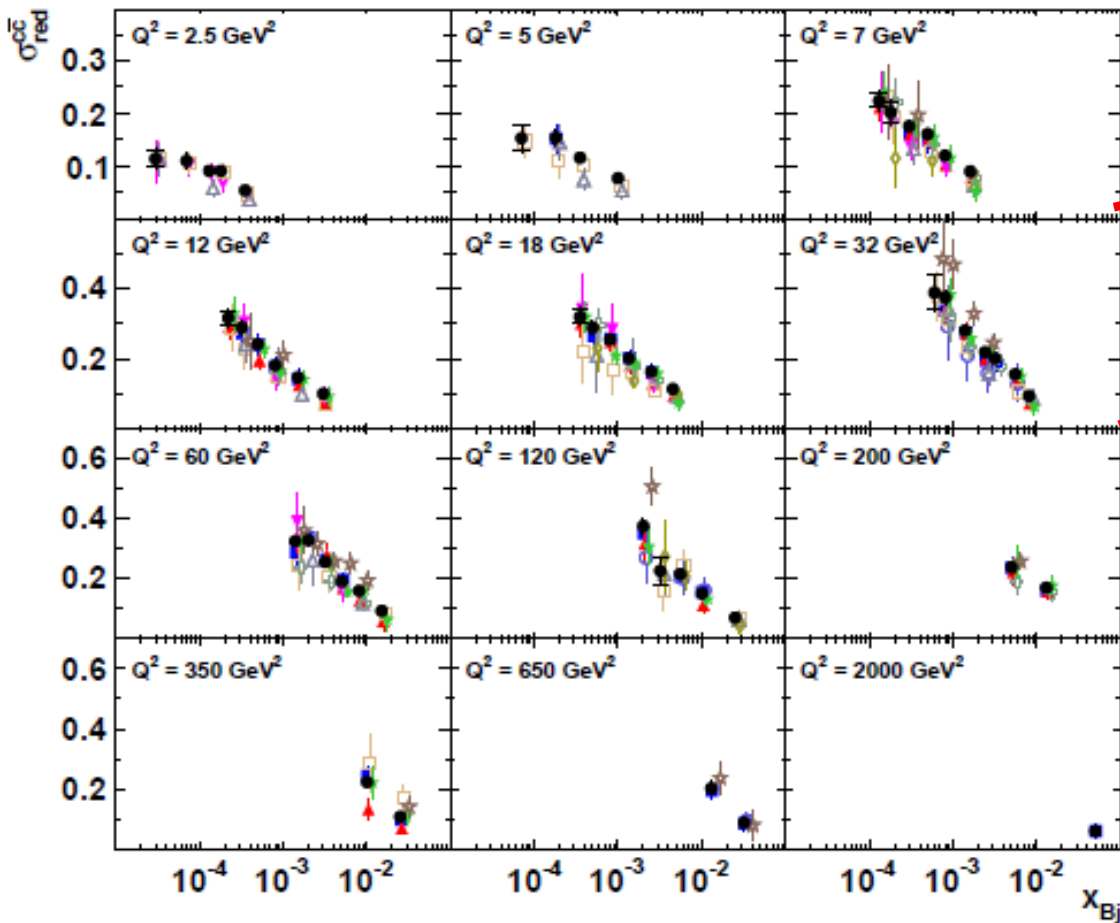


209 → 52 data points

3 HERA II data sets added

- HERA
- ▼ H1 D\* HERA-I
- △ ZEUS D\* 96-97
- ★ ZEUS D\* HERA-II
- H1 VTX
- ZEUS μ 2005
- ◇ ZEUS D<sup>0</sup>
- ☆ ZEUS VTX
- ▲ H1 D\* HERA-II
- ZEUS D\* 98-00
- ⊕ ZEUS D<sup>+</sup>

## H1 and ZEUS



# Beauty combination

arXiv:1804.01019

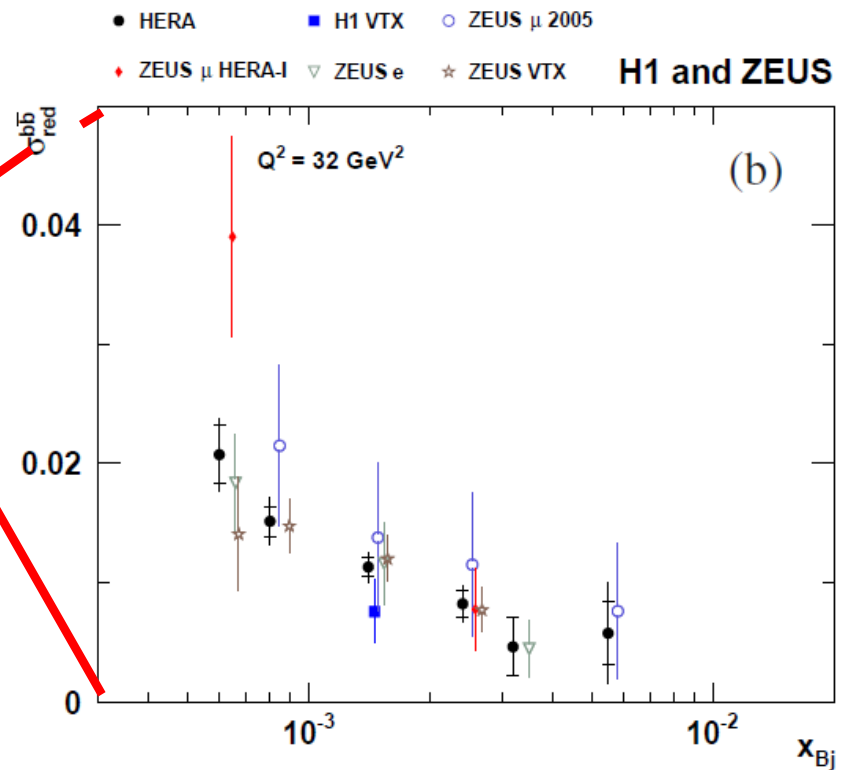
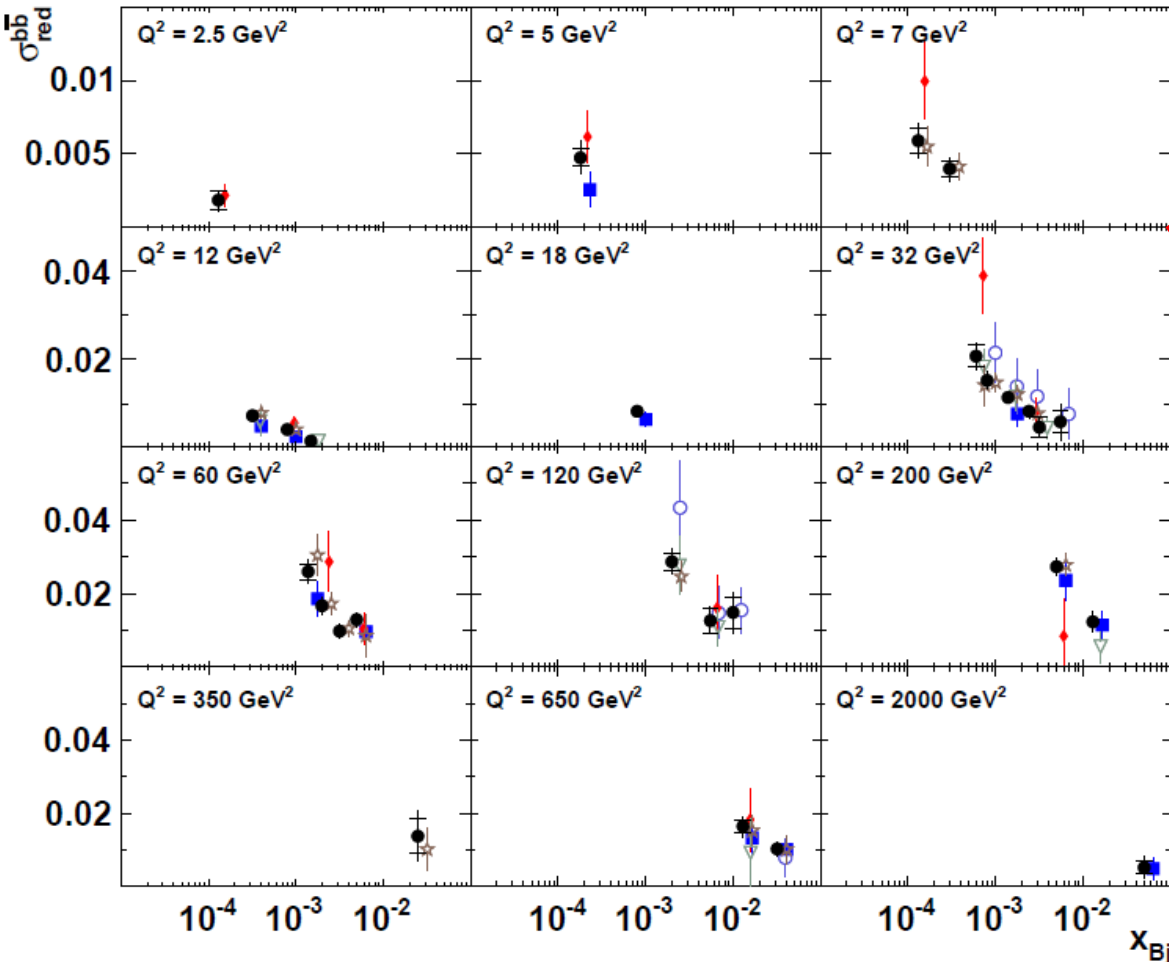


57 → 27 data points

combined for the first time

## H1 and ZEUS

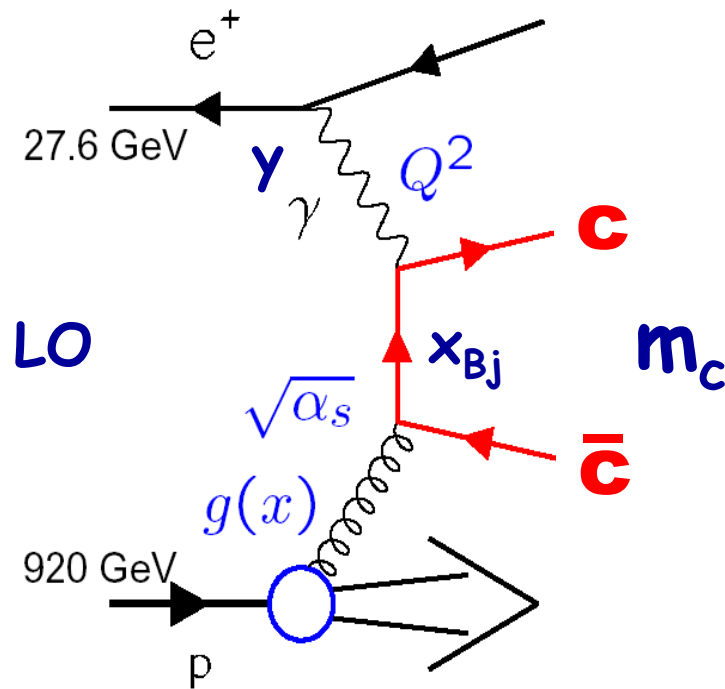
- HERA
- ◆ ZEUS μ HERA-I
- H1 VTX
- ▽ ZEUS e
- ZEUS μ 2005
- ☆ ZEUS VTX



# Fixed Flavour Number Scheme (FFNS)



example: charm



- no charm in proton

- full kinematical treatment of charm mass ☺

(multi-scale problem:  
 $Q^2, p_T, m_c \rightarrow$  logs of ratios)

+ NLO (+partial NNLO) corrections,

- no resummation of logs ☹

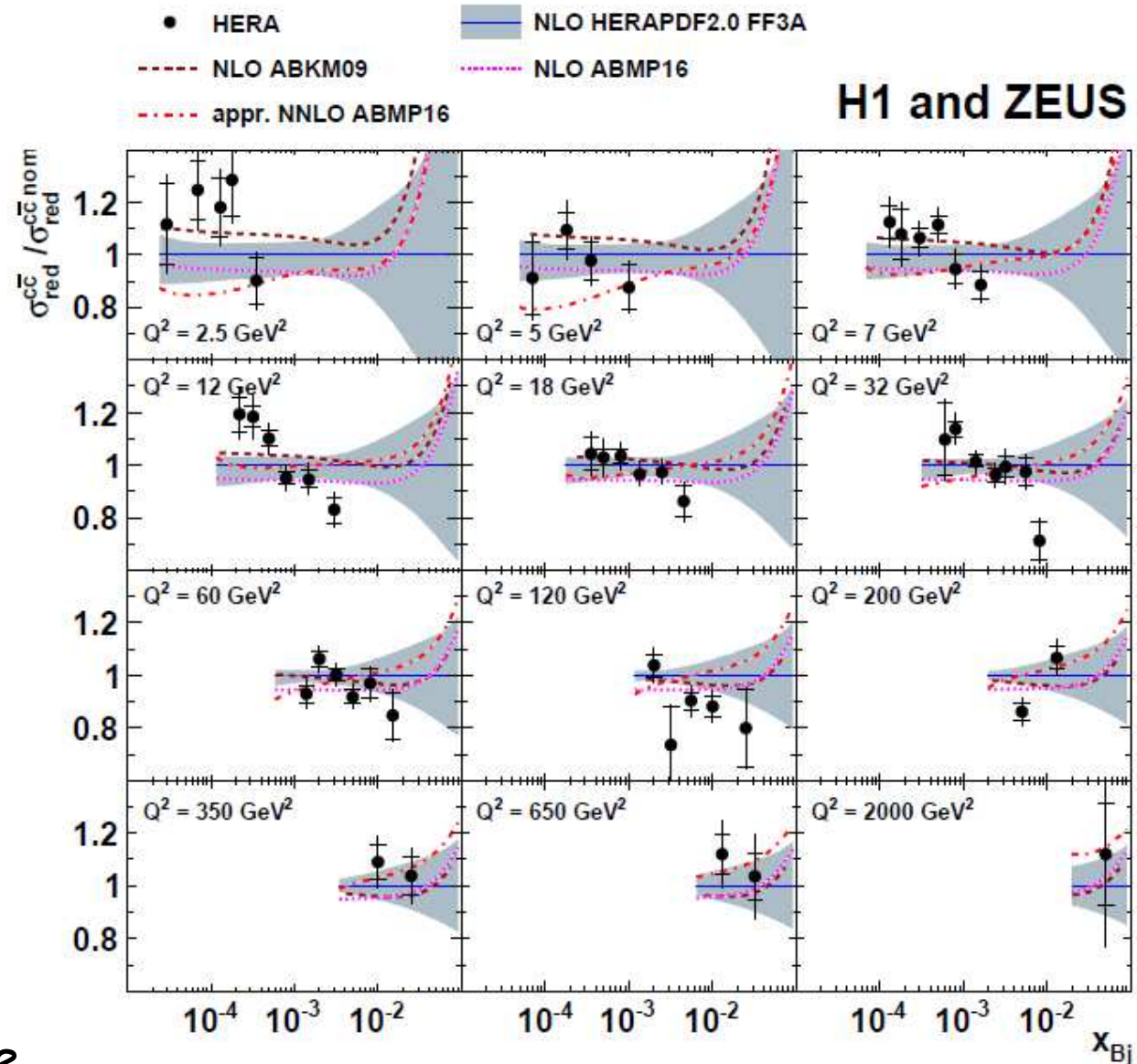
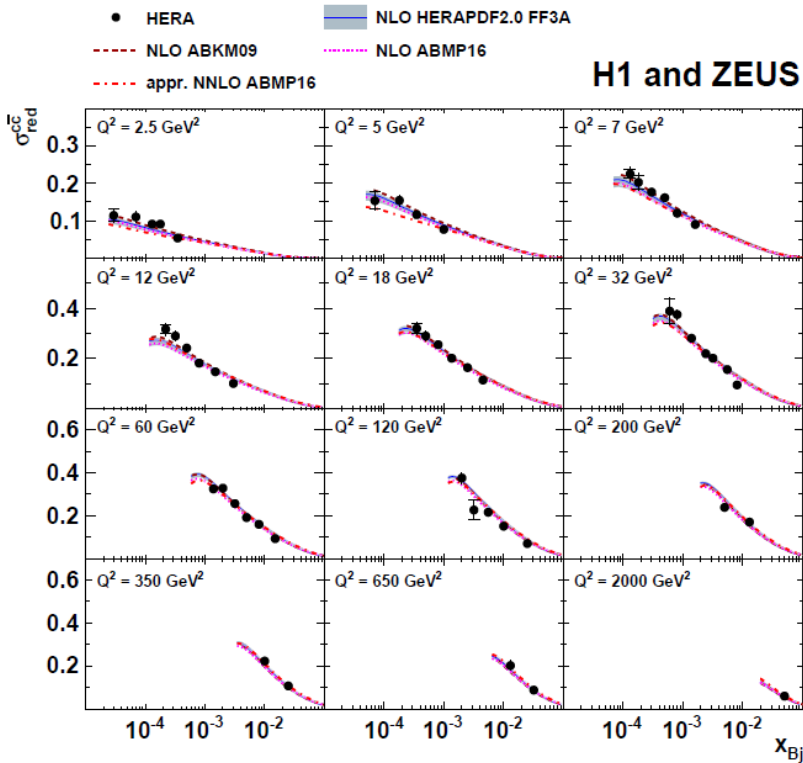
“natural” scale:  
 $\mu^2 = Q^2 + 4m_c^2$

- no extra matching parameters ☺



# Comparison to FFNS QCD predictions

arXiv:1804.01019



data reasonably described  
 best: HERAPDF2.0 FF  
 and ABKM09NLO  
 ~3 $\sigma$  tension with  $x_{Bj}$  slope  
 appr. NNLO does not improve

18. 04. 18

A. Geiser, DIS2018

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# Comparison to VFNS QCD predictions

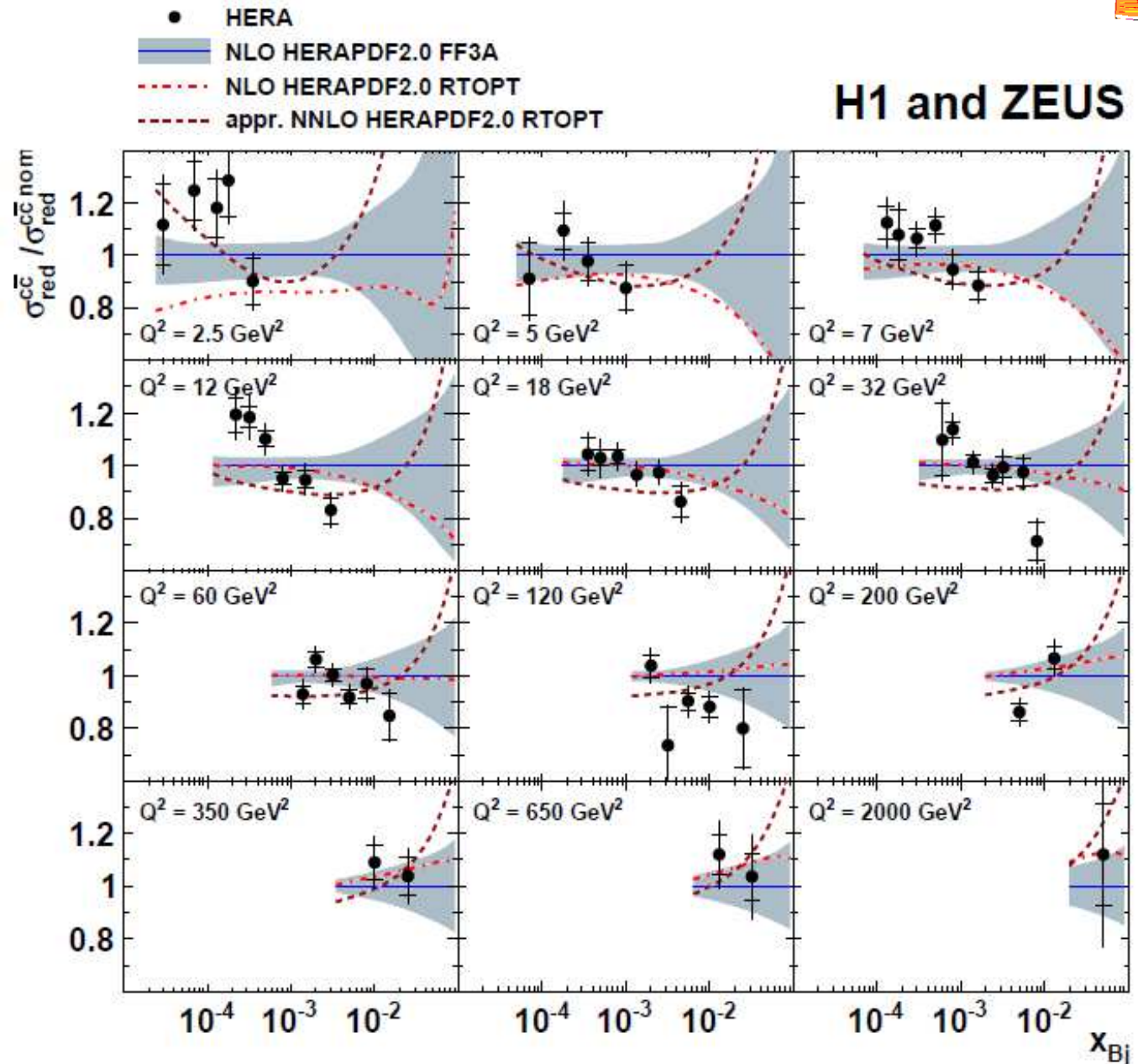
arXiv:1804.01019



data description  
reasonable but  
not better than FF

overall, **NLO**  
better than  
appr. NNLO

beauty in backup:  
larger  
uncertainties  
-> all consistent



# Predictions w/o and with log 1/x resummation

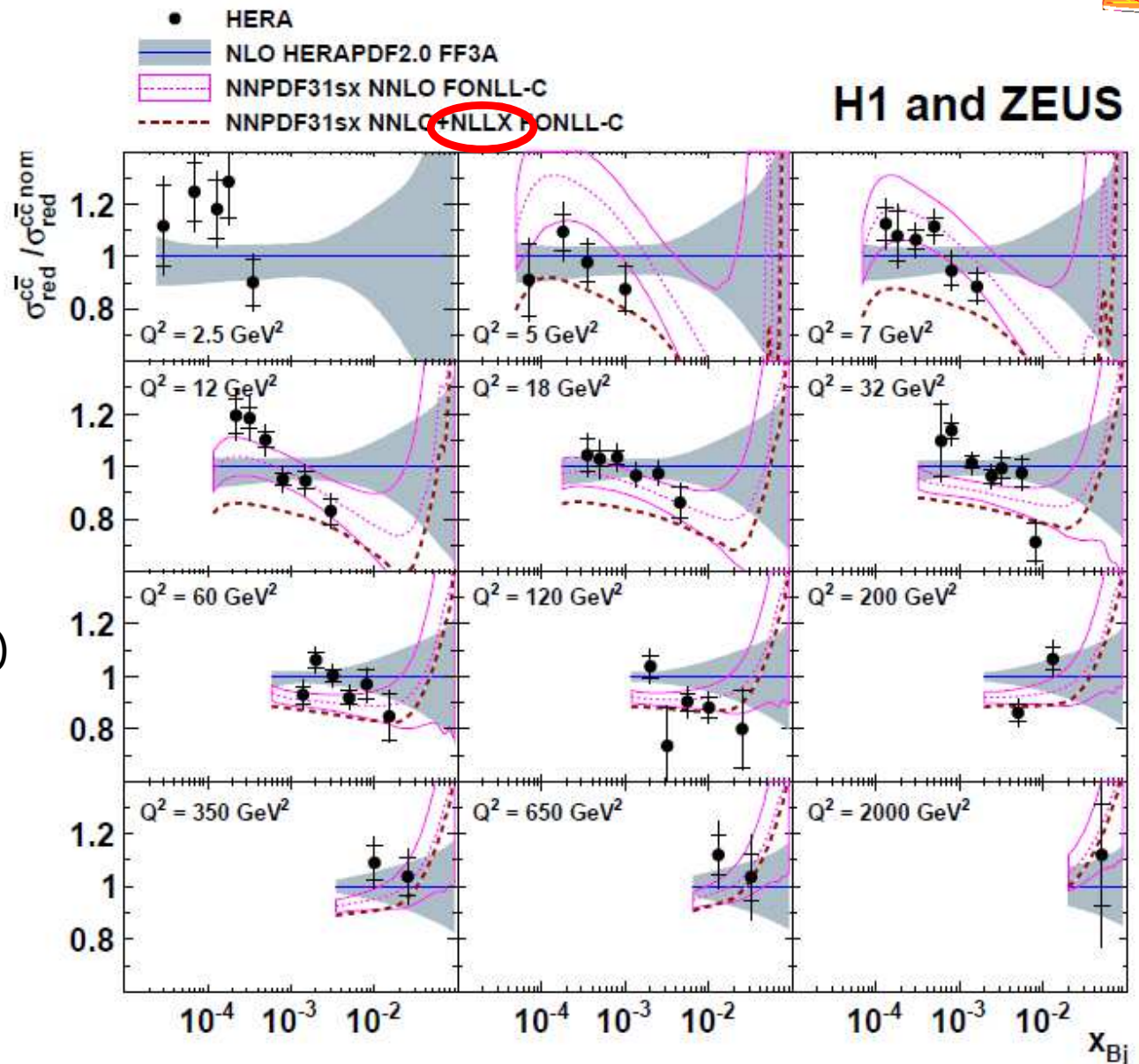


arXiv:1804.01019



NLL resummation of log 1/x terms improves  $x_{Bj}$  slope but deteriorates normalisation

overall, NNPDF3.1sx (fitted charm, arXiv:1710.05935) either with or w/o log 1/x resummation not better than HERAPDF (FONLL-C + NLLx see backup)



# $\chi^2$ and p-values for various QCD predictions

arXiv:1804.01019



central  
predictions

Dataset	PDF (scheme)	$\chi^2$ [p-value]
charm [38] ( $N_{\text{data}} = 52$ )	HERAPDF20_NLO_FF3A (FFNS)	59 [0.23]
	ABKM09 (FFNS)	59 [0.23]
	ABMP16_3_nlo (FFNS)	61 [0.18]
	ABMP16_3_nnlo (FFNS)	70 [0.05]
	HERAPDF20_NLO_EIG (RTOPT)	71 [0.04]
	HERAPDF20_NNLO_EIG (RTOPT)	66 [0.09]
( $N_{\text{data}} = 47$ )	NNPDF31sx NNLO (FONLL-C)	106 [ $1.5 \cdot 10^{-6}$ ]
	NNPDF31sx NNLO+NLLX (FONLL-C)	71 [0.013]
charm, this analysis ( $N_{\text{data}} = 52$ )	HERAPDF20_NLO_FF3A (FFNS)	86 [0.002]
	ABKM09 (FFNS)	82 [0.005]
	ABMP16_3_nlo (FFNS)	90 [0.0008]
	ABMP16_3_nnlo (FFNS)	109 [ $6 \cdot 10^{-6}$ ]
	HERAPDF20_NLO_EIG (RTOPT)	99 [ $9 \cdot 10^{-5}$ ]
	HERAPDF20_NNLO_EIG (RTOPT)	102 [ $4 \cdot 10^{-5}$ ]
( $N_{\text{data}} = 47$ )	NNPDF31sx NNLO (FONLL-C)	140 [ $1.5 \cdot 10^{-11}$ ]
	NNPDF31sx NNLO+NLLX (FONLL-C)	114 [ $5 \cdot 10^{-7}$ ]
beauty, this analysis ( $N_{\text{data}} = 27$ )	HERAPDF20_NLO_FF3A (FFNS)	33[0.20]
	ABMP16_3_nlo (FFNS)	37 [0.10]
	ABMP16_3_nnlo (FFNS)	41 [0.04]
	HERAPDF20_NLO_EIG (RTOPT)	33 [0.20]
	HERAPDF20_NNLO_EIG (RTOPT)	45 [0.016]

previous  
combined  
charm

new combined  
charm

beauty

Table 4: The  $\chi^2$ , p-values and number of data points of the charm and beauty data with respect to the NLO and approximate NNLO calculations using various PDFs as described in the text. The measurements at  $Q^2 = 2.5 \text{ GeV}^2$  are excluded in the calculations of the  $\chi^2$  values for the NNPDF3.1sx predictions, by which the number of data points is reduced to 47, as detailed in the caption of figure 12.

# QCD fit



arXiv:1804.01019



## Simultaneous NLO QCD fit of

- combined **inclusive DIS** data (arXiv:1506.06042),  $Q^2_{\min}=3.5 \text{ GeV}^2$
- new combined **charm and beauty DIS** data (this work)

Simultaneously fit **PDF's** (a la **HERAPDF** FF) in FFNS at NLO and **charm quark and beauty quark "running" masses** in  $\overline{\text{MS}}$  scheme

- using xFitter [www.xfitter.org], 14 parameters ( $\pm 1$ )
- NLO DGLAP [QCDNUM] and matrix elements [OPENQCDRAD],  $n_f = 3$
- $\mu_F = \mu_R = \sqrt{Q^2 + 4m_Q^2}$ , varied by factor 2 (for heavy flavour part only)
- **free  $m_c(m_c)$ ,  $m_b(m_b)$**
- $\alpha_s(M_Z)^{n_f=3} = 0.106$ , equivalent to  $\alpha_s(M_Z)^{n_f=5} = 0.118 \pm 0.002$
- fit uncertainty using  $\Delta\chi^2 = 1$

-> **HERAPDF-HQMASS**

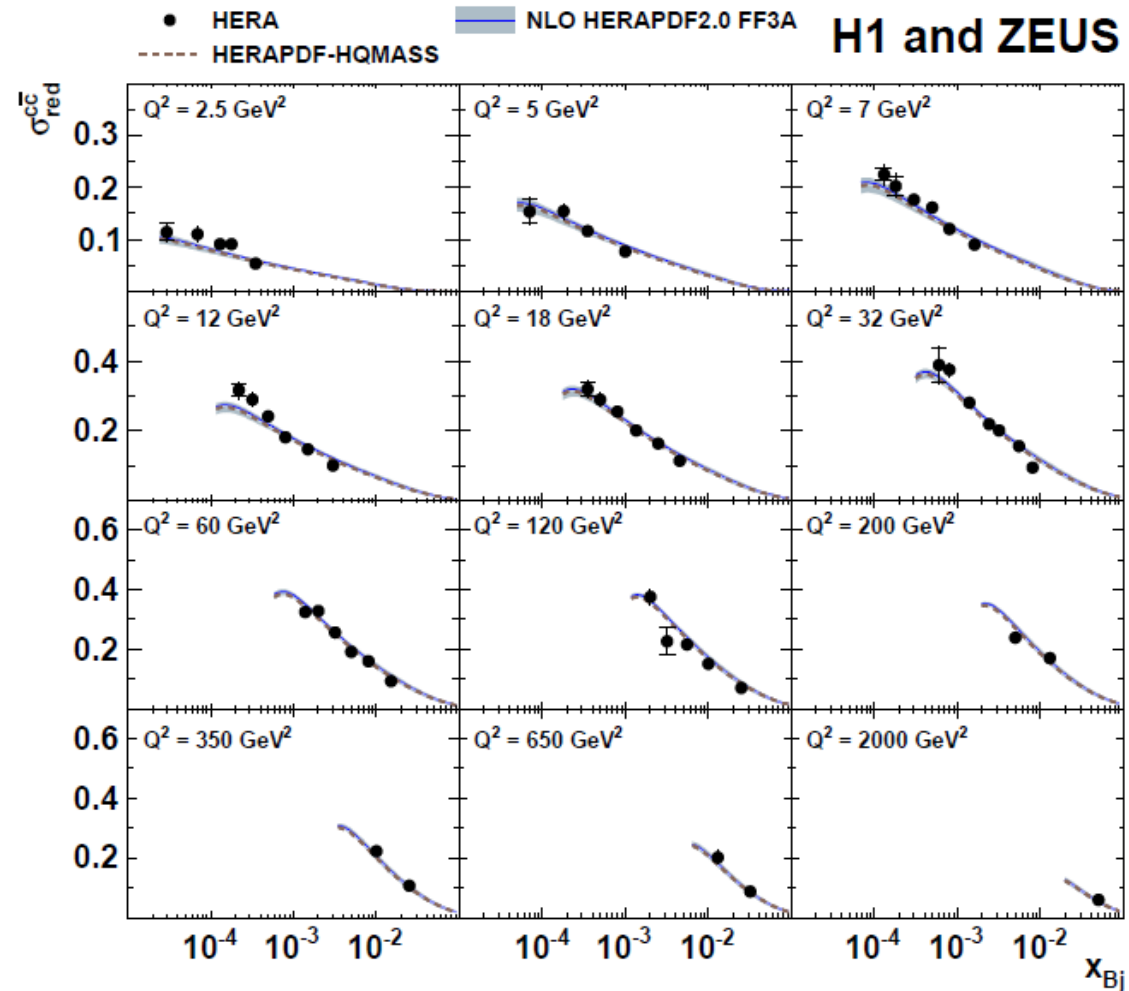
# QCD fit: charm subset

arXiv:1804.01019



fully consistent  
with HERAPDF2.0 FF3A

uncertainty breakdown  
in backup



$$m_c(m_c) = 1.29^{+0.05}_{-0.04 \text{ exp/fit}} \quad +0.06_{-0.01 \text{ mod/scale}} \quad +0.00_{-0.03 \text{ par}} \quad \text{GeV}$$

PDG:  $1.27 \pm 0.03 \text{ GeV}$  (lattice QCD + time-like processes)

# Comparison with other $m_c(m_c)$ determinations

this work:

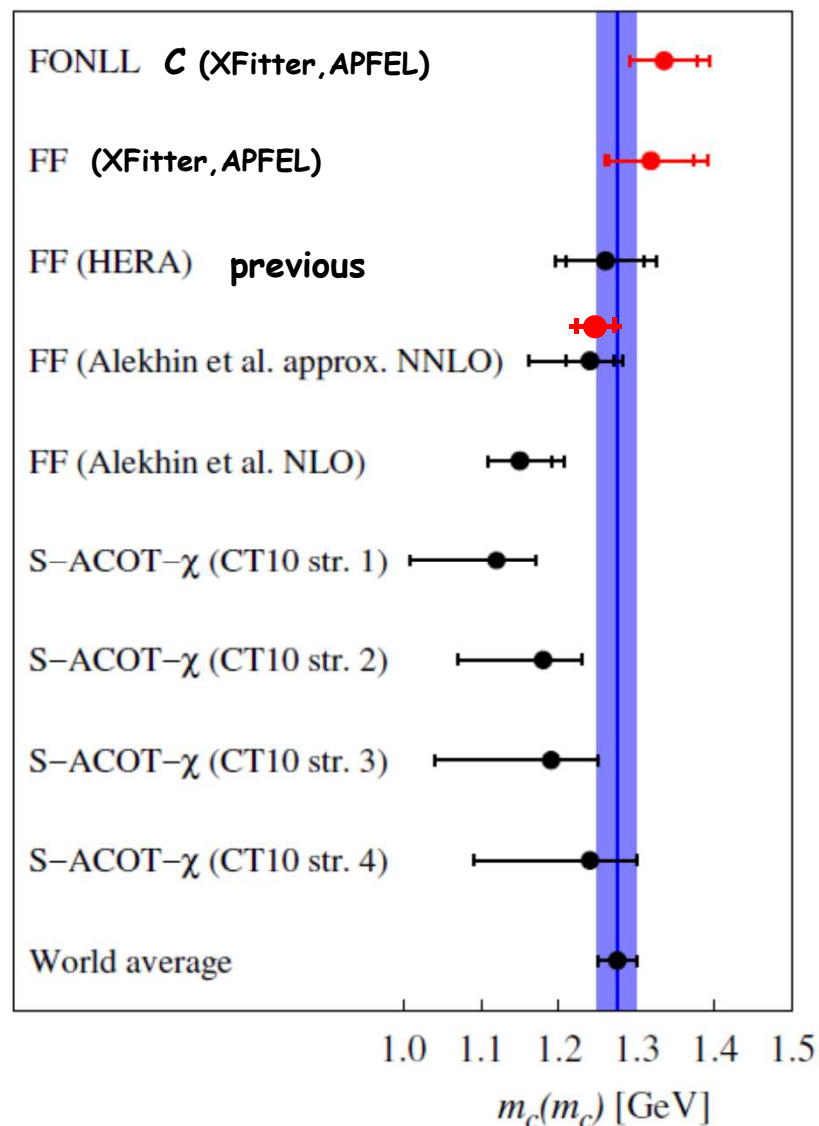
$$m_c(m_c) = 1.29^{+0.05}_{-0.04} \text{ exp/fit} \\ +0.06_{-0.01} \text{ mod/scale} \quad +0.00_{-0.03} \text{ par} \quad \text{GeV}$$

latest ABMP16 result:  $m_c(m_c) = 1.252 \pm 0.018 \pm 0.032 \text{ GeV}$   
 S. Alekhin et al., arXiv:1701.05383,  
 Phys. Rev. D96 (2017) 014011

previous results summarized in  
 V. Bertone et al., arXiv:1605.01946,  
 JHEP 1608 (2016) 050 :

scheme	$m_c(m_c)$ [GeV]
FONLL (this work)	$1.335 \pm 0.043(\text{exp})^{+0.019}_{-0.000}(\text{param})^{+0.011}_{-0.008}(\text{mod})^{+0.033}_{-0.008}(\text{th})$
FFN (this work)	$1.318 \pm 0.054(\text{exp})^{+0.011}_{-0.010}(\text{param})^{+0.015}_{-0.019}(\text{mod})^{+0.045}_{-0.004}(\text{th})$
FFN (HERA) [9]	$1.26 \pm 0.05(\text{exp}) \pm 0.03(\text{mod}) \pm 0.02(\text{param}) \pm 0.02(\alpha_s)$
FFN (Alekhin <i>et al.</i> ) [24]	$1.24 \pm 0.03(\text{exp})^{+0.03}_{-0.02}(\text{scale})^{+0.00}_{-0.07}(\text{th})$ (approx. NNLO) $1.15 \pm 0.04(\text{exp})^{+0.04}_{-0.00}(\text{scale})$ (NLO)
S-ACOT- $\chi$ (CT10) [29]	$1.12^{+0.05}_{-0.11}$ (strategy 1) $1.18^{+0.05}_{-0.11}$ (strategy 2) $1.19^{+0.06}_{-0.15}$ (strategy 3) $1.24^{+0.06}_{-0.15}$ (strategy 4)
World average [53]	$1.275 \pm 0.025$

FF, HERA, this work

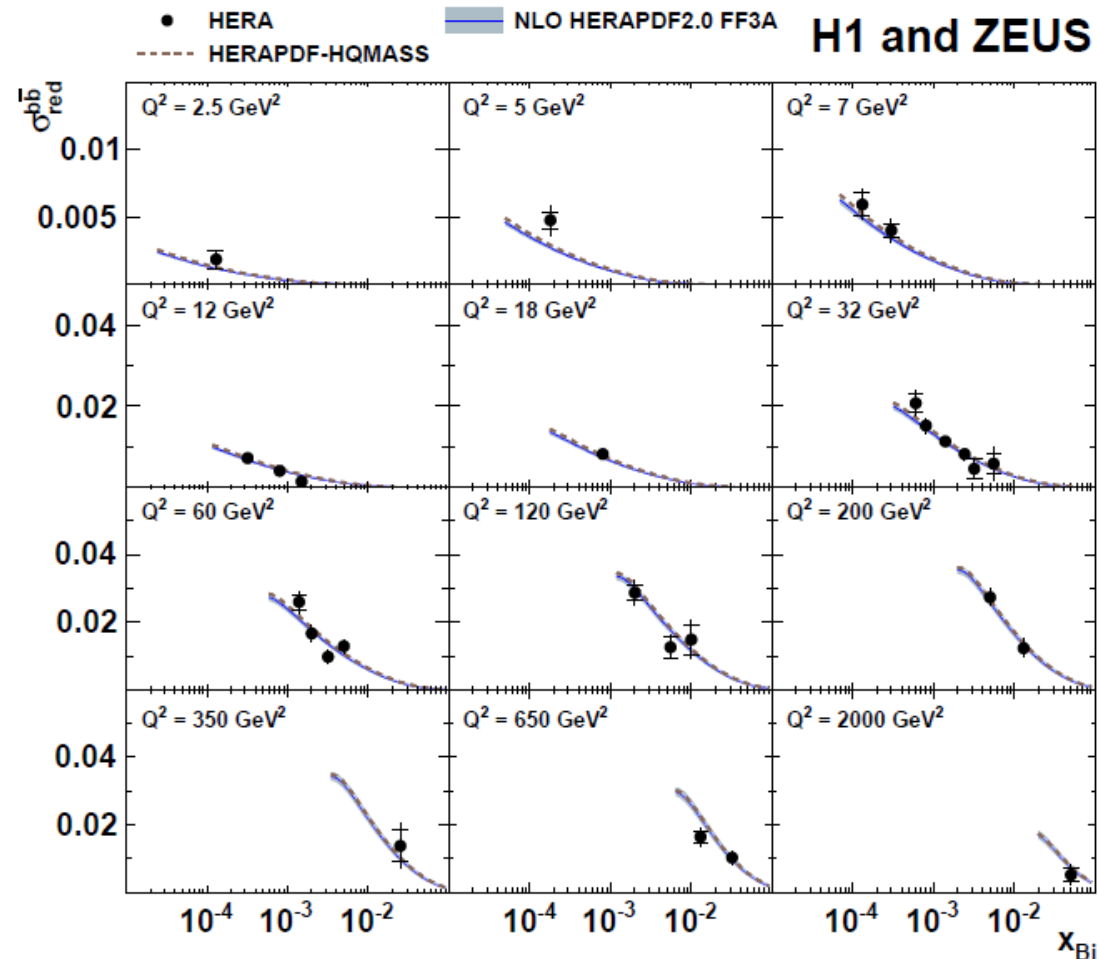


# QCD fit: beauty subset

arXiv:1804.01019



fully consistent with  
HERAPDF FF3A



new:  $m_b(m_b) = 4.05^{+0.10}_{-0.11 \text{ exp/fit}} \quad +0.09_{-0.03 \text{ mod/scale}} \quad +0.00_{-0.03 \text{ par}} \text{ GeV}$

ZEUS:  $m_b(m_b) = 4.07 \pm 0.14_{\text{exp/fit}} \quad +0.08_{-0.08 \text{ mod/scale}} \quad +0.05_{-0.00 \text{ par}} \text{ GeV}$

PDG:  $4.18 \pm 0.03 \text{ GeV}$  (lattice QCD + time-like processes)



# QCD fit: inclusive data subset

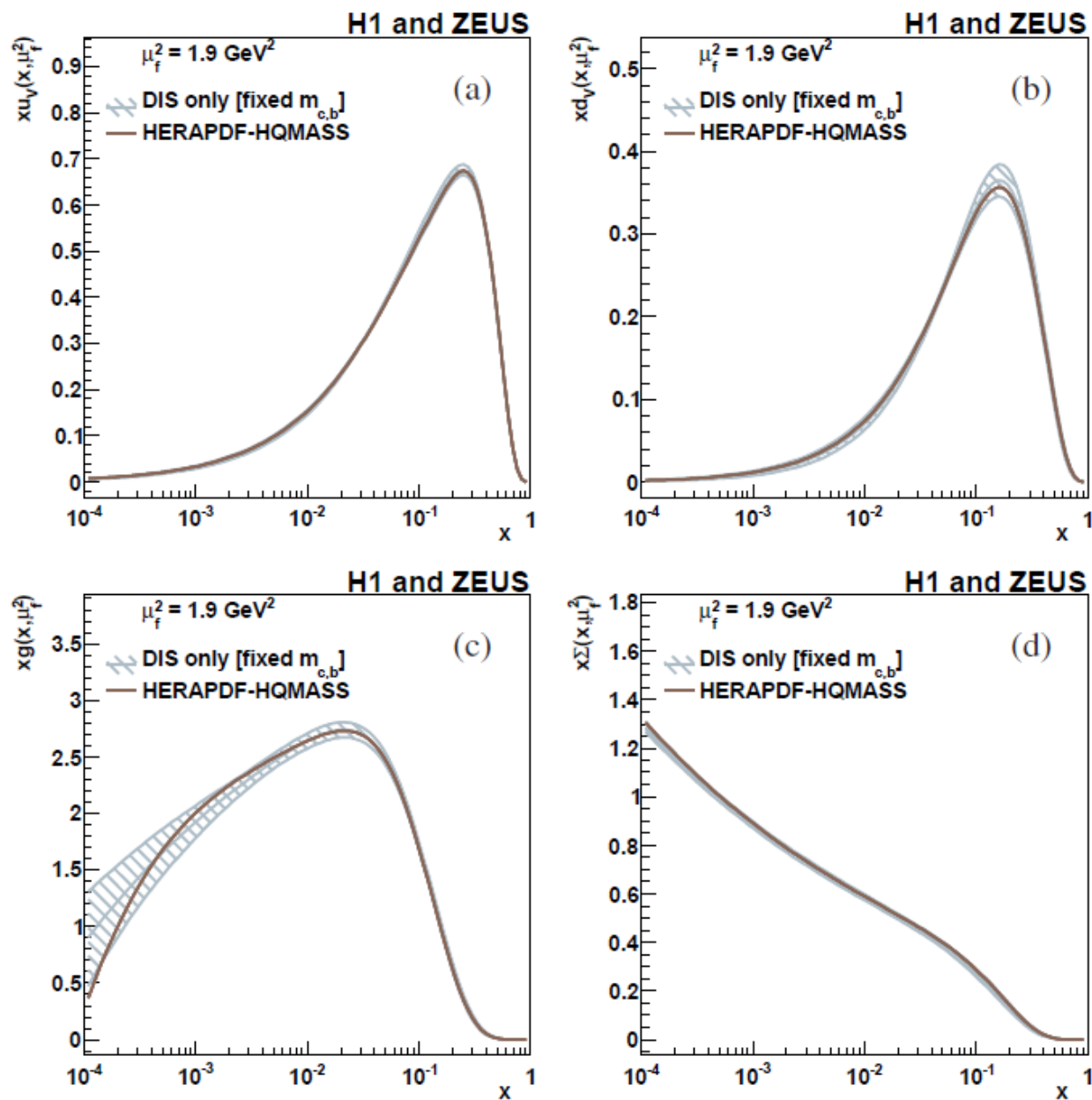


arXiv:1804.01019



PDFs consistent with those of inclusive data only (and  $c, b$  masses fixed to PDG)

-> inclusive data (and  $c, b$  mass values) dominate in fixing PDF



# QCD fit: inclusive data, parametrisation uncert.

- Reminder, **full fit**:  $\Delta\chi^2=1$  scale dom 14p→15p,13p
- $m_c(m_c) = 1290^{+46}_{-41}(\text{exp/fit})^{+62}_{-14}(\text{mod})^{+3}_{-31}(\text{par}) \text{ MeV}$
  - $m_b(m_b) = 4049^{+104}_{-109}(\text{exp/fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par}) \text{ MeV}$

Using **inclusive HERA data only** (14p):

- $m_c(m_c) = 1798^{+144}_{-134}(\text{exp/fit}) \text{ MeV}$
- $m_b(m_b) = 8450^{+2280}_{-1810}(\text{fit}) \text{ MeV}$

no full uncertainty evaluation, but large sensitivity to **PDF parametrisation** (-> 13p):

- $m_c(m_c) = 1798 \rightarrow 1450 \text{ MeV}$ ,
- $m_b(m_b) = 8450 \rightarrow 3995 \text{ MeV}$

dominant effect:

$$\begin{aligned}
 xg(x) &= A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g} \\
 xu_v(x) &= A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2) \\
 xd_v(x) &= A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}} \\
 x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x) \\
 x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}
 \end{aligned}$$

13p:  $E_{u_v} = 0$

- > **inclusive HERA data alone cannot constrain HQ masses reliably**
- > **interplay of PDFs and HQ masses needs careful treatment**

# QCD fit: charm x slope



arXiv:1804.01019

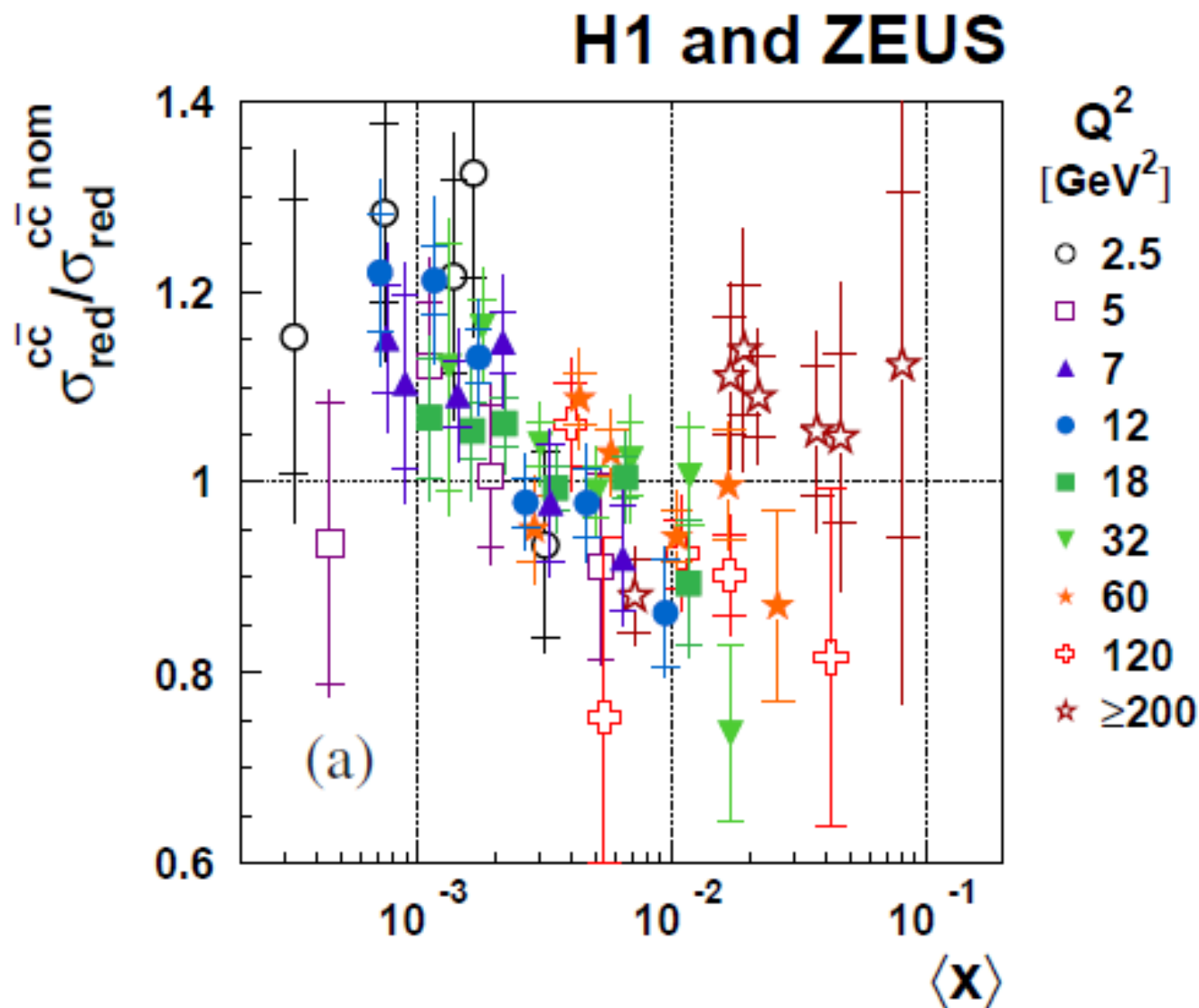


plot **data/fit**  
**vs.  $\langle x \rangle$**  of  
**incoming partons**  
(rather than  $x_{Bj}$ )  
for each data point

$$\text{LO: } x = x_{Bj} \cdot \left(1 + \frac{\hat{s}}{Q^2}\right)$$

$\langle x \rangle$  calculated at **NLO**  
using HVQDIS

-> **common  $\langle x \rangle$  trend**  
**for all  $Q^2$**



# QCD fit with $x_{Bj} > 0.01$ for inclusive data

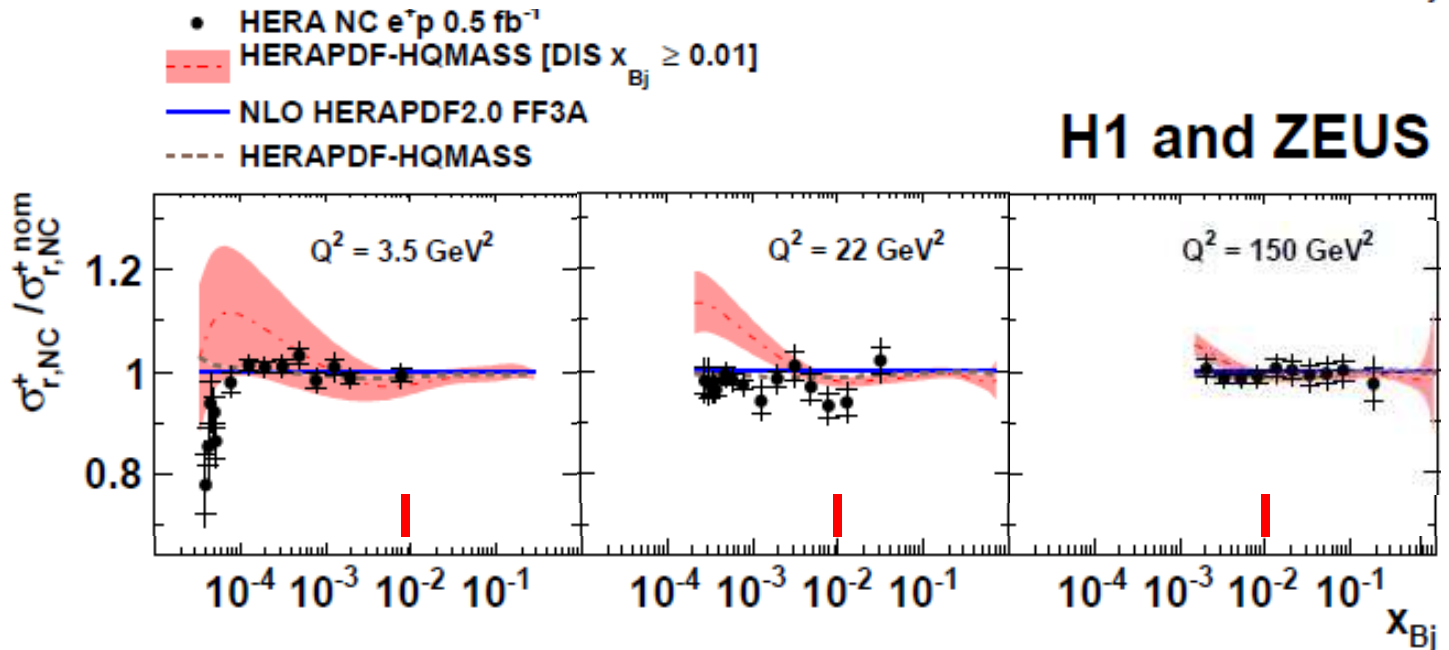
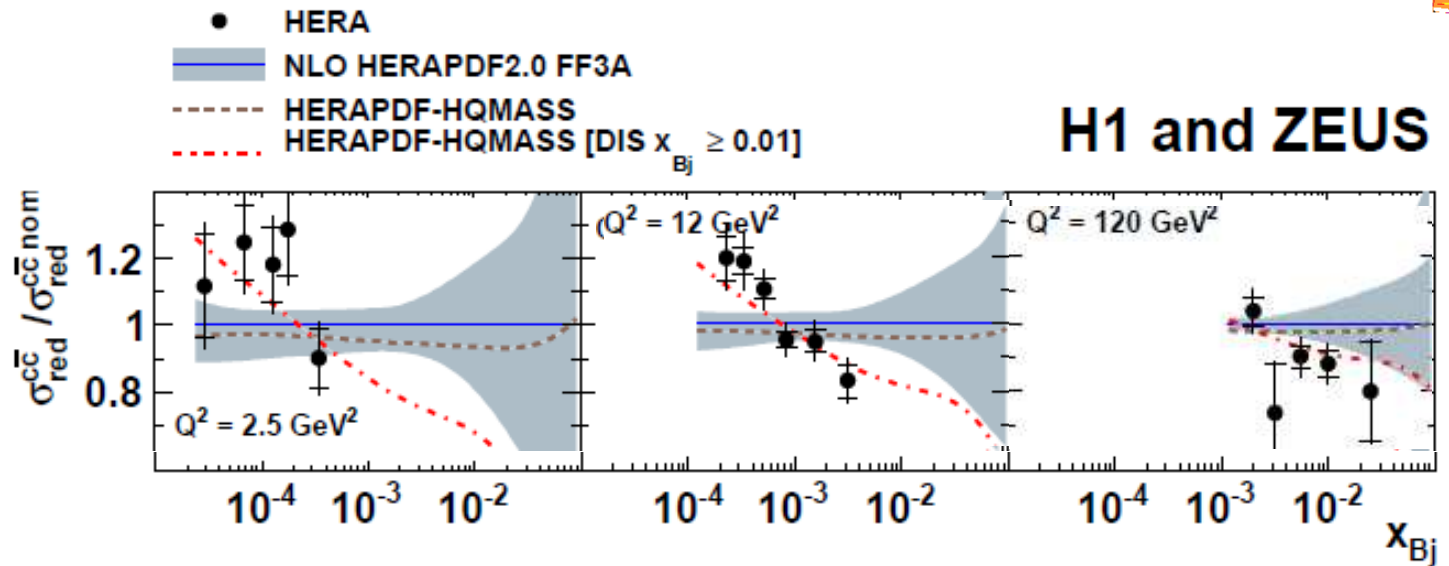


can improve  
low  $x$  charm  
slope

(no longer  
constrained  
by inclusive)

but fails  
to describe  
low  $x$   
inclusive data

-> not a solution  
(but hint)



# Conclusions

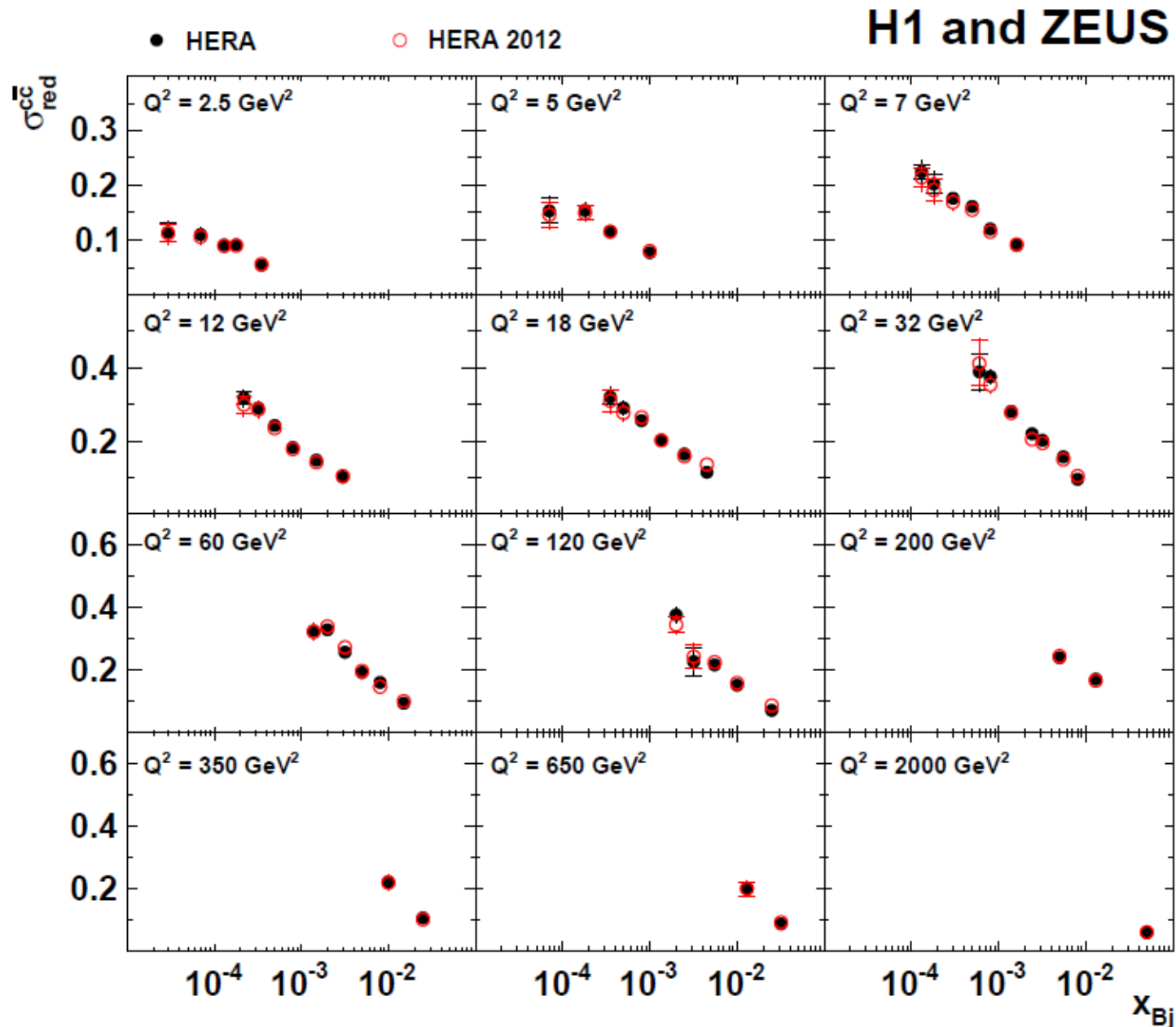
- Final HERA charm and beauty data in DIS have been combined including all correlations  
charm precision improved by  $\sim 20\%$ , beauty combined for the first time
- Data are reasonably described by FFNS (best) and VFNS predictions (NLO better than approx. NNLO), but show  $\sim 3\sigma$  tension in  $x$  slope w.r.t inclusive
- QCD fit of inclusive, charm and beauty data (simultaneous fit of PDFs,  $m_c$  and  $m_b$  in FFNS at NLO) yields
$$m_c(m_c) = 1290^{+46}_{-41}(\text{exp/fit})^{+62}_{-14}(\text{mod})^{+3}_{-31}(\text{par}) \text{ MeV}$$
$$m_b(m_b) = 4049^{+104}_{-109}(\text{exp/fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par}) \text{ MeV}$$
in agreement with world average and previous measurements  
(not affected by  $x$  slope tension within uncertainties)
- More detailed studies of  $x$  slope tension  $\rightarrow$  can not be solved by varying the gluon density, or adding higher orders, or resumming  $\log 1/x$  terms, within the respective pQCD frameworks  
 $\rightarrow$  further investigations useful



# Backup

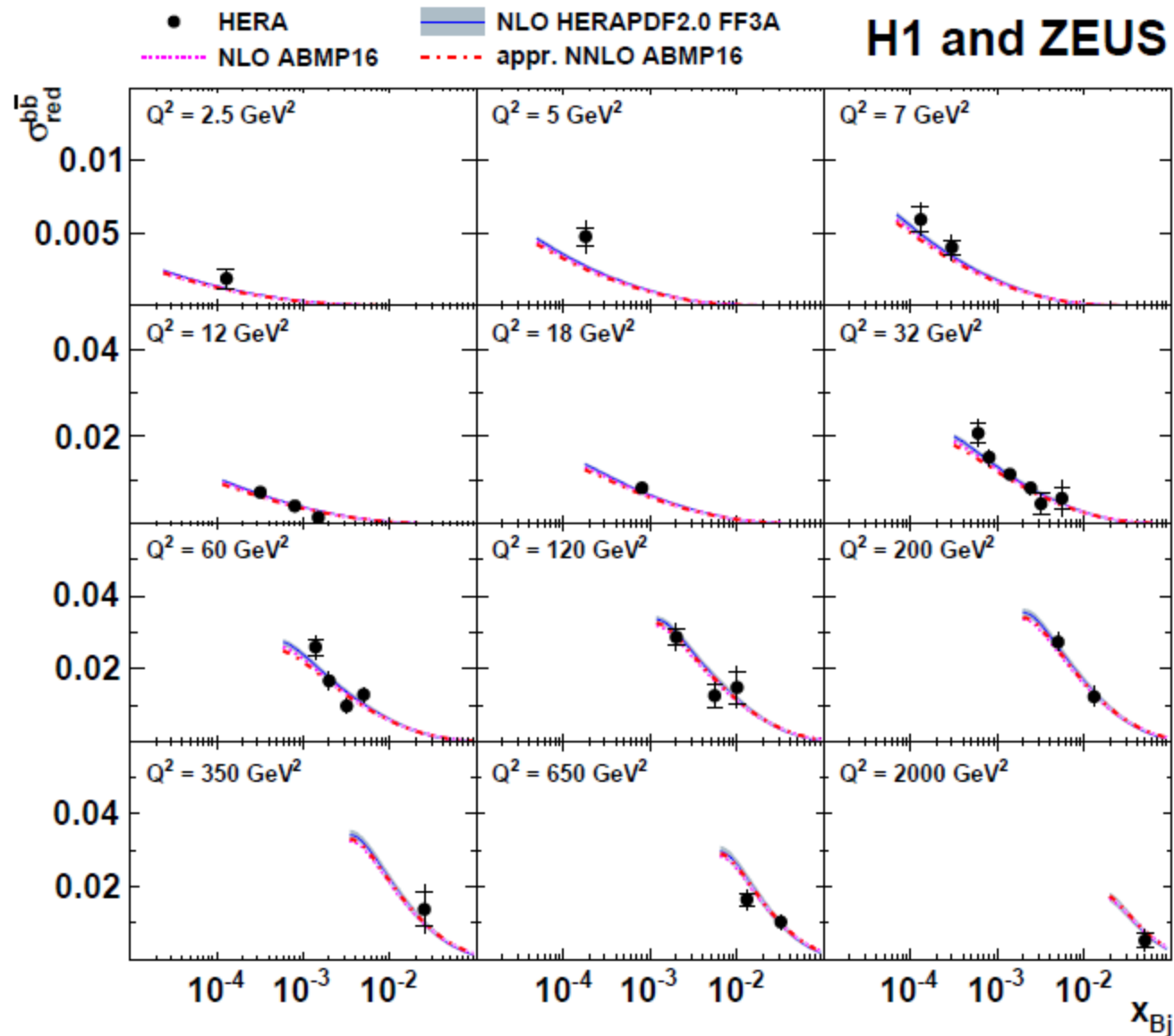
# Comparison to previous charm combination

arXiv:1804.01019



# Comparison to FFNS predictions

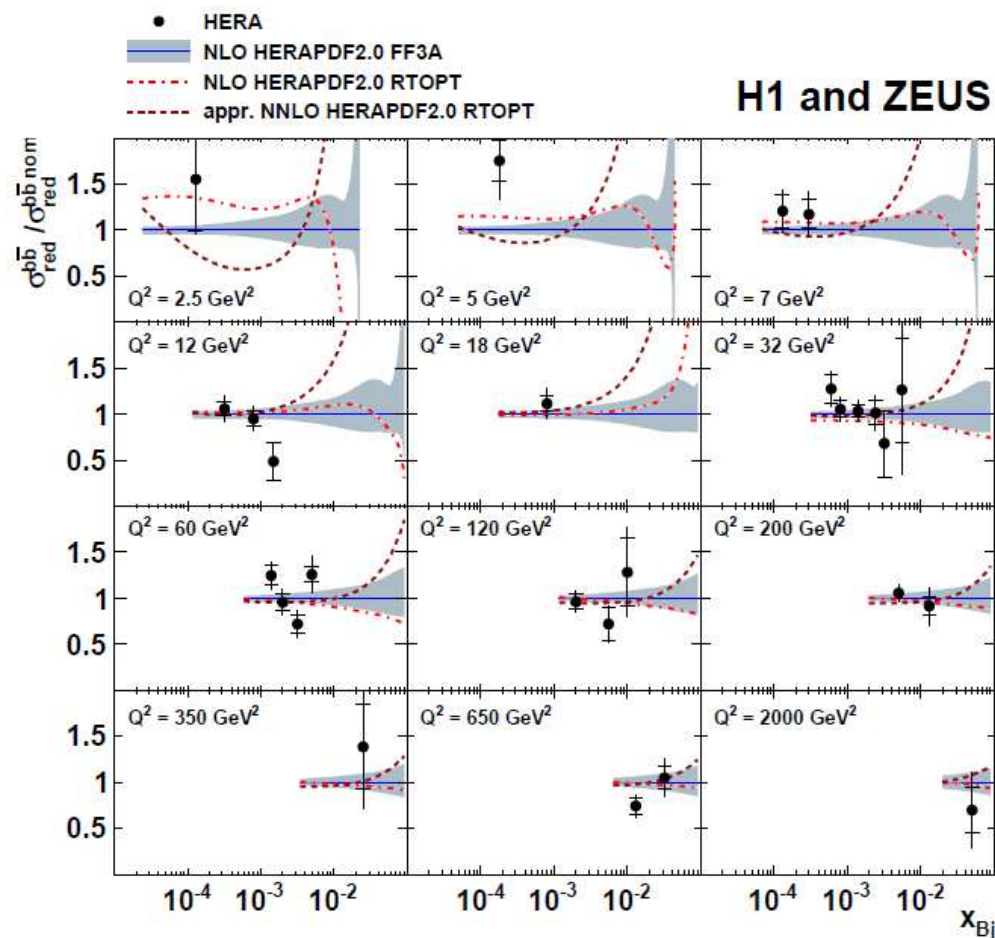
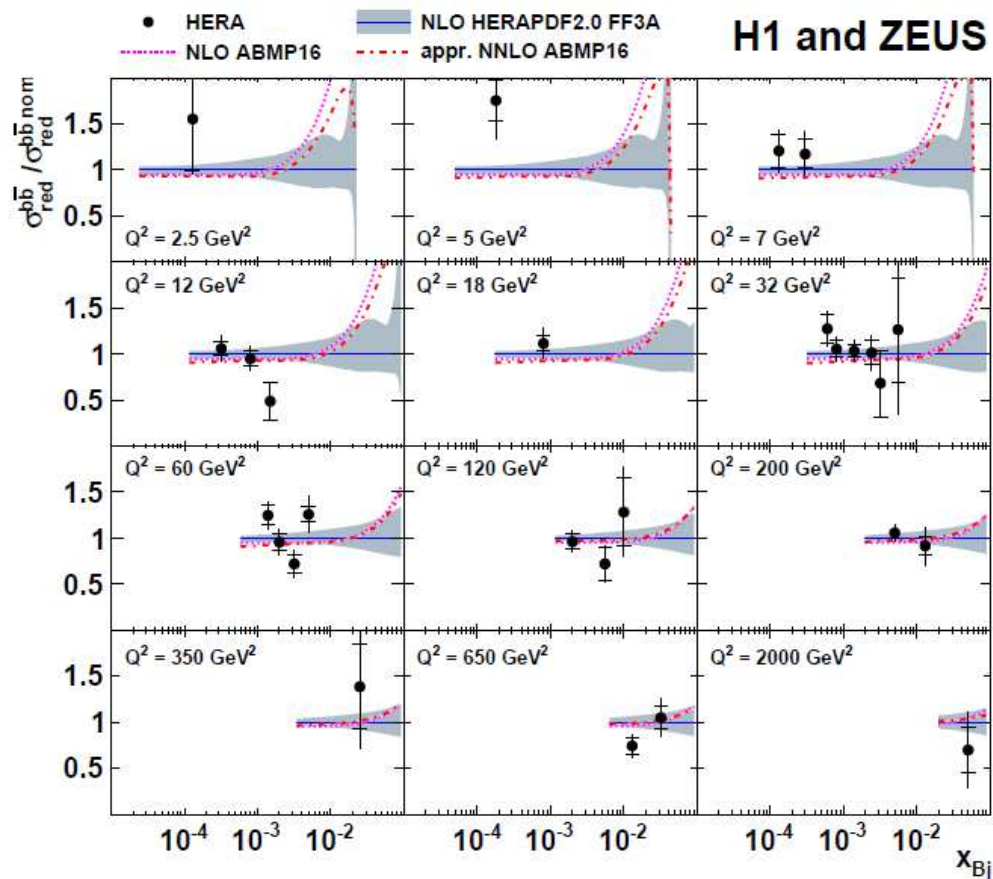
beauty:





# Comparison to FFNS and VFNS predictions

## Beauty:



# QCD fit: systematic uncertainties



arXiv:1804.01019



Parameter	Variation	$m_c(m_c)$ uncertainty [GeV]	$m_b(m_b)$ uncertainty [GeV]
Experimental / Fit uncertainty			
Total	$\Delta\chi^2 = 1$	+0.046 -0.041	+0.104 -0.109
Model uncertainty			
$f_s$	$0.4^{+0.1}_{-0.1}$	-0.003 +0.004	-0.001 +0.001
$Q_{\min}^2$	$3.5^{+1.5}_{-1.0} \text{ GeV}^2$	-0.001 +0.007	-0.005 +0.007
$\mu_{r,f}$	$\mu_{r,f} \times 2.0$ $\mu_{r,f} \times 0.5$	+0.030 +0.060	-0.032 +0.090
$\alpha_s^{n_f=3}(M_Z)$	$0.1060^{+0.0015}_{-0.0015}$	-0.014 +0.011	+0.002 -0.005
Total		+0.062 -0.014	+0.090 -0.032
PDF parameterisation uncertainty			
$\mu_{f,0}^2$	$1.9 \pm 0.3 \text{ GeV}^2$	+0.003 -0.001	-0.001 +0.001
$E_{u\psi}$	set to 0	-0.031	-0.031
Total		+0.003 -0.031	+0.001 -0.031

Table 5: List of uncertainties for the charm- and beauty-quark mass determination. The PDF parameterisation uncertainties not shown have no effect on  $m_c(m_c)$  and  $m_b(m_b)$ .

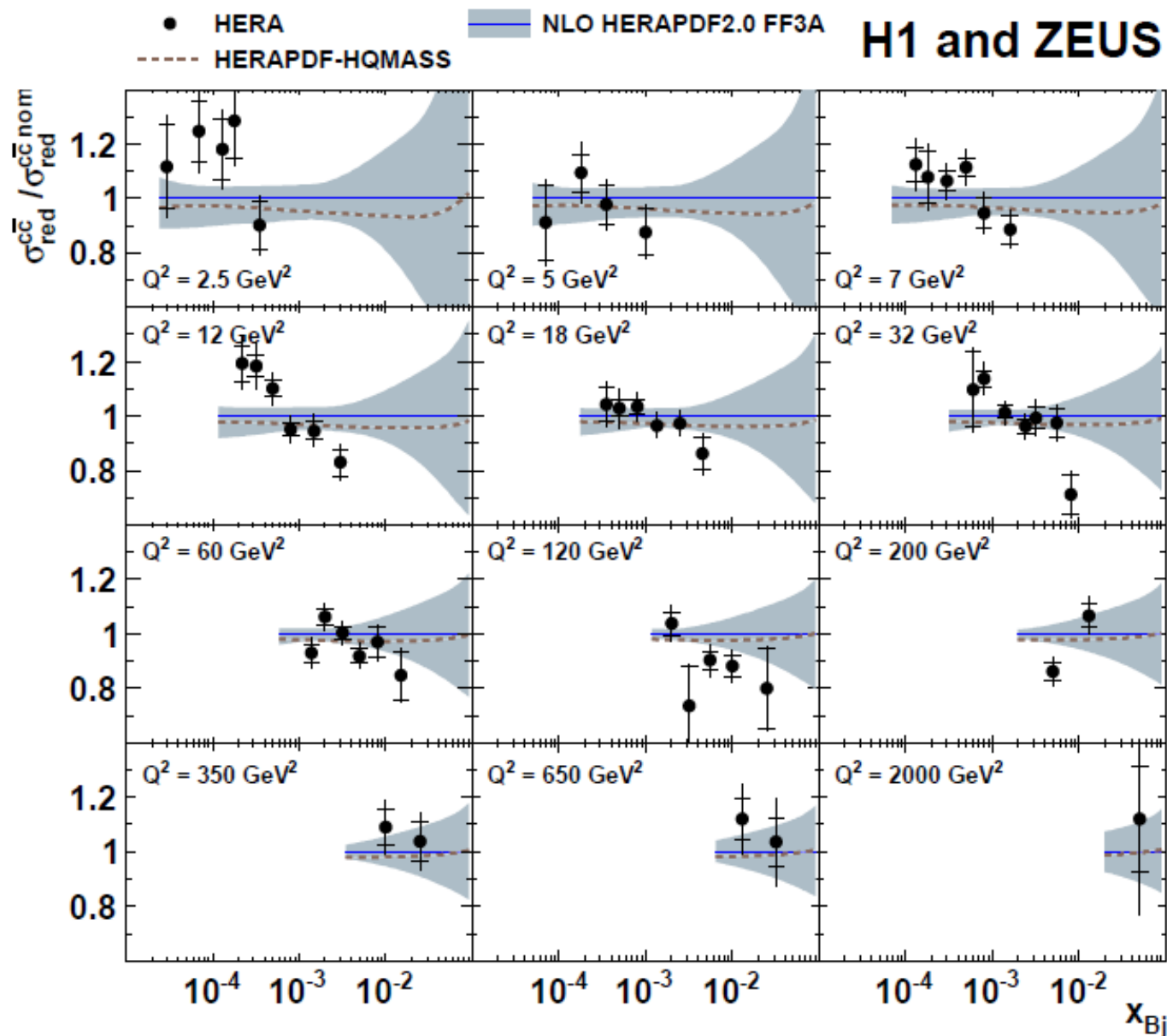
# QCD fit: charm



arXiv:1804.01019



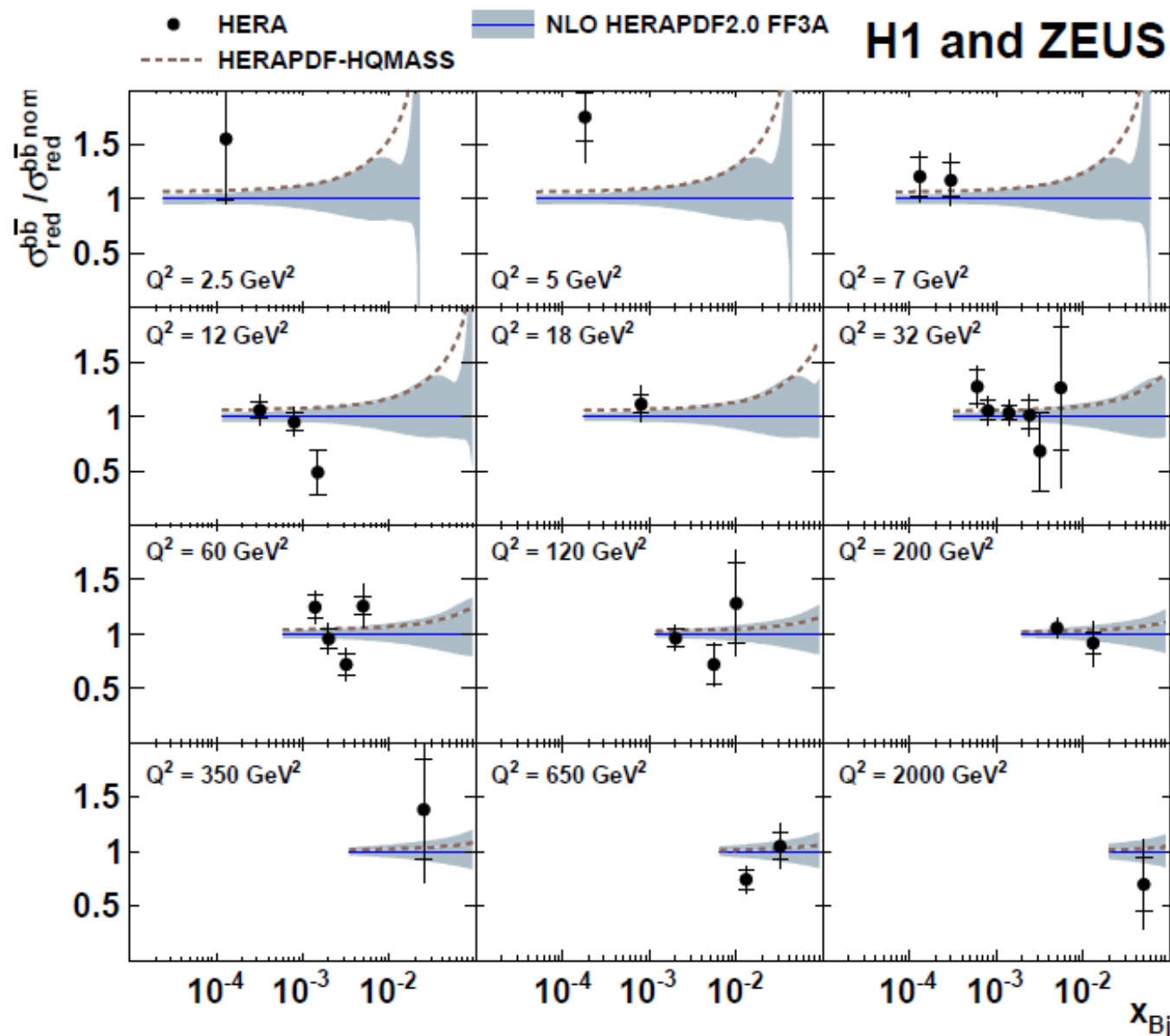
fully consistent  
with  
HERAPDF2.0FF3A



# QCD fit: beauty



arXiv:1804.01019



fully consistent  
with  
HERAPDF2.0FF3A

# QCD fit: beauty x slope



arXiv:1804.01019

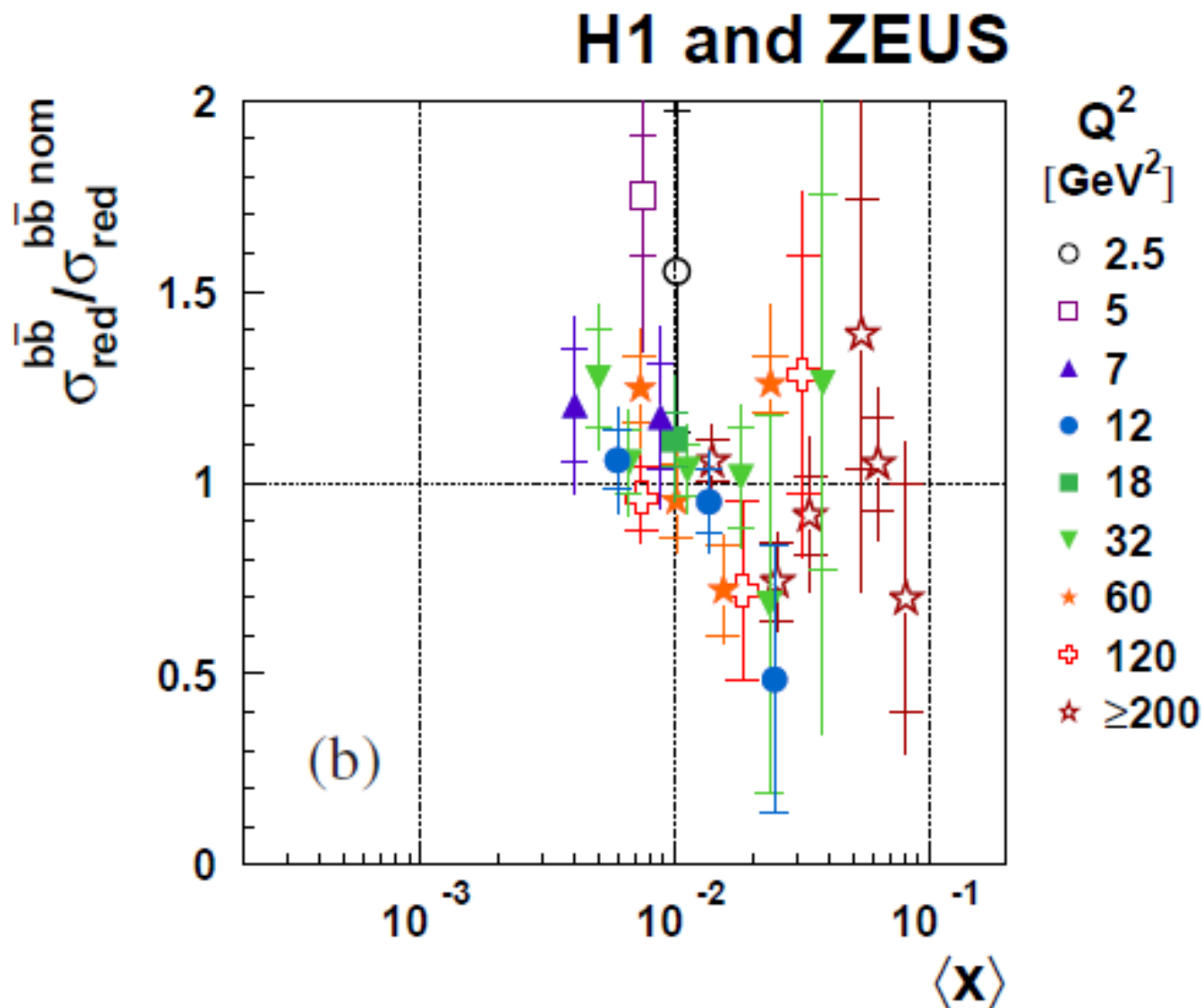


plot data/fit  
vs.  $\langle x \rangle$  of  
incoming partons  
(rather than  $x_{Bj}$ )  
for each data point

$$\text{LO: } x = x_{Bj} \cdot \left(1 + \frac{\hat{s}}{Q^2}\right)$$

$\langle x \rangle$  calculated at NLO  
using HVQDIS

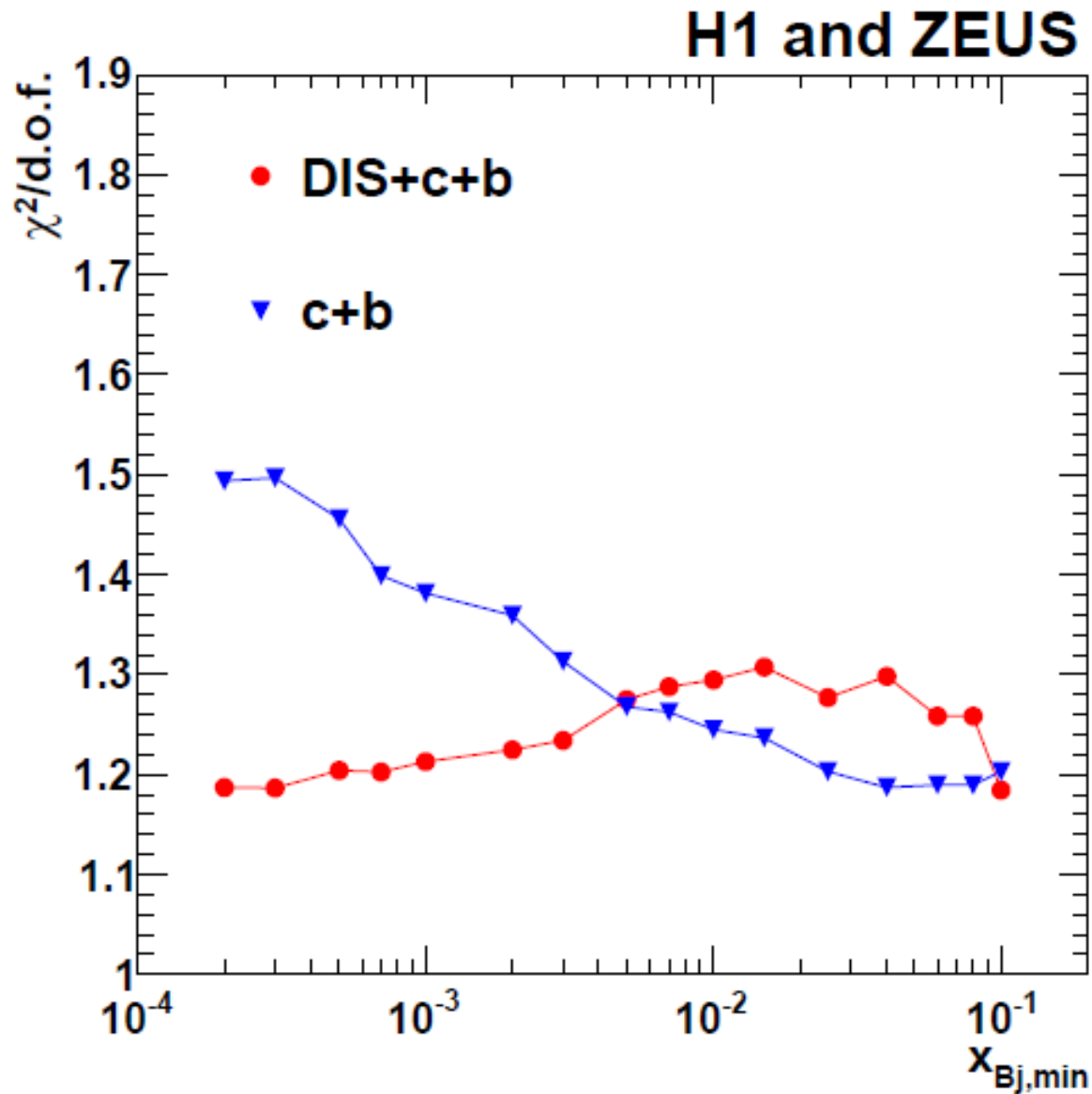
-> beauty consistent  
with charm but does  
not add information



# $\chi^2$ as function of min. $x_{Bj}$ cut



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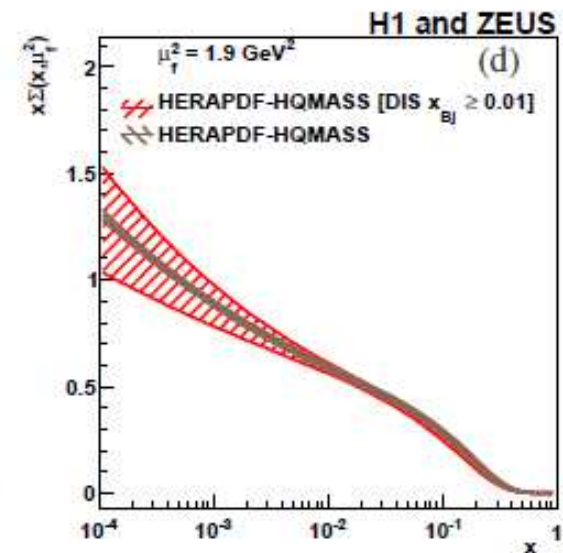
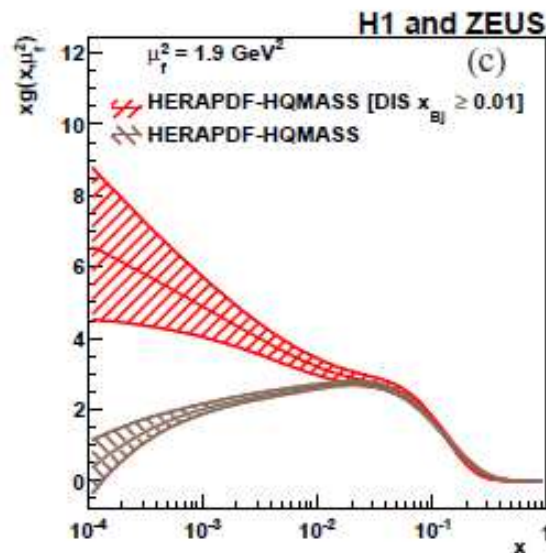
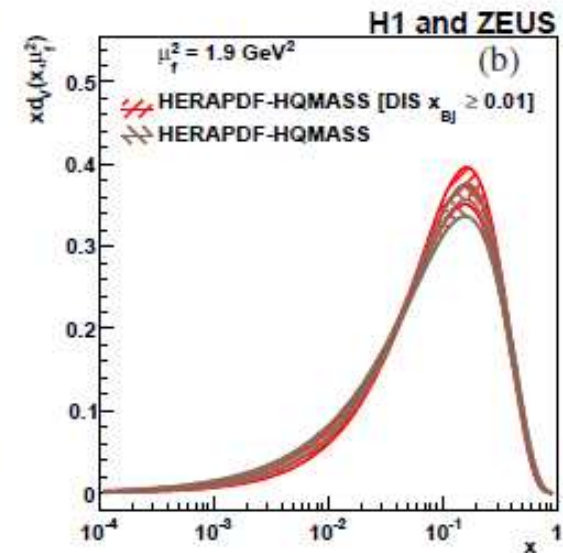
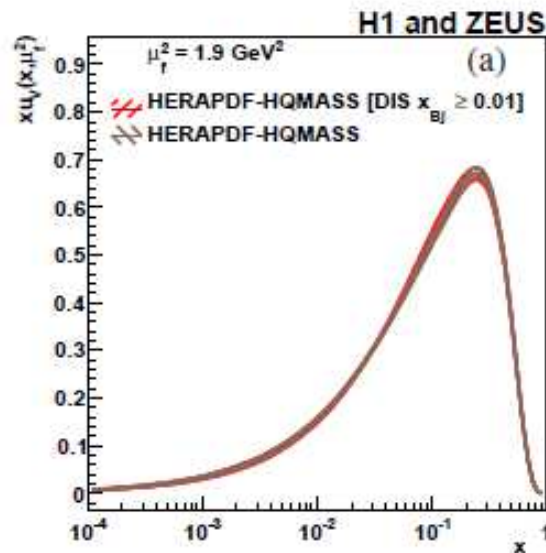
# QCD fit with $x_{Bj} > 0.01$ for inclusive data

arXiv:1804.01019



charm and  
beauty mass  
floating

gluon at  $x < 0.01$   
inconsistent  
with  
inclusive fit



# FONLL-C fit of inclusive data

arXiv:1802.00064 (XFitter team):

**FONLL-C inclusive fit with and without NLLx resummation**

personal remark:

FONLL-C inclusive fit with NLLx qualitatively consistent with FF charm  
+  $x > 0.01$  inclusive fit (compare previous slide)

-> combine both worlds by applying NLLx to light flavours only in FF scheme?

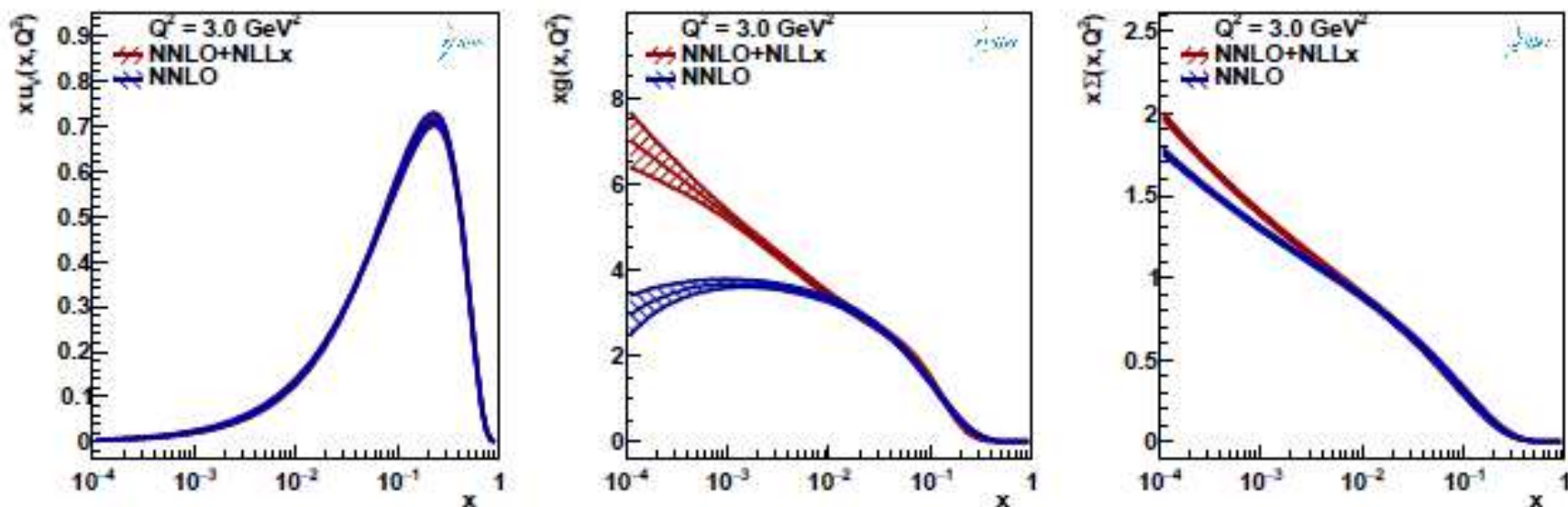


Figure 3 The up valence PDF  $xu_v$ , the gluon PDF  $xg$  and the total singlet PDF  $x\Sigma$  for the final fits with (NNLO+NLLx) and without (NNLO)  $\ln(1/x)$  resummation.



# Comparison of HERAPDF with FONLL-C + NLLx

for inclusive data only

from

arXiv:1802.00064 :

	Step-1	Step-2	Step-3	Step-4
	HERAPDF2.0 NNLO	FONLL-C	Move $Q_0^2$ and charm threshold	include NLLx resummation
HERA $\chi^2$ /d.o.f	1363/1131	1387/1131	1389/1131	1316/1131

Table 1 The  $\chi^2$  per degree of freedom (d.o.f.) for the PDF fits under different conditions, starting from the settings for the HERAPDF2.0 NNLO.

from

arXiv:1506.06042:

HERAPDF	$Q_{\min}^2$ [GeV <sup>2</sup> ]	$\chi^2$	d.o.f.	$\chi^2$ /d.o.f
2.0 NLO	3.5	1357	1131	1.200
2.0HiQ2 NLO	10.0	1156	1002	1.154
2.0 NNLO	3.5	1363	1131	1.205
2.0HiQ2 NNLO	10.0	1146	1002	1.144
2.0 AG NLO	3.5	1359	1132	1.201
2.0HiQ2 AG NLO	10.0	1161	1003	1.158
2.0 AG NNLO	3.5	1385	1132	1.223
2.0HiQ2 AG NNLO	10.0	1175	1003	1.171
2.0 NLO FF3A	3.5	1351	1131	1.195
2.0 NLO FF3B	3.5	1315	1131	1.163
2.0Jets $\alpha_s(M_Z^2)$ fixed	3.5	1568	1340	1.170
2.0Jets $\alpha_s(M_Z^2)$ free	3.5	1568	1339	1.171

Table 4: The values of  $\chi^2$  per degree of freedom for HERAPDF2.0 and its variants.

inclusion of NLLx resummation with FONLL-C achieves similar performance as HERAPDF2.0 FF3B