Appendix A: Reconstruction Of x_γ

The basic resolved photoproduction reaction is shown in figure A.1. Given that the incoming photon four-momentum is **q**, the incoming momentum of the parton from the photon is x_γ**q** (neglecting any primordial p_T of the parton). Similarly, if the proton has incoming four-momentum **p**, the parton from the proton has x_p**p**. The outgoing scattered partons have four-momenta **j** and **k**, the photon remnant **r** and the proton remnant **f**.

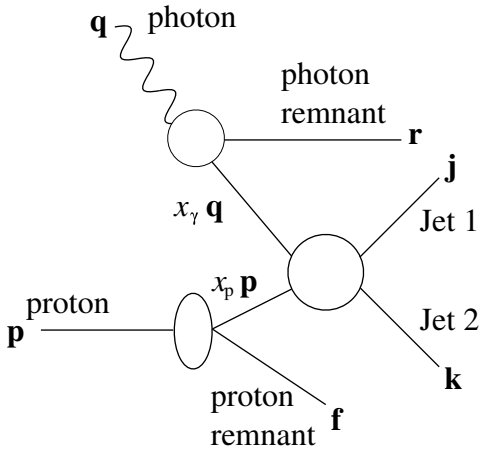

Figure A.1: Kinematics of a resolved photoproduction interaction.

Four-momentum conservation requires that

$$x_\gamma\ \mathbf{q} + x_p\ \mathbf{p} = \mathbf{j} + \mathbf{k}$$

If the assumption is made that the initial p_T is zero, the final p_T must also be zero. Two equations are then obtained, one for energy conservation

$$x_\gamma E_\gamma + x_p E_p = E_j + E_k$$

and one for conservation of the z-component of momentum

$$x_\gamma\ p_{z\gamma} + x_p\ p_{zp} = p_{zj} + p_{zk}$$

where E_i and p_{zi} are the energy and z-component of momentum of four-vector i. Substituting for x_p gives

$$x_\gamma E_\gamma + \frac{E_p}{p_{zp}}\left(p_{zj} + p_{zk} - x_\gamma\ p_{z\gamma}\right) = E_j + E_k$$

If the incoming particles are massless and have negligible p_T, $E_p = p_{zp}$ and $E_\gamma = -p_{z\gamma}$, which gives

$$x_\gamma = \frac{\left(E_j - p_{zj}\right) + \left(E_k - p_{zk}\right)}{2E_\gamma}$$

The transverse energy, E_T, of a massless particle may be re-written as

$$E_T = \sqrt{E^2 - p_z^2} = \sqrt{(E - p_z)(E + p_z)}$$

and by exponentiating rapidity, j, the following expression is obtained

$$\exp(-j) = \exp\left(-\frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right)\right) = \sqrt{\frac{E - p_z}{E + p_z}}$$

Hence,

$$E_T\ \exp(-j) = \sqrt{(E - p_z)(E + p_z)}\sqrt{\frac{E - p_z}{E + p_z}} = E - p_z$$

Substituting pseudo-rapidity, η, as an approximation to rapidity,

$$x_\gamma = \frac{E_{Tj}\ \exp\left(-\eta_j\right) + E_{Tk}\ \exp\left(-\eta_k\right)}{2E_\gamma}$$

An expression for x_γ may also be obtained by using the photon remnant four-vector **r**. Since

$$x_\gamma \mathbf{q} = \mathbf{q} - \mathbf{r}$$

multiplying throughout by the proton four-momentum, **p**, and re-arranging gives

$$x_\gamma = 1 - \frac{\mathbf{r} \cdot \mathbf{p}}{\mathbf{q} \cdot \mathbf{p}}$$

The fraction of the photon's four-momentum taken by the remnant, x_{rem} is therefore given by

$$x_{rem} = \frac{\mathbf{r} \cdot \mathbf{p}}{\mathbf{q} \cdot \mathbf{p}}$$

as the checksum

$$x_\gamma + x_{rem} = 1$$

must be true in order to conserve four-momentum.

Appendix B: Correlation Coefficients

The correlation coefficient, ρ, between two distributions x and y is defined as

$$\rho = \frac{\mathrm{cov}(x\ ,\ y)}{\sigma_x \sigma_y} = \frac{\overline{xy}\ -\ \overline{x}\ \overline{y}}{\sigma_x \sigma_y}$$

where σ_x and σ_y are the sigma values of the x and y distributions, $\mathrm{cov}(x,y)$ is the element of the covariance matrix between the two, and a bar over a quantity indicates the mean of that quantity.

The correlation coefficient ranges from −1.0 to +1.0, with −1.0 or +1.0 indicating no scatter (the two variables being exactly related) and a value of zero indicating that the variables are independent. Some care is necessary in evaluating ρ as the above formula is biased when used as an estimator. The unbiased estimator

$$\hat{\rho} = \frac{\overline{xy}\ -\ \overline{x}\ \overline{y}}{(N\ -\ 1) s_x s_y}$$

was therefore used, where N is the number of events and s_x, s_y are the unbiased estimators of the sigma of the x and y distributions given by

$$s = \sqrt{\frac{1}{N\ -\ 1} \sum\ (x_i\ -\ \overline{x})^2}$$

The error on the correlation coefficient may best be estimated by transforming to the variable z,

$$z = \frac{1}{2}\ \ln\left(\frac{1\ +\ \hat{\rho}}{1\ -\ \hat{\rho}}\right)$$

where the error is approximately a Gaussian of sigma

$$\sigma = \frac{1}{\sqrt{N\ -\ 3}}$$

so long as the distributions of x and y are approximately Gaussian.