DESY 95–185
hep-ph/9511434
November 1995

HECTOR 1.00
A program for the calculation of QED, QCD and electroweak corrections to ep and $l^\pm N$ deep inelastic neutral and charged current scattering *

A. Arbuzov[1#], D. Bardin[1,2], J. Blümlein[2], L. Kalinovskaya[1], T. Riemann[2]

[1]*Bogoliubov Laboratory for Theoretical Physics, JINR, ul. Joliot-Curie 6, RU-141980 Dubna, Russia*
[2]*DESY – Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany*

ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of $\mathcal{NC}$ and $\mathcal{CC}$ deep inelastic charged lepton proton (or lepton deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for applications at HERA or LEP$\otimes$LHC, but may be used also for μN scattering in fixed target experiments. The QED corrections may be calculated in different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading logarithmic approximation up to order $\mathcal{O}(\alpha^2)$, exact $\mathcal{O}(\alpha)$ corrections and inclusive soft photon exponentiation are taken into account. The photoproduction region is also covered.

* Supported by the EC network 'Capital Humain et Mobilite' under grant CHRX–CT92–0004.
# Supported by the Heisenberg-Landau fund.
e–mail: hector@ifh.de.

Contents

List of Tables

List of Figures

HECTOR, son of the Troyan King PRIAM. Troyan war leader. HOMER describes HECTOR to be *open, frank, brave, cheerful in adversity, and tenderly compassionate* in the *Ilyad* [1]. Drawing by J. Flaxman [2].

PROGRAM SUMMARY

Title of program: HECTOR
Version: 1.00 November 1995
Catalogue number:
Program obtainable from:
http://www.ifh.de/theory or on request from e-mail: hector@ifh.de
Licensing provisions: non
Computers: all
Operating system: all
Program language: FORTRAN-77
Memory required to execute with typical data: 4.2 Mb.
No. of bits per word: 64;
the branch TERADLOW uses 128 bits per word for numerical quadric precision
No. of lines in distributed program: 20.000
Other programs called:
libraries KERNLIB [1];
FFREAD – part of PACKLIB [2];
DIZET – an electroweak library [3];
optionally possible: PDFLIB [4].
External files needed:
HECTOR.INP – input cards to be read by the FFREAD package.
Keywords: deep inelastic scattering, QED, electroweak and QCD radiative corrections, structure functions, higher order corrections.
Nature of physical problem:
First and higher order QED radiative corrections to the lepton nucleon deep inelastic scattering; virtual electroweak and QCD corrections to the process; QCD corrections to structure functions. *Method of solution*:
Numerical integration of analytical formulae.
Restrictions on complexity of the problem:
Only selective experimental cuts are possible. Results for full calculations in order $\mathcal{O}(\alpha)$ are not available for all possible kinematical variables.
Typical running time:
The running time strongly depends upon the options used. One finds e.g.: LLA, leptonic variables, $\mathcal{NC}$, 25 points: about 25 sec. (CPU time), full $\mathcal{O}(\alpha)$ + $\mathcal{O}(\alpha^2)$ LLA + soft exponentiation + electroweak nonrunning couplings, leptonic variables, $\mathcal{NC}$, 25 points: about 600 sec.
References:

[1] CERN Program Library Z 001.

[2] R. Brun, et al., FFREAD User Guide and Reference Manual, CERN DD/US/71, CERN Program Library I 302, February 1987.

[3] D. Bardin et al., *Comput. Phys. Commun.* **59** (1990) 303.

[4] H. Plothow–Besch, *Comput. Phys. Commun.* **75** (1993) 396.

LONG WRITE-UP

1 Introduction

HECTOR is a program to calculate radiative corrections (RC) to the charged lepton nucleon neutral and charged current deep inelastic scattering (DIS) $l^{\pm}N \to l'X$. This requires to take into account the full set of virtual electroweak (EW) corrections, QCD corrections, and the complete QED corrections in lowest order, as well as the leading higher order QED corrections. The QCD radiative corrections are implemented up to next-to-leading order (NLO). They depend on the factorization scheme used. The $\mathcal{O}(\alpha)$ QED radiative corrections due to radiation from the lepton lines are known to be large and can reach the order of 100% in some regions of phase space. This makes, at least in parts of the phase space, the inclusion of higher order corrections indispensable. The aim of the project is to obtain a description at the percent level.

The program originates from two different previous codes: `HELIOS` [3] and `TERAD91` [4]. `HELIOS` calculates QED RC in the leading logarithmic approximation (LLA) in the $\mathcal{O}(\alpha)$ order. The program is based on refs. [5]–[7]. The $\mathcal{O}(\alpha^2 L^2)$ corrections and soft photon exponentiation have been included recently [8]. `TERAD91` calculates complete QED and the EW corrections in the $\mathcal{O}(\alpha)$ order. It is based on a series of papers [9]–[14]. The current Fortran program `HECTOR` combines the two methods.

2 Basic notations

2.1 Born cross sections

`HECTOR` allows the calculation of neutral current ($\mathcal{NC}$) and charged current ($\mathcal{CC}$) Born cross sections.

The $\mathcal{NC}$ cross section may be calculated in several ways and approximations: in the ultra-relativistic approximation or exact in all masses; neglecting the longitudinal structure functions or not; without or with electroweak one loop and higher order corrections; the latter case is called improved Born cross section. The Born cross section is calculated in subroutine `sigbrn.f`:

$$\frac{d^2\sigma^{\rm B}_{\rm NC}}{dxdy} = \frac{2\pi\alpha^2(Q^2)S}{Q^4}\left\{\left[2\,(1-y) - 2xy\frac{M^2}{S} + \left(1-2\frac{m^2}{Q^2}\right)\left(1+4x^2\frac{M^2}{Q^2}\right)\frac{y^2}{1+R}\right]\mathcal{F}_2(x,Q^2) + xy(2-y)\mathcal{F}_3(x,Q^2)\right\}. \tag{2.1}$$

We also introduce a slightly modified notion for later use:

$$\frac{d^2\sigma^{\rm B}_{\rm NC}}{dxdy} = \frac{2\pi\alpha^2(Q^2)S}{Q^4}\left\{\left[\frac{1}{S^2}\mathcal{S}^{\rm B}_2(y,Q^2) + \frac{1}{Q^2}\mathcal{S}^{\rm B}_1(y,Q^2)\left(1+4x^2\frac{M^2}{Q^2}\right)\frac{y^2}{1+R}\right]\mathcal{F}_2(x,Q^2) + \frac{1}{2S^2}\mathcal{S}^{\rm B}_3(y,Q^2)\mathcal{F}_3(x,Q^2)\right\}, \tag{2.2}$$

with

$$\begin{aligned}
\mathcal{S}_1^{\mathrm{B}}(y,Q^2) &= Q^2 - 2m^2, && (2.3)\\
\mathcal{S}_2^{\mathrm{B}}(y,Q^2) &= 2[(1-y)S^2 - M^2Q^2], && (2.4)\\
\mathcal{S}_3^{\mathrm{B}}(y,Q^2) &= 2Q^2(2-y)S. && (2.5)
\end{aligned}$$

Here, M and m are the proton and lepton masses, $s = -(k_1+p_1)^2 = S + m^2 + M^2 \approx 4E_eE_p$, x and y denote the Bjorken variables, and $q^2 = -Q^2$ is the four momentum transfer squared. These variables will be discussed in more detail in appendix A. The generalized structure functions are denoted by $\mathcal{F}_i(x,Q^2)$ and R is the ratio of the cross sections with virtual exchange of longitudinal and transverse photons, respectively:

$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \left(1 + 4x^2\frac{M^2}{Q^2}\right)\frac{\mathcal{F}_2(x,Q^2)}{2x\mathcal{F}_1(x,Q^2)} - 1. \tag{2.6}$$

These relations are thoroughly used by **HECTOR**. We mention the ultra-relativistic approximation of the Born cross section:

$$\frac{d^2\sigma^{\mathrm{B}}}{dxdy} = \frac{2\pi\alpha^2(Q^2)S}{Q^4}\left[Y_+\mathcal{F}_2(x,Q^2) + Y_-x\mathcal{F}_3(x,Q^2) - y^2\mathcal{F}_L(x,Q^2)\right], \tag{2.7}$$

with

$$Y_\pm(y) = 1 \pm (1-y)^2. \tag{2.8}$$

For the $\mathcal{CC}$ reaction, a similar cross section formula holds:

$$\frac{d^2\sigma^{\mathrm{B}}_{CC}}{dxdy} = \frac{G_\mu^2 S}{4\pi}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2\left\{Y_+\mathcal{W}_2(x,Q^2) + Y_-\mathcal{W}_3(x,Q^2) - y^2\mathcal{W}_L(x,Q^2)\right\}. \tag{2.9}$$

The generalized structure functions for the $\mathcal{NC}$ case ($\mathcal{F}$) and for the $\mathcal{CC}$ case ($\mathcal{W}$) are defined in section 2.2.1, while the relations of the basic input parameters follow in the next section, and the description of the running of α in section 2.1.2.

In a lowest order description one may assume the validity of Callan–Gross relations [15] for $\mathcal{F}_{1,2}$ and $\mathcal{W}_{1,2}$:

$$\begin{aligned}
\mathcal{F}_L(x,Q^2) &\equiv \mathcal{F}_2(x,Q^2) - 2x\mathcal{F}_1(x,Q^2) = 0,\\
\mathcal{W}_L(x,Q^2) &\equiv \mathcal{W}_2(x,Q^2) - 2x\mathcal{W}_1(x,Q^2) = 0. && (2.10)
\end{aligned}$$

2.1.1 Input parameters

HECTOR takes into account that not all the physical parameters are independent of each other in the Standard Model. There are three sets of input values foreseen. The default set is chosen in file `hecset.f` with flag setting **IMOMS**=1. For this, the input is expressed by the following parameters:

- fine structure constant $\alpha = 1/137.0359895$; its running is discussed in section 2.1.2;
- muon decay constant $G_\mu = 1.166388 \cdot 10^{-5}$ GeV^{-2};
- Z boson mass $M_Z = 91.175$ GeV;

- fermion masses, including the top quark mass $m_t = 180$ GeV;
- Higgs boson mass M_H, for which the default value of 300 GeV is assumed;
- QCD parameter $\Lambda_{\overline{\rm MS}}$, which is chosen in file `alpqcd.f` in accordance with the pre-selected `PDF` set (set of parton distribution functions).

The default values of constants are set in file `hecset.f`. The strong coupling constant α_s is calculated from $\Lambda_{\overline{\rm MS}}$ as described in section 2.3.1.

Several additional parameters may be calculated from the above input. The most important ones are the W boson mass:

$$M_W = M_Z \left\{ \frac{1}{2} + \frac{1}{2} \left[1 - \left(\frac{74.562}{M_Z} \right)^2 \frac{1}{1-\Delta r} \right]^{1/2} \right\}^{1/2}, \tag{2.11}$$

and the weak mixing angle:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}. \tag{2.12}$$

Details on the definition of Δr within `HECTOR` may be found in [14, 16]. It depends on the chosen electroweak parameters and on the fermion masses.

If one wants to use the W mass as an input parameter instead of M_Z, the default setting is:

- $M_W = 80.22$ GeV.

Then, one has to determine M_Z by iteratively solving the equation:

$$M_Z = M_W \left[1 - \left(\frac{74.562}{M_W} \right)^2 \frac{1}{1-\Delta r} \right]^{-1/2}. \tag{2.13}$$

This is done when using `IMOMS`=2.

A third possibility, `IMOMS`=3, is to take both M_Z and M_W together with α as an input and to calculate G_μ:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 \sin^2\theta_W (1-\Delta r)}. \tag{2.14}$$

For any setting of the flag `IMOMS`, it is also possible to use an effective weak mixing angle[1] *instead* of the definition (2.12). This is done by choosing the flag `IWEA`=0, which switches off the calculation of virtual weak corrections. The weak mixing angle as defined in (2.12) is not used for the calculation of weak neutral couplings in this case:

$$\sin^2\theta_W \to \sin^2\theta_W^{\rm eff}. \tag{2.15}$$

In file `hecset.f` a default value is given:

$$\sin^2\theta_W^{\rm eff} = 0.232. \tag{2.16}$$

[1] For more details, see the discussion in [16] on the definition of $\sin^2\theta_W^{\rm eff}$.

2.1.2 The running electromagnetic coupling $\alpha(Q^2)$

The running of the QED coupling is taken into account if IVPL=1. Then, instead of α, the value

$$\alpha(Q^2) = \frac{\alpha}{1-\Delta\alpha} \tag{2.17}$$

is used. The correction consists of the pieces:

$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha_{udcsb} + \Delta\alpha_t \tag{2.18}$$

corresponding to the charged leptons, the lighter quarks (u-b), and the top quark.

The value of $\Delta\alpha$ is calculated in function XFOTF1 of the package DIZET. The contribution due to leptons is:

$$\Delta\alpha_l = \sum_{f=e,\mu,\tau} Q_f^2 N_f \Delta F_f(Q^2), \tag{2.19}$$

$$\Delta F_f(Q^2) = \frac{\alpha}{\pi}\left\{-\frac{5}{9}+\frac{4}{3}\frac{m_f^2}{Q^2}+\frac{1}{3}\beta_f\left(1-\frac{2m_f^2}{Q^2}\right)\ln\frac{\beta_f+1}{\beta_f-1}\right\}, \tag{2.20}$$

$$\beta_f = \sqrt{1+\frac{4m_f^2}{Q^2}}. \tag{2.21}$$

The color factor, N_f, and the charge squared, Q_f^2, are unity for leptons. In the weak library, ΔF_f is calculated by the function XI3: $\Delta F_f = 2$ XI3. The contribution from the light quarks has been parametrised in two different ways. It may be calculated by (2.19) setting IHVP=2. In this case, we use effective quark masses [17, 18]: $m_u = m_d = 0.041, m_s = 0.15, m_c = 1.5, m_b = 4.5$ GeV. The preferred, default approach (with IHVP=1) uses a parameterization of the hadronic vacuum polarization [19], which is contained in file hadr5.f. An older parameterization [20] may also be accessed choosing IHVP=3.

Finally, the t quark corrections are:

$$\Delta\alpha_t = Q_t^2 N_t \Delta F_t(Q^2) + \Delta\alpha^{2\mathrm{loop},\alpha\alpha_s}. \tag{2.22}$$

The last term contains higher order corrections and is calculated by the functions ALQCDS (approximate) or ALQCD (exact):

$$\Delta\alpha^{2\mathrm{loop},\alpha\alpha_s} = \frac{\alpha\alpha_s}{3\pi^2}Q_t^2\frac{m_t^2}{Q^2}\left\{\mathcal{R}e\Pi_t^{VF}(Q^2)+\frac{45}{4}\frac{Q^2}{m_t^2}\right\}. \tag{2.23}$$

$\Pi_t^{VF}(Q^2)$ is a two loop self energy function [21, 22]. For IQCD=0 (default value), this tiny correction is neglected. For IQCD=1 approximate and for IQCD=2 exact calculations according to (2.23) are made. The first one is a crude approximation only, the second, on the other hand, results in a very time consuming computation.

2.2 Structure functions

2.2.1 Generalized structure functions

The generalized structure functions describe the electroweak interactions of leptons with beam particle charge Q_l and lepton beam polarization ξ via the exchange of a photon, Z boson, or W boson with unpolarized nucleons.

The generalized structure functions are calculated for the $\mathcal{NC}$ cross sections in file **genstf.f** or, for the low Q^2 branch `TERADLOW`, in file **gsflow.f**, and for the $\mathcal{CC}$ cross sections in file **gccstf.f**. The exact treatment of virtual electroweak corrections (for the choice `IWEAK`=1) will be described in section 2.2.3. Here we assume `IWEAK`=0. Then, the generalized $\mathcal{NC}$ structure functions $\mathcal{F}_i(x,Q^2)$ are expressed in terms of the usual *hadronic* structure functions F_i, G_i, H_i by the following equations:

$$\begin{aligned}
\mathcal{F}_{1,2}(x,Q^2) &= F_{1,2}(x,Q^2) + 2|Q_e|\,(v_e+\lambda a_e)\,\chi(Q^2)G_{1,2}(x,Q^2) \\
&\quad + 4\left(v_e^2+a_e^2+2\lambda v_e a_e\right)\chi^2(Q^2)H_{1,2}(x,Q^2), \qquad (2.24)\\
x\mathcal{F}_3(x,Q^2) &= -2\ \mathrm{sign}(Q_l)\Big\{|Q_e|\,(a_e+\lambda v_e)\,\chi(Q^2)xG_3(x,Q^2) \\
&\quad + \left[2v_e a_e + \lambda\left(v_e^2+a_e^2\right)\right]\chi^2(Q^2)xH_3(x,Q^2)\Big\}, \qquad (2.25)
\end{aligned}$$

with $Q_e=-1$ and

$$\begin{aligned}
\lambda &= \xi\,\mathrm{sign}(Q_l), &(2.26)\\
v_e &= 1-4\sin^2\theta_W^{\mathrm{eff}}, &(2.27)\\
a_e &= 1, &(2.28)
\end{aligned}$$

and

$$\chi(Q^2) = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha(Q^2)}\frac{Q^2}{Q^2+M_Z^2}. \qquad (2.29)$$

The structure functions $F_{1,2}, G_{1,2}, H_{1,2}$ and G_3, H_3 are defined in the next section.

The charged current generalized structure functions for `IWEAK`=0 are:

$$\begin{aligned}
\mathcal{W}_2(x,Q^2) &= \frac{1+\lambda}{2}W_2^{Q_l}(x,Q^2), &(2.30)\\
\mathcal{W}_3(x,Q^2) &= -\mathrm{sign}(Q_l)\frac{1+\lambda}{2}xW_3^{Q_l}(x,Q^2). &(2.31)
\end{aligned}$$

They are as well defined in the subsequent section.

For the case of an exact inclusion of the electroweak virtual corrections, we refer to section 2.2.3. There we will see that the notion of the generalized structure functions holds further on, but that of hadronic structure functions will be lost.

The photon exchange parts of the generalized structure functions have to vanish at low Q^2. This is ensured by non-perturbative modifications of the corresponding structure functions as described in section 2.4.

2.2.2 Structure functions

There are several ways to choose structure functions in `HECTOR`. This will be described in detail in section 3.

One option is to derive structure functions from parton distribution functions (PDF). The different structure functions are in a lowest order approach[2]

$$
\begin{aligned}
F_2(x,Q^2) &= x\sum_q |Q_q|^2\,[q(x,Q^2)+\bar{q}(x,Q^2)], && (2.32)\\
G_2(x,Q^2) &= x\sum_q |Q_q|\,v_q\,[q(x,Q^2)+\bar{q}(x,Q^2)], && (2.33)\\
H_2(x,Q^2) &= x\sum_q \frac{1}{4}\left(v_q^2+a_q^2\right)[q(x,Q^2)+\bar{q}(x,Q^2)], && (2.34)\\
xG_3(x,Q^2) &= x\sum_q |Q_q|\,a_q\,[q(x,Q^2)-\bar{q}(x,Q^2)], && (2.35)\\
xH_3(x,Q^2) &= x\sum_q \frac{1}{2}v_q a_q\,[q(x,Q^2)-\bar{q}(x,Q^2)], && (2.36)
\end{aligned}
$$

where $q(x,Q^2)$ ($\overline{q}(x,Q^2)$) denote the quark (antiquark) distributions. The neutral current couplings are in our notations:

$$
\begin{aligned}
v_f &= 1-4|Q_f|\sin^2\theta_W^{\text{eff}}, && (2.37)\\
a_f &= 1. && (2.38)
\end{aligned}
$$

Here, θ_W^{eff} is the effective weak mixing angle introduced in section 2.1.1 and $s_e|Q_e| = Q_e = -1$ is the electron charge.

The $\mathcal{CC}$ structure functions are:

$$
\begin{aligned}
W_2^+(x,Q^2) &= 2x\sum_i \left[d_i(x,Q^2)+\bar{u}_i(x,Q^2)\right], && (2.39)\\
W_2^-(x,Q^2) &= 2x\sum_i \left[u_i(x,Q^2)+\bar{d}_i(x,Q^2)\right], && (2.40)\\
xW_3^+(x,Q^2) &= 2x\sum_i \left[d_i(x,Q^2)-\bar{u}_i(x,Q^2)\right], && (2.41)\\
xW_3^-(x,Q^2) &= 2x\sum_i \left[u_i(x,Q^2)-\bar{d}_i(x,Q^2)\right], && (2.42)
\end{aligned}
$$

with u_i and d_i denoting the up (u,c,t) and down quark (d,s,b) densities, respectively.

The structure functions $\mathcal{S}_1(x,Q^2) = F_1(x,Q^2), G_1(x,Q^2), H_1(x,Q^2), W_1^-(x,Q^2)$ are calculated in `HECTOR` by

$$
2x\mathcal{S}_1 = \mathcal{S}_2 - \mathcal{S}_L, \qquad (2.43)
$$

and analogously for the other distributions. The longitudinal structure functions vanish in lowest order of QCD.

[2]NLO corrections are delt with in section 2.3.

2.2.3 Generalized structure functions for improved Born cross sections

For the introduction of the generalized structure functions in the presence of virtual weak interactions, we use a slightly different notation compared to that of section 2.2.1 in order to make the relation between **HECTOR** and [9] and [10] sufficiently transparent. To these references (and for later improvements also to [16]) we refer for all the details which cannot be described here.

In the $\mathcal{NC}$ case the structure functions are given by:

$$\mathcal{F}_2^{\mathcal{SM}}(x,Q^2) = \sum_{B=\gamma,I,Z} \sum_{i=1}^{3} \sum_{q=Q,\bar{Q}} \mathcal{K}(B,p)\,\mathcal{V}(B,p)\,q_i(x,Q^2), \tag{2.44}$$

$$\mathcal{F}_3^{\mathcal{SM}}(x,Q^2) = \sum_{B=\gamma,I,Z} \sum_{i=1}^{3} \sum_{q=Q,\bar{Q}} p\,\mathcal{K}(B,p)\,\mathcal{A}(B,p)\,q_i(x,Q^2). \tag{2.45}$$

Here, index i denotes the quark generation. $B=\gamma$ stands for the pure photon exchange contribution, Z for the Z exchange and I for their interference. The parameter p depends on the relative charge assignments of the beam particle and the struck quark:

$$\begin{aligned} p &= 1 && \text{for} : e^-Q_i,\ e^+\bar{Q}_i, \\ p &= -1 && \text{for} : e^+Q_i,\ e^-\bar{Q}_i. \end{aligned} \tag{2.46}$$

The function $\mathcal{K}$ contains overall normalizations:

$$\mathcal{K}(\gamma,p) = (Q_e\,Q_Q)^2\,, \tag{2.47}$$

$$\mathcal{K}(I,p) = 2\,|Q_e\,Q_Q|\,\chi(Q^2)\,\rho_Z(p), \tag{2.48}$$

$$\mathcal{K}(Z,p) = [\chi(Q^2)\rho_Z(p)]^2\,. \tag{2.49}$$

The function $\chi(Q^2)$ has been defined in (2.29). The real-valued weak form factor $\rho_Z(p)$ was introduced as $\rho_{(I)}^{Z}(p)$ in [9]. The form factors depend on a variety of variables:

$$\begin{aligned} \rho_Z(+1) &= \rho_Z(xS,Q^2,xU;e,q), \\ \rho_Z(-1) &= \rho_Z(xU,Q^2,xS;e,q), \end{aligned} \tag{2.50}$$

$$U = Q^2 - S. \tag{2.51}$$

An explicit expression for $\rho_Z(S,Q^2,U;a,b)$ in $\mathcal{O}(\alpha)$ including the ZZ,WW box terms, which are crucial here, is given by eq. (A.1) in [9].

The dependence on the weak mixing angle and its radiative corrections, but also that on a longitudinal beam polarization ξ, is contained in the functions $\mathcal{V}$ and $\mathcal{A}$:

$$\begin{aligned} \mathcal{V}(\gamma,p) &= 1, \\ \mathcal{A}(\gamma,p) &= 0, \\ \mathcal{V}(I,p) &= v_{eq} + \lambda a_e v_q, \\ \mathcal{A}(I,p) &= (a_e + \lambda v_e)\,a_q, \\ \mathcal{V}(Z,p) &= \left(v_e^2 + a_e^2\right) a_q^2 a_e^2 v_q^2 + v_{eq}^2 + 2\lambda a_e \left(v_e a_q^2 + v_{eq} v_q\right), \\ \mathcal{A}(Z,p) &= 2\left[a_e a_q \left(v_e v_q + v_{eq}\right) + \lambda a_q \left(v_e v_{eq} + a_e^2 v_q\right)\right]. \end{aligned} \tag{2.52}$$

The vector and axial vector couplings contain finite radiative corrections from the real-valued weak form factors $\kappa_a(p)$:

$$
\begin{aligned}
a_e &= 1, \\
a_q &= 1, \\
v_e &= 1 - 4\sin^2\theta_W |Q_e| \kappa_e(p), \\
v_q &= 1 - 4\sin^2\theta_W |Q_q| \kappa_q(p), \\
v_{eq} &= v_e + v_q - 1 + 16\sin^4\theta_W |Q_e Q_q| \kappa_{eq}(p).
\end{aligned} \tag{2.53}
$$

The weak form factors $\kappa_a(p)$, $a = e, q, eq$, are defined in eqs. (A.2) and (A.3) of [9]:

$$
\begin{aligned}
\kappa_a(+1) &= \kappa_a(xS, Q^2, xU; e, q), \\
\kappa_a(-1) &= \kappa_a(xU, Q^2, xS; e, q).
\end{aligned} \tag{2.54}
$$

Some universal higher order corrections to ρ_Z and κ_a may be found in [16]. The form factors are calculated in `DIZET`, file `rokap.f`, and transferred to other parts of the package as elements of a vector `XFF`.

An approximation to the exact electroweak expressions in terms of an effective weak mixing angle may be introduced as follows:

$$
\begin{aligned}
\rho &\to 1, \\
\kappa_{eq} &\to \kappa_e \kappa_q, \\
\kappa_e \approx \kappa_q &\to \kappa,
\end{aligned} \tag{2.55}
$$

together with the definition

$$
\sin^2\theta_W^{\mathrm{eff}} = \kappa \sin^2\theta_W. \tag{2.56}
$$

(cf. sections 2.1.1, 2.2.1, and 2.2.2). In `HECTOR` a numerical value may be chosen for $\sin^2\theta_W^{\mathrm{eff}}$. This value may be identified by that used in the description of the Z resonance physics at LEP 1:

$$
\sin^2\theta_W^{\mathrm{eff}} = \kappa_e \sin^2\theta_W, \tag{2.57}
$$

where the form factor κ_e describes the decay of the Z boson into two charged leptons. Thus, a certain scale has been chosen ($Q^2 = -M_Z^2$), there are no contributions from box diagrams included etc. For details see [16, 23, 24].

For the e^-p CC scattering, the corresponding formulae are:

$$
\mathcal{W}_2^{-\mathcal{SM}}(x, Q^2) = \frac{1-\xi}{2} x \sum_{i=1}^{3} \left[\rho_C^2(1) u_i(x, Q^2) + \rho_C^2(-1) \bar{d}_i(x, Q^2)\right], \tag{2.58}
$$

$$
x\mathcal{W}_3^{-\mathcal{SM}}(x, Q^2) = \frac{1-\xi}{2} x \sum_{i=1}^{3} \left[\rho_C^2(1) u_i(x, Q^2) - \rho_C^2(-1) \bar{d}_i(x, Q^2)\right], \tag{2.59}
$$

whereas, for the e^+p scattering:

$$
\mathcal{W}_2^{+\mathcal{SM}}(x, Q^2) = \frac{1+\xi}{2} x \sum_{i=1}^{3} \left[\rho_C^2(-1) d_i(x, Q^2) + \rho_C^2(1) \bar{u}_i(x, Q^2)\right], \tag{2.60}
$$

$$
x\mathcal{W}_3^{+\mathcal{SM}}(x, Q^2) = \frac{1+\xi}{2} x \sum_{i=1}^{3} \left[\rho_C^2(-1) d_i(x, Q^2) - \rho_C^2(1) \bar{u}_i(x, Q^2)\right]. \tag{2.61}
$$

For IMOMS=1,2, the real-valued weak form factors ρ_C are defined as follows:

$$\begin{aligned} \rho_C(+1) &= 1+\delta\rho_W(xS, Q^2, xU; e, q), \\ \rho_C(-1) &= 1+\delta\rho_W(xU, Q^2, xS; e, q), \end{aligned} \tag{2.62}$$

where the arguments e, q indicate the isospin and charge contents of the scattering particles. For IMOMS=3, the definition is instead:

$$\begin{aligned} \rho_C(+1) &= \frac{[1+\delta\rho_W(xS, Q^2, xU; e, q)]}{1-\Delta r}, \\ \rho_C(-1) &= \frac{[1+\delta\rho_W(xU, Q^2, xS; e, q)]}{1-\Delta r}. \end{aligned} \tag{2.63}$$

The weak correction $\delta\rho_W$ is a part of the electroweak virtual CC corrections, which include also electromagnetic contributions. The latter have to be combined with real photonic bremsstrahlung in order to cancel infrared divergent intermediary terms. As is well-known, the separation of a weak and an electromagnetic part of the virtual corrections,

$$\rho_W = 1+\delta\rho_W+\delta\rho_W^{\mathrm{QED}}, \tag{2.64}$$

is not unique for the charged current. For practical purposes, a recipe was proposed in [10], which allows a reasonable definition of both parts. In this reference, further details and explicit expressions for the form factors may be found in appendix A.

For all the weak form factors ρ and κ, the masses of the light leptons and quarks may be neglected while the t quark mass has to be taken into account. The exception is $\Delta\alpha$, which depends on the light fermion masses logarithmically. To the QED part of the corrections, which is discussed later, terms of the order $\ln\left[\{Q^2, S, U\}/m_f^2\right]$ will also contribute. For details we again refer to the quoted literature.

2.3 Parton distributions and QCD corrections to structure functions

Nucleons and hadrons will be assumed to be built out of quarks. The quark distributions are labelled as follows:

$$q_i = (u, d, s, c, b, t), \qquad \text{for i} = 1, \ldots, 6, \tag{2.65}$$

$$\bar{q}_i = (\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}, \bar{t}), \qquad \text{for i} = 1, \ldots, 6. \tag{2.66}$$

These quark distributions contain both the valence and sea quark contributions. The explicit expression for xW_3^-, which was introduced in (2.42) may serve as an example:

$$xW_3^-(x, Q^2) = 2\left[q_{\bar{1}2}^-(x, Q^2)+q_{4\bar{3}}^-(x, Q^2)+q_{\bar{6}5}^-(x, Q^2)\right]. \tag{2.67}$$

Here, we use the following notations for the parton densities:

$$q_{ij}^{\pm}(x, Q^2) = x\left[q_i(x, Q^2) \pm \bar{q}_j(x, Q^2)\right]. \tag{2.68}$$

The combinations $q_{ij}^L(x, Q^2)$ are defined in eq. (2.80). The quark densities q_i are functions of x and Q^2. In HECTOR, there are three approaches to the parton distributions available:

- Leading order (LO) distributions;
- Next to leading order (NLO) distributions in the $\overline{\mathrm{MS}}$ scheme;
- Next to leading order distributions in the DIS scheme.

The NLO corrections are only active if the flag `IWEA = 0` is set. Otherwise only LO distributions are accessed.

2.3.1 The strong coupling constant

For the renormalization of the strong coupling constant, α_s, the $\overline{\mathrm{MS}}$ scheme is choosen. The expressions for α_s in leading and next to leading order are

$$\alpha_s^{\mathrm{LO}}(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \tag{2.69}$$

$$\alpha_s^{\mathrm{NLO}}(Q^2) = \alpha_s^{\mathrm{LO}}(Q^2)\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln(Q^2/\Lambda^2)}{[\ln(Q^2/\Lambda^2)]^2}\right], \tag{2.70}$$

with

$$\beta_0 = 11 - \frac{2}{3}N_f, \tag{2.71}$$

$$\beta_1 = 102 - \frac{38}{3}N_f. \tag{2.72}$$

Here, $\Lambda \equiv \Lambda_{\overline{\mathrm{MS}}}$ denotes the QCD parameter. $\Lambda_{\overline{\mathrm{MS}}}$ depends on the number of active flavours, N_f. On passing the respective flavour thresholds the value of Λ is changed leaving α_s continuous. In the case of distributions contained in `PDFACT` this treatment of α_s is provided. For other cases the corresponding function `ALPQCD(ISSET,Q)` has to be added by the user. The mass thresholds in the case of the distributions [25] were choosen as in [26].

2.3.2 Parton distributions in the $\overline{\mathrm{MS}}$ factorization scheme

Several types of parton densities emerge in the structure functions and were defined in (2.68): $q_{ij}^+(x,Q^2)$, $q_{ij}^-(x,Q^2)$, and $q_{ij}^L(x,Q^2)$.

The distribution q_{ij}^+ reads

$$\begin{aligned} q_{ij}^+(x,Q^2) &= q_{ij}^+(x,Q^2)_{\overline{\mathrm{MS}}} + \frac{\alpha_s(Q^2)}{2\pi}\left\{\int_x^1 \frac{dy}{y}\left[\frac{x}{y}C_q\left(\frac{x}{y}\right)\left[q_{ij}^+(y,Q^2)_{\overline{\mathrm{MS}}} - q_{ij}^+(x,Q^2)_{\overline{\mathrm{MS}}}\right]\right.\right. \\ &\left.\left. + 2\frac{x}{y}C_g\left(\frac{x}{y}\right)\mathcal{G}(y,Q^2)_{\overline{\mathrm{MS}}}\right] - q_{ij}^+(x,Q^2)_{\overline{\mathrm{MS}}}\int_0^x dy C_q(y,Q^2)\right\}, \end{aligned} \tag{2.73}$$

where

$$\mathcal{G} \equiv xG, \tag{2.74}$$

$$C_q(z) = C_F\left[\frac{1+z^2}{1-z}\left(\ln\frac{1-z}{z} - \frac{3}{4}\right) + \frac{1}{4}(9+5z)\right], \tag{2.75}$$

$$C_g(z) = \frac{1}{2}\left\{\left[z^2 + (1-z)^2\right]\ln\frac{1-z}{z} + 8z(1-z) - 1\right\}, \tag{2.76}$$

with $C_F = (N_c^2 - 1)/(2N_c) \equiv 4/3$.

Correspondingly q_{ij}^- is:

$$q_{ij}^-(x, Q^2) = x\left[q_i(x, Q^2) - \bar{q}_j(x, Q^2)\right]. \tag{2.77}$$

For this combination one has:

$$\begin{aligned} q_{ij}^-(x, Q^2) &= q_{ij}^-(x)_{\overline{\rm MS}} + \frac{\alpha_s(Q^2)}{2\pi}\left\{\int_x^1 \frac{dy}{y}\frac{x}{y}\left[C_3\left(\frac{x}{y}\right) q_{ij}^-(y, Q^2)_{\overline{\rm MS}} - C_q\left(\frac{x}{y}\right) q_{ij}^-(x, Q^2)_{\overline{\rm MS}}\right]\right. \\ &\left. - \ q_{ij}^-(x, Q^2)_{\overline{\rm MS}} \int_0^x dy C_q(y)\right\}, \end{aligned} \tag{2.78}$$

with:

$$C_3(z) = C_q(z) - (1+z)C_F. \tag{2.79}$$

Finally, for the longitudinal structure function, the correction reads:

$$q_{ij}^L(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left\{2C_F q_{ij}^+(y, Q^2)_{\overline{\rm MS}} + 4\left(1 - \frac{x}{y}\right)\mathcal{G}(y, Q^2)_{\overline{\rm MS}}\right\}. \tag{2.80}$$

In a series of parameterizations all active quark flavours are delt with as massless. In these cases the above relations (and those given in section 2.3.3) yield the complete description. On the other hand, some parameterizations account for mass effects explicitely (e.g. [26]) leading to coefficient functions different from (2.73, 2.78, 2.80) for heavy flavour contributions $q = c, b, t$. In general these contributions are process dependent. `HECTOR` foresees a simple approach for parameterizations contained in `PDFACT` in the present version: the light flavours (u, d, s) are parametrized according to [26], while the heavy flavours (c, b) are taken from [27], which is some suitable approximation (cf. [26]).

In all other cases (including the use of `PDFLIB`) the user has to provide the correct mappings her/himself.

The distributions $[q_i(x, Q^2) \pm \bar{q}_i(x, Q^2)]_{\overline{\rm MS}}$ and $G(y, Q^2)_{\overline{\rm MS}}$ have to be taken from one of the corresponding libraries.

In the above we have chosen Q^2 as the factorization scale always.

2.3.3 Parton distributions in the DIS factorization scheme

The + distributions (2.68) are associated with the coefficient function

$$\delta(1-z) \tag{2.81}$$

in the DIS scheme. Thus the generalized structure functions $\mathcal{F}_2, \mathcal{G}_2, \mathcal{H}_2, \mathcal{W}_2$, are just linear combinations of the quark quark distributions in the DIS scheme. The generalized structure functions $\mathcal{G}_3, \mathcal{H}_3, \mathcal{W}_3$, on the other hand, constitute of linear combinations of

$$q_{ij}^-(x, Q^2) = q_{ij}^-(x, Q^2)_{\rm DIS} + \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\frac{x}{y}C_3^{\rm DIS}\left(\frac{x}{y}\right) q_{ij}^-(y, Q^2)_{\rm DIS}. \tag{2.82}$$

with

$$C_3^{\rm DIS}(z) = -C_F(1+z). \tag{2.83}$$

The expression for the longitudinal structure functions are identical in the $\overline{\rm MS}$ and DIS schemes.

Again the distributions $[q_i(x, Q^2) \pm \bar{q}_i(x, Q^2)]_{\rm DIS}$ and $G(y, Q^2)_{\rm DIS}$ have to be taken from one of the corresponding libraries. The heavy flavour distributions are delt with as in section 2.3.2.

2.3.4 Structure functions in different schemes

The physical expressions of the structure functions (2.32)–(2.36), (2.39)–(2.43) in the different schemes are obtained by linear combinations of the functions:

$$q^{+}_{ij,\overline{\mathrm{MS}}(\mathrm{DIS})}(x,Q^2), \tag{2.84}$$

$$q^{-}_{ij,\overline{\mathrm{MS}}(\mathrm{DIS})}(x,Q^2), \tag{2.85}$$

$$q^{L}_{ij,\overline{\mathrm{MS}}(\mathrm{DIS})}(x,Q^2), \tag{2.86}$$

replacing the corresponding leading order expressions there.

2.4 Low Q^2 modifications of structure functions and parton distributions

In order to ensure the vanishing of the photonic parts of the generalized structure functions at low Q^2, dedicated modifications of the photonic structure functions are foreseen in `HECTOR`. They are realized by a global suppression factor `FVAR` in the files `disepm.f`, `genstf.f`, `prodis.f`, and `gsflow.f`. The default setting is `IVAR`=0 corresponding to [28]. The structure functions are

$$\mathcal{F}_{1,2}(x,Q^2) \;\rightarrow\; F_{var}\,\mathcal{F}_{1,2}(x,Q^2), \tag{2.87}$$

$$F_{var} \;=\; [1-\exp(-aQ^2)], \qquad a = 3.37\,\mathrm{GeV}^{-2}. \tag{2.88}$$

Other approaches have a similar form:

$$F_{var}(Q^2) \;=\; \left[1 - W_2^{el}(Q^2)\right]. \tag{2.89}$$

Parameterizations of the elastic form factor W_2^{el},

$$W_2^{el}(Q^2) \;=\; \frac{G_e^2+\tau G_m^2}{1+\tau}, \tag{2.90}$$

$$\tau \;=\; \frac{Q^2}{4M^2}, \tag{2.91}$$

may be chosen (cf. e.g. [30]) as is listed in section 4.4. The other structure functions are continued by their value at Q_0^2 to values in the range $Q^2 < Q_0^2$ at a given value of x.

In figure 1 we show as an example the combination of the structure function F_2 from [31] with a low Q^2 parameterization as proposed in [29, 30].

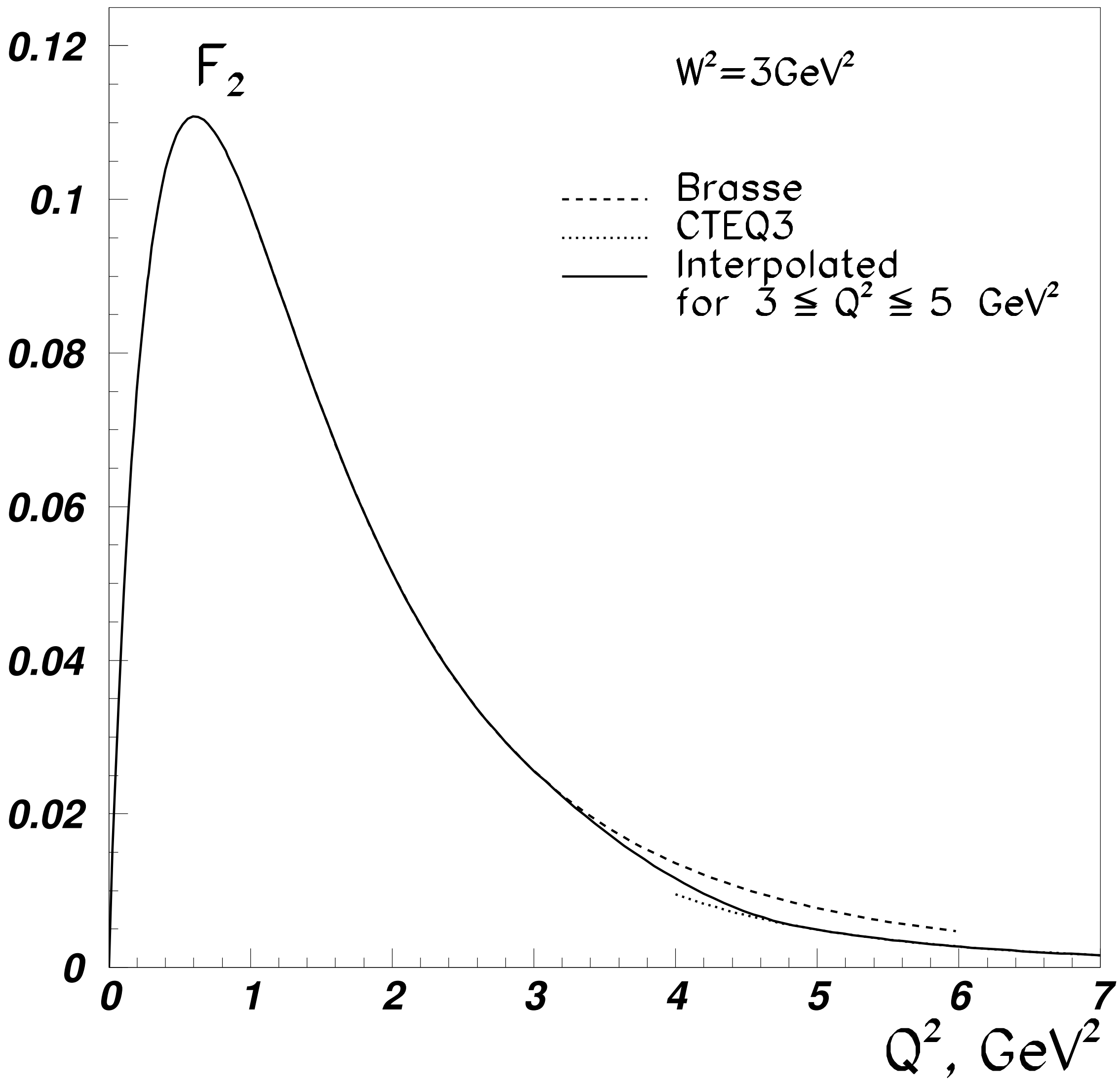

Figure 1: *Low Q^2 interpolation for the structure function $F_2(x, Q^2)$ between $F_2 = F_2^{LO}(CTEQ3)$ for $Q^2 \geq 5\mathrm{GeV}^2$ (*`ISCH = 0, ISSE =1`*), and the low Q^2 extension seleced by the flag* `IVAR=2` *for $Q^2 \leq 3\mathrm{GeV}^2$.*

2.5 QED corrections. Introduction

The inclusion of photonic corrections,

$$e(k_1) + p(p_1) \to e(k_2) + X(p_2) + n\gamma(k), \tag{2.92}$$

is crucial for a correct analysis of deep inelastic scattering. The radiative corrections may *differ considerably* comparing different sets of outer kinematical variables. If Born kinematics is valid, the scaling variable y, e.g., may be defined from momentum measurements of the leptons or from the hadrons with no difference:

$$y_l = \frac{p_1(k_1 - k_2)}{p_1 k_1}, \tag{2.93}$$

$$y_h = \frac{p_1(p_2 - p_1)}{p_1 k_1}. \tag{2.94}$$

As may be seen from figure 2 [11], the two definitions are quite different if a photon with non-vanishing momentum k is emitted. The same holds true for the variables x and Q^2, or y.

In fixed target experiments mostly leptonic variables are used. At ep colliders, a lot of different sets of variables may be measured: the scattering angles of the lepton or the hadronic system, the energies of the scattered lepton or the hadrons, the transverse hadronic momentum, the kinetic energy of the hadrons etc. (cf. refs. [32]-[36]). Those definitions of x, y, Q^2, which may be used in **HECTOR** are introduced in appendix A.

The **HELIOS** part of **HECTOR** allows the calculation of leading logarithmic approximations (LLA) including higher order corrections to $\mathcal{NC}$ and $\mathcal{CC}$ scattering in a large variety of different variable sets.

With the **TERAD** part one may calculate complete $\mathcal{O}(\alpha)$ QED corrections, including soft photon exponentiation, in a model independent approach. The structure functions are not

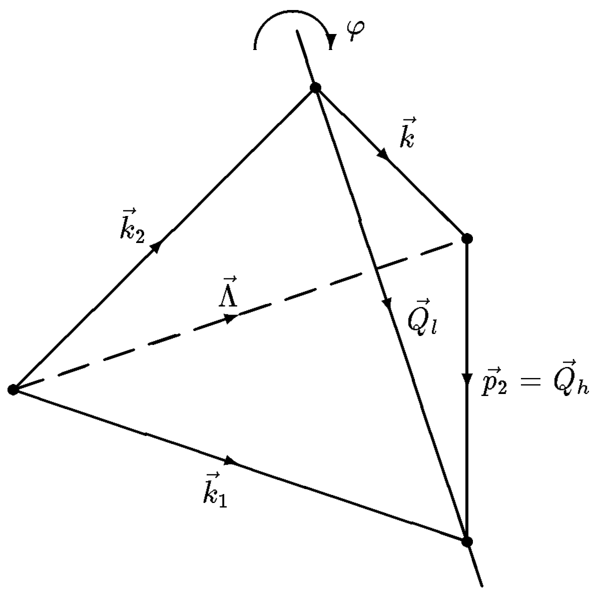

Figure 2: *Configuration of the 3–momenta in the reaction* $e(k_1)+p(p_1) \to l'(k_2)+X(p_2)+\gamma(k)$ *in the proton rest system,* $\vec{p}_1 = 0$.

necessarily calculated in the quark parton model and the kinematics has been worked out exact in the proton mass and ultra-relativistic in the electron mass.

The **TERAD** part of **HECTOR** may also be used for the calculation of complete $\mathcal{O}(\alpha)$ QED corrections, including soft photon exponentiation, in the quark parton model. This makes use of the **DISEP** branch. In the quark parton model, one may, in addition to the leptonic corrections, also calculate the lepton quark interference terms and treat the corrections from the quark lines.

The complete calculations are available for both $\mathcal{NC}$ and $\mathcal{CC}$ scattering for a series of sets of kinematical variables.

For $\mathcal{NC}$ scattering at very low Q^2, the complete leptonic corrections described in leptonic variables are available from the **TERADLOW** branch. Here, the exact kinematics both in the electron and proton masses is taken into account.

The *combined* use of both parts **HELIOS** and **TERAD** is also possible. Then, the complete $\mathcal{O}(\alpha)$ corrections of the **TERAD** part, without its soft photon exponentiation, are used together with the complete higher order corrections of the **HELIOS** part.

Details on the general structure of **HECTOR** may be seen in figure 6 and on the availability of variables in figure 9.

2.6 HELIOS: The leading logarithmic approximation

The LLA corrections are calculated by the **HELIOS** branch of **HECTOR**. They are related to logarithmic mass singularities, which distinguish them from the remaining corrections within a given order of perturbation theory. In leading logarithmic approximation, one may separate the terms due to initial and final state radiation. The QED corrections are composed out of several contributions. In **HECTOR**, the LLA cross section is[3]:

$$\begin{aligned}\frac{d^2\sigma^{LLA}}{dxdy} &= \frac{d^2\sigma^0}{dxdy} + \frac{d^2\sigma^{ini,1loop}}{dxdy} + \frac{d^2\sigma^{ini,2loop}}{dxdy} + \frac{d^2\sigma^{ini,>2,soft}}{dxdy} + \frac{d^2\sigma^{ini,e^-\to e^+}}{dxdy} \\ &+ \frac{d^2\sigma^{fin,1loop}}{dxdy} + \frac{d^2\sigma^{C}}{dxdy}. \end{aligned} \tag{2.95}$$

The Born contribution has been discussed in section 2.1. It is calculated in **sigbrn.f** and may or may not include weak virtual corrections depending on flag **IWEAK** and QCD corrections to the parton distributions depending on flag **ISCH**. For neutral current scattering both initial and final state LLA corrections may contribute, while for charged current scattering there is only initial state radiation. For the $\mathcal{NC}$ case, the corresponding Born diagram is shown in figure 3(a). The virtual corrections to order $\mathcal{O}(\alpha)$ arise from the diagram in figure 3(b) for the $\mathcal{NC}$ case. For $\mathcal{CC}$ scattering, it is replaced by a diagram with a photon exchange between incoming electron and virtual W boson. The $\mathcal{O}(\alpha)$ diagrams with real photon emission are shown in figure 4.

The two first order LLA corrections have a common generic structure:

$$\frac{d^2\sigma^{ini(fin),1loop}}{dxdy} = \frac{\alpha}{2\pi}L_e\int_0^1 dzP_{ee}^{(1)}\left\{\theta(z-z_0)\mathcal{J}(x,y,Q^2)\left.\frac{d^2\sigma^0}{dxdy}\right|_{x=\hat{x},y=\hat{y},S=\hat{S}} - \frac{d^2\sigma^0}{dxdy}\right\},$$

[3]There are also leading logarithmic corrections related to the emission of photons from the initial or final state quarks. They dependend on $\ln(Q^2/m_q^2)$ iff calculated in the on-mass-shell scheme. These terms deserve a special treatment. One can absorb these corrections into parton distributions [37, 5, 38, 13, 39] or fragmentation functions [5] leading to modified expressions of the scaling violations of these quantities.

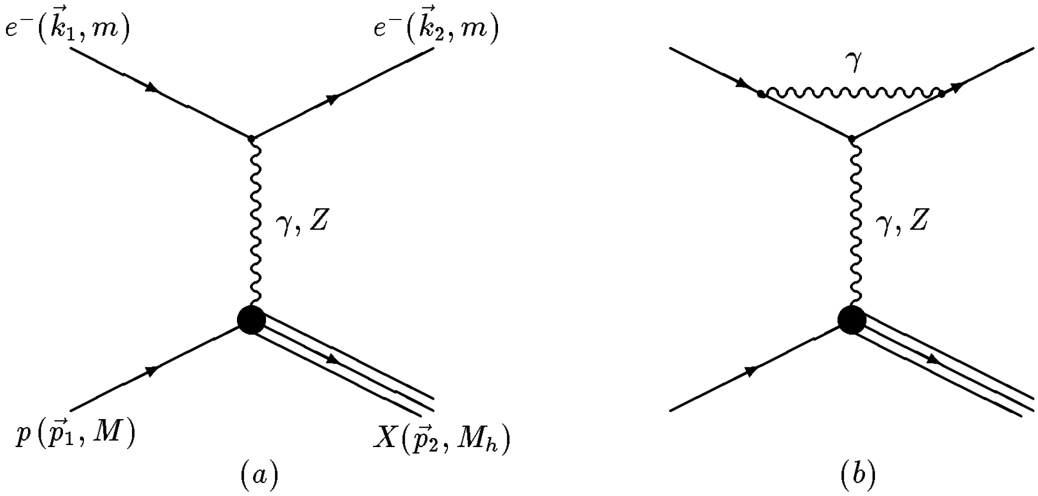

Figure 3: *NC deep inelastic scattering of electrons off protons: (a) Born diagram, (b) leptonic QED vertex correction*

$$(2.96)$$

$$P_{ee}^{(1)} = \frac{1+z^2}{1-z}. \tag{2.97}$$

Here, we introduced the notion

$$L_e = \ln\frac{Q^2}{m_e^2} - 1. \tag{2.98}$$

The definition (2.98) reproduces the soft photon terms of complete calculations in leptonic variables (see e.g. [11]).

The expressions (2.96) for initial and final state radiation differ by the scaling properties of the kinematical variables, the resulting Jacobian, $\mathcal{J}$, and the integration bounds. In table 1, for the initial state corrections these parameters are listed for a variety of different sets of variables. Table 2 contains the same for final state corrections if they exist. For some variables, e.g. the Jaquet-Blondel variables, the final state is treated completely inclusive. Then, in accordance with the Kinoshita-Lee-Nauenberg theorem [40] there is no LLA correction.

Furthermore, there are certain second order corrections from the initial state which may be non-negligible [8]:

$$\begin{aligned}\frac{d^2\sigma^{ini,2loop}}{dxdy} &= \left[\frac{\alpha}{2\pi}L_e\right]^2 \int_0^1 dz P_{ee}^{(2,1)}(z) \left\{ \theta(z-z_0)\mathcal{J}(x,y,z) \left.\frac{d^2\sigma^0}{dxdy}\right|_{x=\hat{x},y=\hat{y},S=\hat{S}} - \frac{d^2\sigma^0}{dxdy} \right\} \\ &+ \left(\frac{\alpha}{2\pi}\right)^2 \int_{z_0}^1 dz \left\{ L_e^2 P_{ee}^{(2,2)}(z) + L_e \sum_{f=l,q} \ln\frac{Q^2}{m_f^2} P_{ee,f}^{(2,3)}(z) \right\} \mathcal{J}(x,y,z) \left.\frac{d^2\sigma^0}{dxdy}\right|_{x=\hat{x},y=\hat{y},S=\hat{S}}.\end{aligned} \tag{2.99}$$

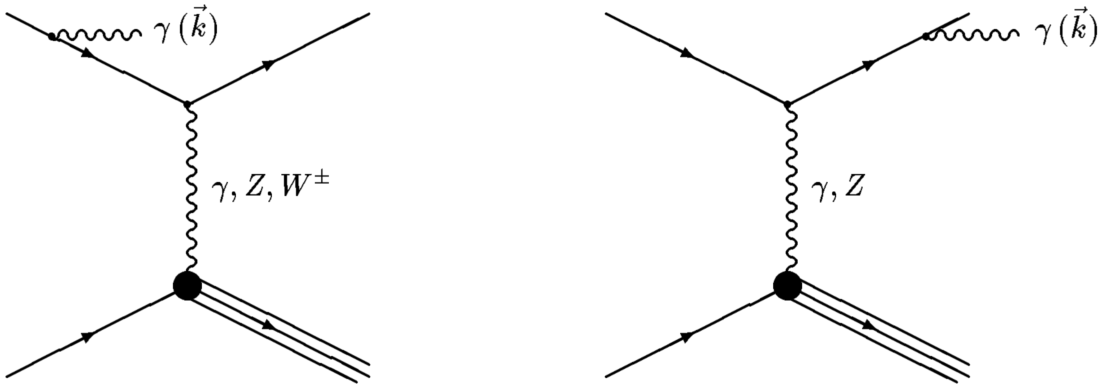

Figure 4: *Leptonic QED bremsstrahlung diagrams*

The second order splitting functions are:

$$\begin{aligned}
P_{ee}^{(2,1)}(z) &= \frac{1}{2}\left[P_{ee}^{(1)} \otimes P_{ee}^{(1)}\right](z) \\
&= \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z), \qquad (2.100) \\
P_{ee}^{(2,2)}(z) &= \frac{1}{2}\left[P_{e\gamma}^{(1)} \otimes P_{\gamma e}^{(1)}\right](z) \\
&\equiv (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3), \qquad (2.101) \\
P_{ee,f}^{(2,3)}(z) &= N_c(f) Q_f^2 \frac{1}{3} P_{ee}^{(1)}(z)\theta\left(1 - z - \frac{2m_f}{E_e}\right). \qquad (2.102)
\end{aligned}$$

The symbol $\otimes$ in (2.100) and (2.101) denotes the Mellin convolution:

$$A(x) \otimes B(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2). \qquad (2.103)$$

Q_f is the fermion charge, and $N_c(f) = 3$ for quarks, $N_c(f) = 1$ for leptons, respectively. In (2.102), we parametrize $\alpha_{_{QED}}(s, m_{f_i})$ in terms of effective quark masses and use the parameters $m_u = 62$ MeV, $m_d = 83$ MeV, $m_s = 215$ MeV, $m_c = 1.5$ GeV, and $m_b = 4.5$ GeV as obtained in [17, 18].

The **HELIOS** part of **HECTOR** allows to include higher order soft photon corrections into cross section calculations:

$$\frac{d^2\sigma^{(>2,soft)}}{dxdy} = \int_0^1 dz P_{ee}^{(>2)}(z, Q^2)\left\{\theta(z - z_0)\mathcal{J}(x,y,z)\left.\frac{d^2\sigma^{(0)}}{dxdy}\right|_{x=\hat{x}, y=\hat{y}, S=\hat{S}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}, \qquad (2.104)$$

with [8]:

$$\begin{aligned}
P_{ee}^{>2}(z, Q^2) &= D_{NS}(z, Q^2) - \frac{\alpha}{2\pi} L_e \frac{2}{1-z}\left\{1 + \frac{\alpha}{2\pi} L_e \left[\frac{11}{6} + 2\ln(1-z)\right]\right\}, \qquad (2.105) \\
D_{NS}(z, Q^2) &= \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2} - 2\gamma_E\right)\right]}{\Gamma(1+\zeta)}, \qquad (2.106) \\
\zeta &= -3\ln\left[1 - (\alpha/3\pi)L_e\right]. \qquad (2.107)
\end{aligned}$$

Equation (2.106) was derived in [41] already. If the charge of the final state electron is not recorded, also the conversion cross section:

$$\frac{d^2\sigma^{(2,e^-\to e^+)}}{dxdy} = \int_{z_0}^{1} dz P(z,Q^2;e^-\to e^+)\mathcal{J}(x,y,z)\left.\frac{d^2\sigma^{(0)}}{dxdy}\right|_{x=\hat{x},y=\hat{y},S=\hat{S}}, \tag{2.108}$$

with the conversion rate:

$$P(z,Q^2;e^-\to e^+) = \left(\frac{\alpha}{2\pi}\right)^2 L_e^2 P_{ee}^{(2,2)}(z) \tag{2.109}$$

has to be included into the radiative corrections.

The explicit expressions for the splitting functions have been taken from [8] where additional useful comments on the higher order corrections may be found.

Finally, we come to the Compton correction. It has to be taken into account in the case of leptonic variables:

$$\frac{d^2\sigma^{C1}}{dx_l dy_l} = \frac{\alpha^3}{x_l S}\left[1+(1-y_l)^2\right]\ln\left(\frac{Q_l^2}{M^2}\right)\int_{x_l}^{1}\frac{dz}{z^2}\frac{z^2+(x_l-z)^2}{x_l(1-y_l)}\sum_f\left[q_f(z,Q_l^2)+\bar{q}_f(z,Q_l^2)\right]. \tag{2.110}$$

This expression may be extracted from the complete calculation of the $\mathcal{O}(\alpha)$ corrections [5, 11] and is accessed setting the flag ICMP=1. An alternative description of the Compton peak has been derived in [7] (ICMP=2):

$$\frac{d^2\sigma^{C_2}}{dx_l dy_l} = \int_0^1 \frac{dz}{z} D_{\gamma/p}(z,Q_l^2)\left.\frac{d^2\hat{\sigma}(e\gamma\to e\gamma)}{d\hat{x}dy_l}\right|_{\hat{s}=zs,\hat{x}=x_l/z}, \tag{2.111}$$

with:

$$\frac{d^2\hat{\sigma}(e\gamma\to e\gamma)}{d\hat{x}dy_l} = \frac{2\pi\alpha^2}{\hat{s}}\frac{1+(1-y_l)^2}{1-y_l}\delta(1-\hat{x}), \tag{2.112}$$

$$D_{\gamma/p}(x_l,Q_l^2) = \frac{\alpha}{2\pi}\int_{x_l}^1 dz\int_{Q^2_{h,min}}^{Q_l^2}\frac{dQ_h^2}{Q_h^2}\frac{z}{x_l}\left[\frac{1+(1-z)^2}{z^2}F_2\left(\frac{x_l}{z},Q_h^2\right) - F_L\left(\frac{x_l}{z},Q_h^2\right)\right]. \tag{2.113}$$

In principle, the Compton correction has to be taken into account also in the case of mixed variables. But, although being logarithmically enhanced, it is not larger than the rest of the $\mathcal{O}(\alpha)$ corrections in this case [11].

2.6.1 Cuts

In the LLA approach, it is possible to apply a cut on the reconstructed electron energy by cutting the integration variable z. The variable ZCUT is defined by:

$$[E_J(1-\cos\theta_J)+E_{e'}(1-\cos\theta_{e'})] \geq \mathtt{ZCUT}. \tag{2.114}$$

It is also possible to reject the production of final states with a Q_h^2 and/or an invariant hadronic mass W_h^2 below some bound. See also the description of flag IHCU in section 4.2.

Initial state radiation					
	$\hat{S}$	$\hat{Q}^2$	$\hat{y}$	z_0	$\mathcal{J}(x,y,z)$
leptonic variables	Sz	$Q_l^2 z$	$\frac{z+y_l-1}{z}$	$\frac{1-y_l}{1-x_l y_l}$	$\frac{y_l}{z+y_l-1}$
mixed variables	Sz	$Q_l^2 z$	$\frac{y_{\mathrm{JB}}}{z}$	y_{JB}	1
hadronic variables	Sz	Q_h^2	$\frac{y_h}{z}$	y_h	$\frac{1}{z}$
JB variables	Sz	$\frac{Q^2_{\mathrm{JB}}(1-y_{\mathrm{JB}})z}{z-y_{\mathrm{JB}}}$	$\frac{y_{\mathrm{JB}}}{z}$	$\frac{y_{\mathrm{JB}}}{1-x_{\mathrm{JB}}(1-y_{\mathrm{JB}})}$	$\frac{1-y_{\mathrm{JB}}}{z-y_{\mathrm{JB}}}$
double angle method	Sz	$Q^2_{\mathrm{DA}} z^2$	y_{DA}	0	z
$\theta_l, y_{\mathrm{JB}}$	Sz	$Q^2_{\theta\mathrm{y}} \frac{z(z-y_{\mathrm{JB}})}{1-y_{\mathrm{JB}}}$	$\frac{y_{\mathrm{JB}}}{z}$	y_{JB}	$\frac{z-y_{\mathrm{JB}}}{1-y_{\mathrm{JB}}}$
Σ method	Sz	Q^2_{Σ}	y_{Σ}	x_{Σ}	$\frac{1}{z}$
$e\Sigma$ method	Sz	$Q_l^2 z$	$y_{e\Sigma} z$	x_{Σ}	1

Table 1: *Scaling properties of various sets of kinematical variables for leptonic initial state radiation. The different parameters are defined in appendix A.*

Final state radiation					
	$\hat{S}$	$\hat{Q}^2$	$\hat{y}$	z_0	$\mathcal{J}(x,y,z)$
leptonic variables	S	$\frac{Q_l^2}{z}$	$\frac{z+y_l-1}{z}$	$1-y_l(1-x_l)$	$\frac{y_l}{z(z+y_l-1)}$
mixed variables	S	$\frac{Q_l^2}{z}$	y_{JB}	x_m	$\frac{1}{z}$
Σ method	S	$Q_\Sigma^2 \frac{1-y_\Sigma(1-z)}{z^2}$	$\frac{y_\Sigma z}{1-y_\Sigma(1-z)}$	$z_0^{\Sigma,f}$	$\frac{1}{z^2}$
$e\Sigma$ method	S	$\frac{Q_l^2}{z}$	$\frac{y_{e\Sigma} z^2}{[1-y_{e\Sigma}(1-z)]^2}$	$z_0^{\Sigma,f}$	$\frac{1+y_{e\Sigma}(1-z)}{[1-y_{e\Sigma}(1-z)]z}$

Table 2: *Scaling properties of various sets of kinematical variables for leptonic final state radiation. The different parameters are defined in appendix A.*

2.7 TERAD: Complete $\mathcal{O}(\alpha)$ leptonic corrections in the model independent approach for $\mathcal{NC}$ scattering

If one wants to improve the LLA approximation of QED corrections, a natural step is to calculate the contributions from figures 3(b) and 4, which lead to the leptonic corrections, exact in order $\mathcal{O}(\alpha)$. This is much more involved than the LLA calculation alone and has been done so far for a rather limited set of variables only. A comprehensive presentation of the calculation and of the numerical details may be found in [11] and in references quoted therein. Here we restrict ourselves to a short collection of formulae, which are realized in HECTOR.

Let us remind the Born cross section as given in (2.2):

$$\frac{d^2\sigma^{\mathrm{B}}}{dydQ^2} = \frac{2\pi\alpha^2}{\lambda_S}\frac{S}{Q^4}\sum_{i=1}^{3}\mathcal{A}_i(x,Q^2)\mathcal{S}_i^{\mathrm{B}}(y,Q^2), \tag{2.115}$$

with $\lambda_S = S^2 - 4m^2M^2$. The factorized kinematical functions $\mathcal{S}_i^{\mathrm{B}}$ are introduced in (2.3)–(2.5). The functions $\mathcal{A}_i$ are related to the generalized structure functions by:

$$\begin{aligned}
\mathcal{A}_1(x,Q^2) &= 2\mathcal{F}_1(x,Q^2),\\
\mathcal{A}_2(x,Q^2) &= \frac{1}{yS}\mathcal{F}_2(x,Q^2),\\
\mathcal{A}_3(x,Q^2) &= \frac{1}{2Q^2}\mathcal{F}_3(x,Q^2).
\end{aligned} \tag{2.116}$$

In the TERAD part of HECTOR, the QED corrections have a structure quite similar to that of the Born cross section (2.115):

$$\frac{d^2\sigma^{QED}}{d\mathcal{E}} = \frac{2\alpha^3S^2}{\pi\lambda_S}\int d^2\mathcal{I}\,\frac{1}{Q_h^4}\,S(\mathcal{E},\mathcal{I}), \tag{2.117}$$

$$S(\mathcal{E},\mathcal{I}) = \sum_{i=1}^{3}\mathcal{A}_i(x,Q^2)\,\mathcal{S}_i(\mathcal{E},\mathcal{I}). \tag{2.118}$$

Here, $\mathcal{E}$ is a set of two variables on which $d^2\sigma$ finally depends. In the case of leptonic, mixed, hadronic variables, e.g.:

$$\mathcal{E} = \{y_l,Q_l^2\},\{y_h,Q_l^2\},\{y_h,Q_h^2\}.$$

The $\mathcal{I}$ is a corresponding set of two variables to be integrated over:

$$\mathcal{I} = \{y_h,Q_h^2\},\{y_l,Q_h^2\},\{y_l,Q_l^2\}.$$

The functions $\mathcal{S}_i(\mathcal{E},\mathcal{I})$ in (2.118) mark the difference to the Born cross section and have to be calculated. They are results of an one-dimensional analytical integration. In some of the variable sets additional integrations have been performed analytically.

We only mention here that in the exact calculations the correct treatment of the kinematically allowed regions of the integration variables may be crucial. A careful and exhaustive analysis of this may be found in appendix B of [11].

The cross sections may be represented in the following generic form:

$$\frac{d^2\sigma^{QED}}{d\mathcal{E}} = \left[1+\frac{\alpha}{\pi}\,\delta_{VS}^{expon}(\mathcal{E})\right]\frac{d^2\sigma^B}{d\mathcal{E}} + \frac{d^2\sigma^{brems}}{d\mathcal{E}}. \tag{2.119}$$

The term:

$$\frac{\alpha}{\pi}\,\delta_{VS}^{expon}(\mathcal{E}) = \exp\left[\frac{\alpha}{\pi}\delta_{inf}(\mathcal{E})\right] - 1 + \frac{\alpha}{\pi}\left[\delta_{VR}(\mathcal{E}) - \delta_{inf}(\mathcal{E})\right] \tag{2.120}$$

contains the (exponentiated) virtual and soft photon part of the corrections and $d^2\sigma^{brems}$ the finite part of real bremsstrahlung. Exponentiation may be taken into account with flag `IEXP`.

The Born cross section is called from subroutine `sigbrn.f`, while the non-factorizing part of the QED corrections directly uses file `genstf.f`. The corrections may or may not include weak virtual corrections depending on the flag `IWEAK` and QCD corrections to the parton distributions depending on the flag `ISCH`. These two types of corrections are not yet thoroughly compatible in the present version of `HECTOR` as mentioned in section 2.3.

2.7.1 Leptonic variables

In *leptonic* variables one has:

$$\begin{aligned}\delta_{VR}(\mathcal{E}_l) &= \delta_{inf}(\mathcal{E}_l) - \frac{1}{2}\ln^2\left[\frac{1-y_l(1-x_l)}{1-y_l x_l}\right] + \mathrm{Li}_2\left[\frac{1-y_l}{(1-y_l x_l)[1-y_l(1-x_l)]}\right] \\ &+ \frac{3}{2}\ln\frac{Q_l^2}{m^2} - \mathrm{Li}_2(1) - 2\ ,\end{aligned} \tag{2.121}$$

where

$$\delta_{inf}(\mathcal{E}_l) = \left(\ln\frac{Q_l^2}{m^2} - 1\right)\ln\left[\frac{y_l^2(1-x_l)^2}{(1-y_l x_l)[1-y_l(1-x_l)]}\right]. \tag{2.122}$$

Here, $\mathrm{Li}_2(x)$ denotes the Euler dilogarithmic function

$$\mathrm{Li}_2(x) = -\int_0^1 dz\frac{\ln(1-xz)}{z}. \tag{2.123}$$

The finite bremsstrahlung correction has a factorizing and a non-factorizing part:

$$\begin{aligned}\frac{d^2\sigma^{brems}}{dy_l dQ_l^2} &= \frac{2\alpha^3 S^2}{\lambda_S}\int dy_h dQ_h^2 \sum_{i=1}^{3}\Bigg[\mathcal{A}_i(x_h,Q_h^2)\frac{1}{Q_h^4}\mathcal{S}_i(y_l,Q_l^2,y_h,Q_h^2) \\ &- \mathcal{A}_i(x_l,Q_l^2)\frac{1}{Q_l^4}\ \mathcal{S}_i^B(y_l,Q_l^2)\ \mathcal{L}^{\mathrm{IR}}(y_l,Q_l^2,y_h,Q_h^2)\Bigg],\end{aligned} \tag{2.124}$$

The radiators $\mathcal{S}_i(\mathcal{E},\mathcal{I})$ of the non-factorizing part are:

$$\begin{aligned}\mathcal{S}_1(\mathcal{E},\mathcal{I}) &= \Bigg\{\frac{1}{\sqrt{C_2}}\left[2m^2 - \frac{1}{2}(Q_h^2+Q_l^2) + \frac{Q_h^4-4m^4}{Q_h^2-Q_l^2}\right] \\ &- m^2(Q_h^2-2m^2)\frac{B_2}{C_2^{3/2}}\Bigg\} + \frac{1}{\sqrt{A_2}} - \Big\{(1)\leftrightarrow -(1-y_l)\Big\},\end{aligned} \tag{2.125}$$

$$\begin{aligned}\mathcal{S}_2(\mathcal{E},\mathcal{I}) &= \Bigg\{\frac{1}{\sqrt{C_2}}\left[M^2(Q_h^2+Q_l^2) - y_h(1-y_l)S^2\right] \\ &+ \frac{1}{(Q_h^2-Q_l^2)\sqrt{C_2}}\Bigg[Q_h^2\Big[(1)\left[(1)-y_h\right]S^2 \\ &+ (1-y_l)\left[(1-y_l)+y_h\right]S^2 - 2M^2(Q_h^2+2m^2)\Big] \\ &+ 2m^2S^2\Big[\left[(1)-y_h\right]\left[(1-y_l)+y_h\right] + (1)(1-y_l)\Big]\Bigg]\end{aligned}$$

$$
\begin{aligned}
& - 2m^2\frac{B_2}{C_2^{3/2}}\Big[(1)\left[(1)-y_h\right]S^2 - M^2Q_h^2\Big]\Big\} - \frac{2M^2}{\sqrt{A_2}} \\
& - \Big\{(1)\leftrightarrow -(1-y_l)\Big\}, \qquad (2.126)\\
\mathcal{S}_3(\mathcal{E},\mathcal{I}) \;=\; & \Big\{\frac{S}{\sqrt{C_2}}\Big[\frac{2Q_h^2(Q_h^2+2m^2)\left[(1)+(1-y_l)\right]}{Q_h^2-Q_l^2} - 2(1-y_l)Q_h^2 - y_h(Q_h^2+Q_l^2)\Big] \\
& - 2m^2SQ_h^2\frac{B_2}{C_2^{3/2}}\left[2(1)-y_h\right]\Big\} + \Big\{(1)\leftrightarrow -(1-y_l)\Big\}, \qquad (2.127)
\end{aligned}
$$

with the following abbreviations:

$$
\begin{aligned}
A_2 &= {y_l}^2S^2 + 4M^2Q_l^2,\\
B_2 &= \left\{2M^2Q_l^2(Q_l^2-Q_h^2) + (1-y_l)(y_lQ_h^2 - y_hQ_l^2)S^2 + S^2(1)Q_l^2(y_l-y_h)\right\}\\
&\equiv \; - B_1\left\{(1)\leftrightarrow -(1-y_l)\right\},\\
C_2 &= \Big\{(1-y_l)Q_h^2 - Q_l^2\left[(1)-y_h\right]\Big]^2S^2 + 4m^2\Big[(y_l-y_h)(y_lQ_h^2-y_hQ_l^2)S^2 - M^2(Q_h^2-Q_l^2)^2\Big\}\\
&\equiv \; C_1\left\{(1)\leftrightarrow -(1-y_l)\right\}.
\end{aligned}
$$

The expression for the correction factorizing off the Born term, $\mathcal{L}^{\rm IR}(y_l,Q_l^2,y_h,Q_h^2)$, reads (exact in both masses m and M):

$$
\mathcal{L}^{IR}(y_l,Q_l^2,y_h,Q_h^2) \;=\; \frac{Q_l^2+2m^2}{Q_l^2-Q_h^2}\Big(\frac{1}{\sqrt{C_1}} - \frac{1}{\sqrt{C_2}}\Big) \; - m^2\Big(\frac{B_1}{C_1^{3/2}} + \frac{B_2}{C_2^{3/2}}\Big). \qquad (2.128)
$$

The boundaries for the two-dimensional numerical integration in (2.124) are derived in appendix B.2.3 of [11].

The above formulae are realized in file `teradl.f`.

2.7.2 Mixed variables

In *mixed* variables, the following functions have to be used in (2.120):

$$
\delta_{VR}(\mathcal{E}_m) \;=\; \delta_{inf}(\mathcal{E}_m) - \frac{1}{2}\ln^2\left(\frac{1-y_h}{1-x_m}\right) - \;\mathrm{Li}_2\left[\frac{x_m(1-y_h)}{x_m-1}\right] + \frac{3}{2}\ln\frac{Q_l^2}{m^2} - 2\,, \qquad (2.129)
$$

where

$$
\delta_{inf}(\mathcal{E}_m) = \left(\ln\frac{Q_l^2}{m^2} - 1\right)\ln[(1-y_h)(1-x_m)]. \qquad (2.130)
$$

The final formula for $d^2\sigma^{brems}/dy_hdQ_l^2$ can be written in the form:

$$
\begin{aligned}
\frac{d^2\sigma^{brems}}{dy_hdQ_l^2} \;=\; & \frac{2\alpha^3S^2}{\lambda_S}\Big\{\int_{Q_h^{2\,\min}}^{Q_l^2} dQ_h^2\sum_{i=1}^{3}\Big[\mathcal{A}_i(x_h,Q_h^2)\frac{1}{Q_h^4}\mathcal{S}_i^{\rm I}(y_l,Q_l^2,y_h,Q_h^2)\\
& - \mathcal{A}_i(x_m,Q_l^2)\frac{1}{Q_l^4}\mathcal{S}_i^{\rm B}(y_h,Q_l^2)\mathcal{L}_{\rm I}^{\rm IR}(y_l,Q_l^2,Q_h^2)\Big]\\
& + \int_{Q_l^2}^{y_hS} dQ_h^2\sum_{i=1}^{3}\Big[\;\mathcal{A}_i(x_h,Q_h^2)\frac{1}{Q_h^4}\mathcal{S}_i^{\rm II}(Q_l^2,y_h,Q_h^2)\\
& - \mathcal{A}_i(x_m,Q_l^2)\frac{1}{Q_l^4}\mathcal{S}_i^{\rm B}(Q_l^2,y_h)\mathcal{L}_{\rm II}^{\rm IR}(Q_l^2,y_h,Q_h^2)\Big]\;\Big\}. \qquad (2.131)
\end{aligned}
$$

The integration boundary $Q_h^{2\,\min}$ for the remaining one-dimensional integration may be found in appendix B.3.3 of [11]. The integration region must be split into two regions I and II as long as the condition $Q_l^2 < y_h S$ is fulfilled, which corresponds to $x_m \leq 1$. If instead $x_m > 1$, there is only the region I. The factorizing part of the cross section differs in the two kinematical regions:

$$\mathcal{L}_I^{IR}(Q_l^2, y_h, Q_h^2) = \frac{1}{S(Q_l^2 - Q_h^2)} \left[-1 - \frac{Q_l^2}{Q_h^2}(\mathrm{L}_t + \mathrm{L}_2) + (\mathrm{L} + \mathrm{L}_t + \mathrm{L}_1) \right], \tag{2.132}$$

$$\mathcal{L}_{II}^{IR}(Q_l^2, y_h, Q_h^2) = \frac{1}{S(Q_l^2 - Q_h^2)} \left[\frac{Q_l^2}{Q_h^2} + \mathrm{L}_1 - \frac{Q_l^2}{Q_h^2}(\mathrm{L} + \mathrm{L}_2) \right]. \tag{2.133}$$

The following abbreviations are used:

$$\mathrm{L} = \ln\frac{Q_l^2}{m^2}, \qquad \mathrm{L}_t = \ln\frac{Q_l^2}{Q_h^2}, \qquad \mathrm{L}_T = \ln\frac{1}{y_h}, \tag{2.134}$$

$$\mathrm{L}_1 = \ln\frac{Q_h^2 - y_h Q_l^2}{\mid Q_l^2 - Q_h^2 \mid}, \qquad \mathrm{L}_2 = \ln\frac{(1-y_h)Q_h^2}{\mid Q_l^2 - Q_h^2 \mid}. \tag{2.135}$$

The non-factorizing hard bremsstrahlung functions in the two integration regions are:

$$\begin{aligned} \mathcal{S}_1^I(Q_l^2, y_h, Q_h^2) &= \frac{1}{S}\Bigg[\frac{1}{2}\frac{Q_l^2}{Q_l^2 - Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)(\mathrm{L} + \mathrm{L}_1 - \mathrm{L}_2 - 1) \\ &\quad - \frac{1}{2}\frac{Q_l^2}{Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)(\mathrm{L}_t + \mathrm{L}_2) + \mathrm{L}_T - \mathrm{L}_t + \frac{1}{2}\left(1 - \frac{Q_h^2}{Q_l^2}\right)\Bigg], \end{aligned} \tag{2.136}$$

$$\begin{aligned} \mathcal{S}_2^I(Q_l^2, y_h, Q_h^2) &= S\Bigg\{ \frac{1}{Q_l^2}\frac{Q_h^2 - y_h Q_l^2}{Q_l^2 - Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)(\mathrm{L} + \mathrm{L}_1 - \mathrm{L}_2 - 1) \\ &\quad + \frac{1}{Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)\left[y_h\left(1 + \frac{Q_l^2}{Q_h^2}\right) - \left(1 + \frac{Q_l^2}{Q_h^2} + \frac{Q_h^2}{Q_l^2}\right)\right](\mathrm{L}_t + \mathrm{L}_2) \\ &\quad + \frac{1}{Q_h^2}\left(1 + \frac{3}{2}\frac{Q_h^2}{Q_l^2} + 3\frac{Q_h^4}{Q_l^4} + 2\frac{Q_h^6}{Q_l^6}\right) \\ &\quad - \frac{y_h}{Q_h^2}\left(\frac{Q_l^2}{Q_h^2} + 2 + 4\frac{Q_h^2}{Q_l^2} + 3\frac{Q_h^4}{Q_l^4}\right) + \frac{1}{2}\frac{y_h^2}{Q_h^2}\left(2 + \frac{Q_l^2}{Q_h^2} + 2\frac{Q_h^2}{Q_l^2}\right)\Bigg\}, \end{aligned} \tag{2.137}$$

$$\begin{aligned} \mathcal{S}_3^I(Q_l^2, y_h, Q_h^2) &= \frac{Q_l^2}{Q_l^2 - Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)\left(2\frac{Q_h^2}{Q_l^2} - y_h\right)(\mathrm{L} + \mathrm{L}_1 - \mathrm{L}_2 - 1) \\ &\quad - \frac{Q_l^2}{Q_h^2}\left(1 + \frac{Q_h^4}{Q_l^4}\right)\left(2\frac{Q_h^2}{Q_l^2} - y_h + 2\right)(\mathrm{L}_t + \mathrm{L}_2) \\ &\quad + \frac{Q_l^2}{Q_h^2}\left(1 + \frac{Q_h^2}{Q_l^2}\right)^2\left(2\frac{Q_h^2}{Q_l^2} - y_h\right) - y_h\left(\frac{Q_l^2}{Q_h^2} + 1 + 2\frac{Q_h^2}{Q_l^2}\right), \end{aligned} \tag{2.138}$$

$$\mathcal{S}_1^{II}(Q_l^2, y_h, Q_h^2) = \frac{1}{S}\left[-\frac{1}{2}\frac{Q_l^4}{Q_h^2(Q_l^2 - Q_h^2)}\left(1 + \frac{Q_h^4}{Q_l^4}\right)(\mathrm{L} - \mathrm{L}_1 + \mathrm{L}_2 - 1) \right.$$

$$
\begin{aligned}
& - \frac{1}{2}\frac{Q_l^2}{Q_h^2}\left(1+\frac{Q_h^4}{Q_l^4}\right)\mathrm{L}_1 + \mathrm{L}_T + \frac{1}{2}\left(1-\frac{Q_l^2}{Q_h^2}\right)\Bigg], \qquad (2.139)\\
\mathcal{S}_2^{II}(Q_l^2,y_h,Q_h^2) = {}& S\Bigg\{-\frac{Q_l^4}{Q_h^4}\frac{(1-y_h)}{Q_l^2-Q_h^2}\left(1+\frac{Q_h^4}{Q_l^4}\right)(\mathrm{L}-\mathrm{L}_1+\mathrm{L}_2-1)\\
& + \frac{1}{Q_h^2}\left(1+\frac{Q_h^4}{Q_l^4}\right)\left[y_h\left(1+\frac{Q_l^2}{Q_h^2}\right)-\left(1+\frac{Q_l^2}{Q_h^2}+\frac{Q_h^2}{Q_l^2}\right)\right]\mathrm{L}_1\\
& + \frac{1}{Q_l^2}\left(\frac{3}{2}+\frac{Q_h^2}{Q_l^2}+3\frac{Q_l^2}{Q_h^2}+2\frac{Q_l^4}{Q_h^4}\right)\\
& - \frac{y_h}{Q_h^2}\left(\frac{3Q_l^2}{Q_h^2}+4+2\frac{Q_h^2}{Q_l^2}+\frac{Q_h^4}{Q_l^4}\right)\\
& + \frac{1}{2}\frac{y_h^2}{Q_h^2}\left(2+2\frac{Q_l^2}{Q_h^2}+\frac{Q_h^2}{Q_l^2}\right)\Bigg\}, \qquad (2.140)\\
\mathcal{S}_3^{II}(Q_l^2,y_h,Q_h^2) = {}& -\frac{(2-y_h)}{(Q_l^2-Q_h^2)}\frac{Q_l^4}{Q_h^2}\left(1+\frac{Q_h^4}{Q_l^4}\right)(\mathrm{L}-\mathrm{L}_1+\mathrm{L}_2-1)\\
& + \left(1+\frac{Q_h^4}{Q_l^4}\right)\left[y_h\frac{Q_l^2}{Q_h^2}-2\left(1+\frac{Q_l^2}{Q_h^2}\right)\right]\mathrm{L}_1\\
& + 2(1-y_h)\frac{Q_l^2}{Q_h^2}\left(1+\frac{Q_h^2}{Q_l^2}\right)^2 + y_h\left(1-\frac{Q_l^2}{Q_h^2}\right). \qquad (2.141)
\end{aligned}
$$

The above formulae are realized in file `teradm.f`.

2.7.3 Hadronic variables

There exists a closed expression for the complete leptonic QED corrections in hadronic variables to order $\mathcal{O}(\alpha)$ with soft-photon exponentiation:

$$
\frac{d^2\sigma^{QED}}{dy_h dQ_h^2} = \frac{d^2\sigma^B}{dy_h dQ_h^2}\exp\left[\frac{\alpha}{\pi}\delta_{inf}(y_h,Q_h^2)\right] + \frac{2\alpha^3}{S}\sum_{i=1}^{3}\frac{1}{Q_h^4}\mathcal{A}_i(x_h,Q_h^2)\mathcal{S}_i(y_h,Q_h^2).
$$

The soft photon corrections are:

$$
\delta_{inf}(y_h,Q_h^2) = \left(\ln\frac{Q_h^2}{m^2}-1\right)\ln(1-y_h). \qquad (2.142)
$$

The hard bremsstrahlung corrections are taken into account to order $\mathcal{O}(\alpha)$:

$$
\begin{aligned}
\mathcal{S}_1(y_h,Q_h^2) = {}& Q_h^2\Bigg[\frac{1}{4}\ln^2 y_h - \frac{1}{2}\ln^2(1-y_h) - \ln y_h\ln(1-y_h) - \frac{3}{2}\mathrm{Li}_2(y_h) + \frac{1}{2}\mathrm{Li}_2(1)\\
& -\frac{1}{2}\ln y_h \mathrm{L_h} + \frac{1}{4}\left(1+\frac{2}{y_h}\right)\mathrm{L_{h1}} + \left(1-\frac{1}{y_h}\right)\ln y_h - \left(1+\frac{1}{4y_h}\right)\Bigg],\\
\mathcal{S}_2(y_h,Q_h^2) = {}& S^2\Bigg[-\frac{1}{2}y_h\ln^2 y_h - (1-y_h)\ln^2(1-y_h) - 2(1-y_h)\ln y_h\ln(1-y_h) - y_h\mathrm{Li}_2(1)\\
& - (2-3y_h)\mathrm{Li}_2(y_h) + y_h\ln y_h\mathrm{L_h} + \frac{y_h}{2}(1-y_h)\mathrm{L_{h1}} - \frac{y_h}{2}(2-y_h)\ln y_h\Bigg],
\end{aligned}
$$

$$\mathcal{S}_3(y_h,Q_h^2) = SQ_h^2\Big\{-(2-y_h)\Big[-\frac{1}{2}\ln^2 y_h+\ln^2(1-y_h)+2\ln y_h\ln(1-y_h)+3\mathrm{Li}_2(y_h)$$
$$-\mathrm{Li}_2(1)+\ln y_h+\ln y_h\mathrm{L_h}\Big]+\frac{3}{2}y_h\mathrm{L_{h1}}+y_h-\frac{7}{2}+2\,(1-2y_h)\ln y_h\Big\}, \tag{2.143}$$

with $\mathrm{L_h}=\ln(Q_h^2/m^2)$ and $\mathrm{L_{h1}}=\mathrm{L_h}+\ln[y_h/(1-y_h)]$.

Since the structure functions depend on the hadronic variables and are hardly to be integrated over together with radiative corrections, such a compact result may be obtained in hadronic variables only.

The above formulae are realized in files **teradh.f** and **terhin.f** using two different analytical expressions.

2.7.4 Jaquet-Blondel variables

The net cross section is:

$$\frac{d^2\sigma^{QED}}{dy_{\mathrm{JB}}dQ^2_{\mathrm{JB}}} = \frac{d^2\sigma^B}{dy_{\mathrm{JB}}dQ^2_{\mathrm{JB}}}\left\{\exp\left[\frac{\alpha}{\pi}\delta_{inf}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}})\right]-1+\frac{\alpha}{\pi}\Big[\delta_{VR}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}})-\delta_{inf}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}})\Big]\right\}$$
$$+\frac{d^2\sigma^{brems}}{dy_{\mathrm{JB}}dQ^2_{\mathrm{JB}}}. \tag{2.144}$$

The net factorizing part is:

$$\delta_{VR}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}}) = \delta_{inf}(Q^2_{\mathrm{JB}},y_{\mathrm{JB}})-\ln^2 x_{\mathrm{JB}}+\ln x_{\mathrm{JB}}\left[\ln(1-y_{\mathrm{JB}})-1\right]$$
$$-\frac{1}{2}\ln^2\left[\frac{(1-x_{\mathrm{JB}})(1-y_{\mathrm{JB}})}{x_{\mathrm{JB}}y_{\mathrm{JB}}}\right]$$
$$+\frac{3}{2}\mathrm{L_{JB}}-\mathrm{Li}_2(x_{\mathrm{JB}})-2\mathrm{Li}_2(1-x_{\mathrm{JB}})-1, \tag{2.145}$$

where

$$\delta_{inf}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}})=\ln\frac{(1-x_{\mathrm{JB}})(1-y_{\mathrm{JB}})}{1-x_{\mathrm{JB}}(1-y_{\mathrm{JB}})}\left(\ln\frac{Q^2_{\mathrm{JB}}}{m^2}-1\right). \tag{2.146}$$

The non-factorizing part of the radiative cross section in Jaquet-Blondel variables is:

$$\frac{d^2\sigma^{brems}}{dy_{\mathrm{JB}}dQ^2_{\mathrm{JB}}} = \frac{2\alpha^3}{S}\int d\tau\sum_{i=1}^{3}\Big[\mathcal{A}_i(x_h,Q_h^2)\frac{1}{Q_h^4}\mathcal{S}_i(y_{\mathrm{JB}},Q^2_{\mathrm{JB}},\tau)$$
$$-\mathcal{A}_i(x_{\mathrm{JB}},Q^2_{\mathrm{JB}})\frac{1}{Q^4_{\mathrm{JB}}}\,\mathcal{S}_i^B(y_{\mathrm{JB}},Q^2_{\mathrm{JB}})\,\mathcal{F}^{\mathrm{IR}}(y_{\mathrm{JB}},Q^2_{\mathrm{JB}},\tau)\Big], \tag{2.147}$$

with

$$\mathcal{S}_1(Q^2_{\mathrm{JB}},y_{\mathrm{JB}},\tau) = Q^2_{\mathrm{JB}}\left[\frac{1}{z_2}(\mathrm{L}_\tau-2)+\frac{z_2}{4\tau^2}\right]+\frac{1-8y_{\mathrm{JB}}}{4(1-y_{\mathrm{JB}})}+\mathrm{L}_\tau\left(\frac{z_2}{2Q_\tau^2}+\frac{y_{\mathrm{JB}}}{1-y_{\mathrm{JB}}}\right), \tag{2.148}$$

$$\mathcal{S}_2(Q^2_{\mathrm{JB}},y_{\mathrm{JB}},\tau) = S^2\Big\{2(1-y_{\mathrm{JB}})\left[\frac{1}{z_2}(\mathrm{L}_\tau-2)+\frac{z_2}{4\tau^2}\right]-\frac{1}{Q_\tau^2}\left[2(\mathrm{L}_\tau-2)+\frac{1}{2}(1-y^2_{\mathrm{JB}})\right]$$
$$+\frac{Q^2_{\mathrm{JB}}}{Q_\tau^4}(1-y_{\mathrm{JB}})\left[1-(1+y_{\mathrm{JB}})(\mathrm{L}_\tau-3)\right]-\frac{Q^4_{\mathrm{JB}}}{Q_\tau^6}(1-y_{\mathrm{JB}})^2(\mathrm{L}_\tau-3)\Big\},$$

$$\begin{aligned}\mathcal{S}_3(Q^2_{\rm JB}, y_{\rm JB}, \tau) &= S\Bigg\{2Q^2_{\rm JB}(2-y_{\rm JB})\left[\frac{1}{z_2}({\rm L}_\tau-2)+\frac{z_2}{4\tau^2}\right]+\frac{y_{\rm JB}(1+y^2_{\rm JB})}{1-y_{\rm JB}}{\rm L}_\tau+5 \\ &- \frac{7y_{\rm JB}}{2(1-y_{\rm JB})}-(1-y_{\rm JB})(5+2y_{\rm JB}) \\ &+ \frac{Q^2_{\rm JB}}{Q^2_\tau}(1-y_{\rm JB})\left[(1-y_{\rm JB})\left(3-\frac{2Q^2_{\rm JB}}{Q^2_\tau}\right)({\rm L}_\tau-2)+12-5{\rm L}_\tau\right]\Bigg\}. \end{aligned} \tag{2.149, 2.150}$$

The integral over τ is performed in `HECTOR` numerically since one cannot neglect the difference between $Q^2_{\rm JB}$ and Q^2_h – the structure functions are depending on Q^2_h and thus on τ.

The above formulae are realized in file `teradj.f`.

2.7.5 Cuts

In the `TERAD` approach it is possible to reject the production of final states with a Q^2_h and/or an invariant hadronic mass W^2_h below some bound. This is realized in `HECTOR` for all the sets of kinematical variables. See also the description of flag `IHCU` in section 4.2.

The QED corrections in *leptonic* variables in the `TERAD` part of `HECTOR` allow, in addition, the application of cuts on the photon kinematics. Details of the formulation may be found in section 8.3 and appendix B.1.2 of [11]. In `HECTOR` this is done by the variable `GCUT` which is described in section 4.2.

2.8 DISEP: Complete $\mathcal{O}(\alpha)$ corrections in the quark parton model approach

Complete order $\mathcal{O}(\alpha)$ corrections in the quark-parton approach are calculated in the `DISEP` part of `HECTOR`. They include a large variety of contributions:

(i) Electroweak corrections, including the running QED coupling.

(ii) Photonic corrections due to the emission of photons from the leptons together with the corresponding vertex corrections.

(iii) Photonic corrections due to the emission of photons from the quark lines together with the corresponding vertex corrections. Their treatment deserves some care in view of the unknown quark masses leading to logarithmic mass singularities in the on-mass-shell scheme. We refer for details to the literature [37, 5, 38, 13, 39]. In the branches selected by `IDIS=1,2` these terms do not contribute.

(iv) The interference of photon emission from leptons and quarks together with the corresponding $\gamma\gamma, \gamma Z, \gamma W$ box corrections.

(v) Exponentiation of soft photon corrections due to (ii).

In the present version there are no QCD corrections taken into account in the parton distributions in this part of `HECTOR`.

The additional bremsstrahlung diagrams for $\mathcal{NC}$ scattering are shown in figure 5. The interference of them among themselves is combined with vertex corrections from the analogue

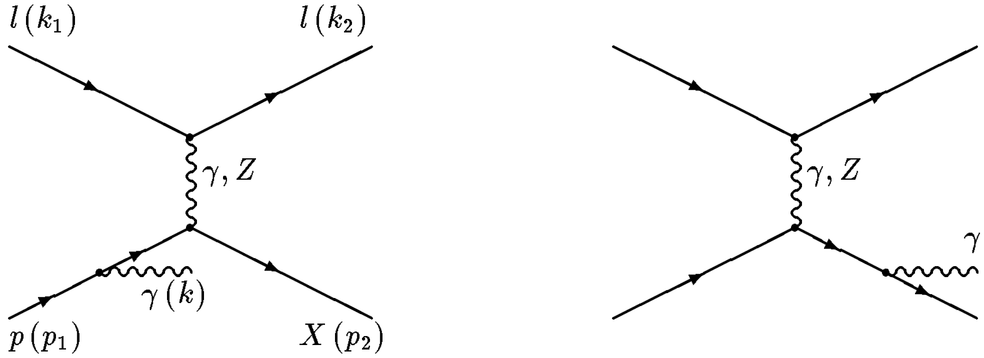

Figure 5: *QED Bremsstrahlung off quarks*

of figure 3(b), and the interference of them with the leptonic radiation (figure 4) has to be combined with the γZ and $\gamma\gamma$ boxes in the $\mathcal{NC}$ case and of γW boxes in the $\mathcal{CC}$ case.

The $\mathcal{CC}$ process has some specific features. The leptonic vertex correction comes from a diagram, where the virtual photon connects the incoming electron with the virtual W boson; the final state neutrino has no photonic interactions. The virtual photonic corrections from the quark side arise analogously. Furthermore, there are contributions from a Feynman diagram with real photon emission from the virtual W boson. All these contributions are taken into account in **HECTOR**. Some details of gauge invariance and other peculiarities may be found in [10].

The electroweak corrections (i) are taken into account by generalized structure functions as is described in section 2.2.3.

If identical structure functions are chosen, the leptonic QED corrections (ii) agree with those calculated in the model independent approach of section 2.7 although the underlying analytical expressions look quite differently. There is no access in the model independent approach to the corrections (iii) and (iv).

The leading logarithmic approximation reduces the complete $\mathcal{O}(\alpha)$ corrections to terms proportional to $\ln(Q^2/m_e^2)$ in (ii), to $\ln(Q^2/m_q^2)$ in (iii). In (iv) no mass singularities are contained.

At present, in **HECTOR** are the following branches with QPM based complete calculations available:

(i) $\mathcal{NC}$ and $\mathcal{CC}$ deep inelastic scattering in *leptonic* variables;

(ii) $\mathcal{NC}$ deep inelastic scattering in *mixed* variables.

2.8.1 $\mathcal{NC}$ scattering in leptonic variables

The QPM based electroweak corrections to $\mathcal{NC}$ scattering in leptonic variables are described in [9].

The virtual corrections are a part of the generalized structure functions in the improved Born approximation, calculated in subroutine `sigbrn.f`. They are described in section 2.2.3[4].

[4]Here we deviate slightly from [9].

In addition, the QED corrections have to be determined. In the DISEP part, they have the following structure:

$$\frac{d^2\sigma^{QED}}{dx_l dy_l} = \frac{2\pi\alpha^2(Q_l^2)Sx_l}{Q_l^4} \sum_B \sum_b \sum_{Q,\bar{Q}} c_b \mathcal{K}(B,p) \left[\mathcal{V}(B,p) R_b^V(B) + p\mathcal{A}(B,p) R_b^A(B)\right], \quad (2.151)$$

where the sum over $Q, \bar{Q}$ extends over quarks and anti-quarks and $B = \{\gamma, I, Z\}$. The sum over b covers bremsstrahlung from leptons, lepton-quark interference, and quarks ($b = \{e, i, q\}$).

The parton distributions are accessed via the subroutine `prodis.f` directly. The explicit use of QCD corrections to the parton densities is not foreseen here.

The parameter p and the couplings $\mathcal{K}(B,p), \mathcal{V}(B,p), \mathcal{A}(B,p)$ are defined in section 2.2.3. The coefficients c_b contain charges:

$$\begin{aligned} c_e &= Q_e^2, & (2.152)\\ c_i &= Q_e Q_q, & (2.153)\\ c_q &= Q_q^2. & (2.154) \end{aligned}$$

The QED corrections are described by the functions $R_b^{V,A}(B)$:

$$R_b^{V,A}(B) = R_b(B; 1, 1-y_l) + s_b^{V,A} R_b(B; y_l - 1, -1), \quad (2.155)$$

with

$$s_e^V = -s_e^A = s_i^A = -s_i^V = s_q^V = -s_q^A = 1. \quad (2.156)$$

The factors $R_b(B; 1, 1-y)$ have the following structure:

$$\begin{aligned} R_c(B;a,b) &= \frac{\alpha}{\pi}\left\{ S_c(B;a,b) f_q(x_l, Q_l^2) + \int_{x_l}^1 dx_h \left[\frac{x_l}{x_h} f_q(x_h, Q_l^2) U_c(B;a,b) \right.\right.\\ &\quad \left.\left. + \frac{r_c x_h f_q(x_h, Q_l^2) - x_l f_q(x_l, Q_l^2)}{x_h - x_l} T_c(B;a,b)\right]\right\}, \quad (2.157) \end{aligned}$$

with

$$r_c = 1/x_h,\ 1,\ 1 \qquad \text{for c} = \text{e}, \text{i}, \text{q}. \quad (2.158)$$

The S, T, U are results of a two-fold analytical integrations over photonic angles. They are the analogues to the kinematical functions $\mathcal{S}_i$ and $\mathcal{L}^{\text{IR}}$ shown in (2.125) to (2.128). Because there are many of them, we refer for explicit expressions to the original literature. The six independent functions are:

$$R_e(\gamma;a,b),\ R_e(I;a,b),\ R_e(Z;a,b),\ R_i(\gamma;a,b),\ R_i(Z;a,b),\ R_q(\gamma;a,b).$$

They are composed in the subroutines `SGGiL, SGZiL, SZZiL`, `i`=`L,I,Q` of `HECTOR` and the constituents S, T, U are calculated in functions `FGGiL, FGZiL, FZZiL`, `i`=`L,I,Q`. The others are expressed by the following identities:

$$\begin{aligned} R_i(I;a,b) &= \frac{1}{2}\left[R_i(\gamma;a,b) + R_i(Z;a,b)\right], & (2.159)\\ R_q(\gamma;a,b) &= R_q(I;a,b) = R_q(Z;a,b). & (2.160) \end{aligned}$$

At the end of this section we should mention that the virtual corrections, by which the QED corrections are accompanied via the weak and electromagnetic couplings, may also be calculated with the use of the actual kinematical variables, i.e. folding them together with the functions T and U. This may be achieved by chosing `ICONV`=1. The resulting computation is very time consuming and leads to numerical values which differ only insignificantly from those obtained without using this option.

In the QPM approach in leptonic variables the virtuality of the parton densities was choosen by the outer variable Q_l^2. For hard photon emission, one actually would have to perform integrations over weighted structure functions at reduced kinematical variables. The structure functions depend on the hadron kinematics, i.e. on x_h and Q_h^2. In the `DISEP` approach Q_h^2 is to be integrated over analytically, which leads, however, to the problem that the quark distribution functions depend on some Q^2, but not on the correct (hadronic) Q_h^2. The same procedure is followed in the LLA calculations. In this sense, the `DISEP` approach has a reduced accuracy in leptonic variables compared to the model independent approach. For a high precision calculation it may be useful to calculate the largest corrections (ii) from `TERAD` and the remaining terms from `DISEP`.

2.8.2 $\mathcal{CC}$ scattering in leptonic variables

The QPM based electroweak corrections in `HECTOR` to $\mathcal{CC}$ scattering in leptonic variables are described in [10]. As in the case of $\mathcal{NC}$ scattering, the improved $\mathcal{CC}$ Born cross section is determined in subroutine `sigbrn.f`.

The QED corrections have access to the parton distributions not through the generalized structure functions of file `gccstf.f` but instead directly through `prodis.f`. The QED corrections are:

$$\frac{d^2\sigma^{QED}}{dx_l dy_l} = \frac{G_\mu^2 S x_l}{\pi}\left[\frac{M_W^2}{Q_l^2+M_W^2}\right]^2 \frac{1+\lambda}{2}\sum_b\sum_{Q,\bar{Q}}\theta(-Q_l Q_q)c_b\rho_c^2(p)\left[\frac{1+p}{2}R_b+\frac{1-p}{2}\bar{R}_b\right]. \tag{2.161}$$

The notations are those being introduced in sections 2.2.3 and 2.8.1. The generic definition of the functions R_b is (2.157). They are calculated in subroutines `SWWL`, `SWWI`, and `SWWQ` of `HECTOR`. Explicit expressions may be found in appendix B of [10].

As was mentioned already in the case of $\mathcal{NC}$ scattering, the QED corrections in leptonic variables in the `DISEP` approach are calculated with parton distributions having a Q^2 dependence on Q_l^2 instead of the correct variable Q_h^2.

2.8.3 $\mathcal{NC}$ scattering in mixed variables

The improved Born cross section is as described in sections 2.1 and 2.2.3.

The QPM based QED corrections to $\mathcal{NC}$ scattering in mixed variables have been described in [13]. The structure of the corrections is the following:

$$\begin{aligned}\frac{d^2\sigma^{QED}}{dx_m dy_m} &= \frac{2\alpha^3 S x_m}{Q_m^{2\ 4}}\sum_{Q,\bar{Q}}\Bigg\{B_0(y_m,1)f_q(x_m,Q_m^2)\sum_{a=e,i,q}c_a S_a(x_m,y_m|m_a^2)\\ &+\sum_{a=e,i,q}\left[\int_1^{1/x_m}dz\left[\bar{B}_a^V(z;x_m,y_m)+p\bar{B}_a^A(z;x_m,y_m)\right]\right.\end{aligned}$$

$$+\int_{y_m}^{1} dz\left[B_a^V(z;x_m,y_m)+pB_a^A(z;x_m,y_m)\right]\Bigg]\Bigg\}+\frac{d^2\sigma^{box}}{dx_m dy_m}, \qquad (2.162)$$

where

$$B_0(y_m,z) \;=\; V_0\,Y_+\left(\frac{y_m}{z}\right)+pA_0Y_+\left(\frac{y_m}{z}\right). \qquad (2.163)$$

Much of the notations can be found in the previous sections and section 2.2.3. In addition, we use for the vector couplings the following modifications:

$$\begin{aligned}
V_0 &= V(1,1),\\
V(a,b) &= \sum_{B=\gamma,I,Z} K(a,b;B,p)\,\mathcal{V}(B,p), \qquad (2.164)\\
K(a,b;B,p) &= \chi_{B_1}(a)\,\chi_{B_2}(b)\,\mathcal{K}(B,p),\\
\chi_B(a) &= \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha(Q^2)}\frac{Q_m^2}{aQ_m^2+M_B^2}.
\end{aligned}$$

The axial couplings $A(a,b)$ are related to the couplings $\mathcal{A}(B,p)$ of (2.52) in the same manner as is explained above for the vector couplings.

The functions S_e, S_q contain factorizing soft photon corrections and the corresponding vertex corrections.

The hard bremsstrahlung functions in (2.162) are:

$$\overset{(-)}{B}{}_a^{V,A}(z;x_m,y_m) \;=\; \left\{\begin{array}{c} V_a \\ A_a \end{array}\right\}\left\{\sum_n Reg\left[\overset{(-)}{F}{}_{an}^{V,A}(z,y_m)f_Q(zx_m)\right]\overset{(-)}{L}{}_{an}+\overset{(-)}{U}{}_a^{V,A}(z,y_m)f_Q(zx_m)\right\}, \qquad (2.165)$$

with the modifications of weak neutral coupling factors; for the vector couplings:

$$V_e \;=\; c_e\,V(z,z), \qquad V_i=c_i\,V(1,z), \qquad V_q=c_q\,V(1,1). \qquad (2.166)$$

The bremsstrahlung axial couplings are defined analogously. For $a=e,i,q$, index n in (2.165) runs from 1 to $2,1,2$. The functions F_{an} are regularized by a subtraction:

$$Reg\left[\overset{(-)}{F}{}_{an}^{V,A}(z,y_m)f_Q(zx_m)\right] \;=\; \frac{y_m}{(1-z)}\left[\overset{(-)}{F}{}_{an}^{V,A}(z,y_m)f_Q(zx_m)-\overset{(-)}{F}{}_{an}^{V,A}(1,y_m)f_Q(x_m)\right]. \qquad (2.167)$$

Explicit expressions for the functions S,F,L,U may be found in [13]. In **HECTOR** they are realized in subroutines **SGGiM, SGZiM, SZZiM**, i=**L,I,Q**.

2.9 TERADLOW: QED corrections at low Q^2

In **HECTOR**, the double differential cross section of the photoproduction process is calculated in leptonic variables with the aid of file `terlow.f`. For details of the derivation of formulae and of the kinematics, which has to be treated exactly in both the electron and proton masses, we refer to [11].

The process kinematics is described by Q_l^2 as defined from the leptonic variables and the invariant mass W of the system consisting both of the photon and the hadrons,

$$\begin{aligned} Q^2 = Q_l^2 &= (k_1 - k_2)^2, \\ W^2 &= -(Q_l + p_1)^2. \end{aligned} \tag{2.168}$$

For a fixed value of $W^2 \ll S$, the minimal value of Q_l^2 may become extremely small, but remains non-vanishing:

$$Q_l^{2\min}(W^2) \approx m^2 \frac{[W^2 - M^2]^2}{S^2} > 0. \tag{2.169}$$

This leaves the integrated deep inelastic cross section finite. The net cross section is:

$$\frac{d^2\sigma_{\rm R}}{dy_l dQ_l^2} = \frac{d^2\sigma^{\rm anom}}{dy_l dQ_l^2} + \frac{d^2\sigma_{\rm R}^{\rm F}}{dy_l dQ_l^2} + \frac{d^2\sigma^{\rm B}}{dy_l dQ_l^2} \left\{ \exp\left[\frac{\alpha}{\pi}\delta^{\rm inf}(\mathcal{E})\right] - 1 + \frac{\alpha}{\pi}\left[\delta^{\rm VR}(\mathcal{E}) - \delta^{\rm inf}(\mathcal{E})\right]\right\}. \tag{2.170}$$

The contribution from the anomalous magnetic moment is:

$$\frac{d^2\sigma^{\rm anom}}{dy_l dQ_l^2} = \frac{2\alpha^3 S}{\lambda_S Q_l^4}\frac{m^2}{x_l Q_l^2}\mathcal{V}^{\rm anom}(\beta)\left[2\beta^2 {y_l}^2 x_l F_1(x_l, Q_l^2) - (2 - y_l)^2 F_2(x_l, Q_l^2)\right], \tag{2.171}$$

where

$$\mathcal{V}^{\rm anom}(\beta) = -\frac{\mathrm{L}_\beta}{\beta}, \tag{2.172}$$

$$\mathrm{L}_\beta = \ln \frac{\beta + 1}{\beta - 1}, \tag{2.173}$$

$$\beta = \sqrt{1 + \frac{4m^2}{Q_l^2}}. \tag{2.174}$$

The factorizing photonic corrections from bremsstrahlung and the usual, exact vertex correction are:

$$\begin{aligned} \delta^{\rm VR}(y_l, Q_l^2) &= \delta^{\rm inf}(y_l, Q_l^2) + \frac{1}{2\beta_1}\ln\frac{1+\beta_1}{1-\beta_1} + \frac{1}{2\beta_2}\ln\frac{1+\beta_2}{1-\beta_2} + \mathcal{S}_\Phi \\ &+ \frac{3}{2}\beta\mathrm{L}_\beta - 2 - \frac{1+\beta^2}{2\beta}\left[\mathrm{L}_\beta\ln\frac{4\beta^2}{\beta^2 - 1} + \mathrm{Li}_2\left(\frac{1+\beta}{1-\beta}\right) - \mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right], \end{aligned} \tag{2.175}$$

with

$$\delta^{\rm inf}(y_l, Q_l^2) = 2\ln\frac{W^2 - (M + m_\pi)^2}{m\sqrt{W^2}}\left(\frac{1+\beta^2}{2\beta}\mathrm{L}_\beta - 1\right), \tag{2.176}$$

and

$$\beta_1 = \sqrt{1 - \frac{4m^2 M^2}{(S - Q_l^2)^2}}, \tag{2.177}$$

$$\beta_2 = \sqrt{1 - \frac{4m^2 M^2}{[S(1 - y_l) + Q_l^2]^2}}, \tag{2.178}$$

$$\mathcal{S}_\Phi = \frac{1}{2}\left(Q^2+2m^2\right)\int_0^1 \frac{d\alpha}{\beta_\alpha(-k_\alpha^2)}\ln\frac{1-\beta_\alpha}{1+\beta_\alpha}. \qquad (2.179)$$

$$k_\alpha = k_1\alpha + k_2(1-\alpha), \qquad (2.180)$$

$$\beta_\alpha = \frac{\left|\vec{k}_\alpha\right|}{k_\alpha^0}. \qquad (2.181)$$

Finally, the infrared finite part of the real bremsstrahlung can be written as follows:

$$\frac{d^2\sigma_{\rm R}^{\rm F}}{dy_l dQ_l^2} = \frac{2\alpha^3 S}{\lambda_S}\int dM_h^2 dQ_h^2 \sum_{i=1}^{3}\Biggl[\mathcal{A}_i(x_h,Q_h^2)\frac{1}{Q_h^4}\mathcal{S}_i(y_l,Q_l^2,y_h,Q_h^2) - \mathcal{A}_i(x_l,Q_l^2)\frac{1}{Q_l^4}\,\mathcal{S}_i^B(y_l,Q_l^2)\,\mathcal{L}^{\rm IR}(y_l,Q_l^2,y_h,Q_h^2)\Biggr], \qquad (2.182)$$

where $\mathcal{S}_i(y_l,Q_l^2,y_h,Q_h^2)$ $(i=1,2,3)$ are given by (2.125)–(2.127), and $\mathcal{L}^{\rm IR}(y_l,Q_l^2,y_h,Q_h^2)$ is defined by (2.133). We would like to remind that these expressions are exact in both masses m and M for the photon exchange contributions.

2.9.1 Cuts

In `TERADLOW` it is possible to reject the production of final states with a Q_h^2 and/or an invariant hadronic mass W_h^2 below some bound, (cf. the description of flag `IHCU` in section 4.2).

2.10 Other reactions

In the LLA approach to leptonic QED corrections the Born cross section is folded with splitting functions at rescaled kinematic variables. This factorization may be used also for exclusive reactions having the same leptonic tensors as in the case of deep inelastic scattering. Examples for this class of reactions are e.g. heavy quark production, deep inelastic scattering including Z' exchange, and Higgs boson production. These processes were studied previously in this context [6, 42, 43].

Sometimes one is interested even in the LLA contributions either due to initial *or* final state radiation *only*. For this case it is sufficient that the subsystem ('Born') cross section has a charged lepton line either in the initial or final state for which the LLA terms shall be calculated.

In both these cases the respective corrections can be obtained setting the flag `IBRN`=0. The user has to supply a subroutine `usrbrn.f` describing the subsystem process.

3 Structure of the Code

The Fortran program HECTOR consists of several subdirectories. They are shown in figure 6. Some of them contain several files, others only one. Some of the files contain several subroutines and functions. Exceptions, which are not shown in figure 6, are: the subdirectory DIZET, which is used for the calculation of the electroweak radiative corrections and the subdirectory HECAUX with some auxiliary routines. Besides the subdirectories, the two files HECTOR.INP and HECTOR.OUT are added in the flowchart.

The subdirectories are used for the following tasks:

CMAIN:	Organization of the program flow for combined use of the branches HELIOS and TERAD
DISEP:	Package for calculation of complete order $\mathcal{O}(\alpha)$ QED corrections with soft photon exponentiation determined in the quark parton model approach
DIZET:	Package for the calculation of weak virtual corrections
GENSTF:	Calculation of the generalized structure functions for HELIOS, TERAD, and DISEP
GSFLOW:	Calculation of generalized structure functions for TERADLOW
HECAUX:	Some auxiliary functions
HECFFR:	Reads input data cards from file HECTOR.INP using FFREAD
HECOUT:	Writes output to file HECTOR.OUT, partly using FFREAD
HECSET:	Setting of initialization parameters
HECTOR:	The main program
HELIOS:	Package for calculation of QED corrections in the leading logarithmic approximation, including higher order corrections and soft photon exponentiation
HMAIN:	Organization of the program flow for branch HELIOS
LOWLIB:	Package for the optional modification of structure functions at low Q^2 values
PRODIS:	Calculation of structure functions for electroweak corrections in all approaches and for QED corrections in package DISEP
PDFLIB:	Standard package of parton distribution functions
PDFACT:	Default package of parton distribution functions
SIGBRN:	Calculation of the (effective) Born cross section
STRUFC:	Calculation of structure functions from parton distribution functions
TERAD:	Package for model independent calculation of complete leptonic order $\mathcal{O}(\alpha)$ QED corrections with soft photon exponentiation
TERADLOW:	Package for model independent calculation of complete leptonic order $\mathcal{O}(\alpha)$ QED corrections with soft photon exponentiation intended for the region of extremely low Q^2
TMAIN:	Organization of the program flow for the branch TERAD, which may call the QED corrections of the packages DISEP, TERAD, or TERADLOW

The user supplied subdirectories are installed containing trivial versions only:

USRINI:	File for input modifications, which go beyond the change of data cards foreseen
USROUT:	Modified output specifications
USRBRN:	User supplied Born cross section
USRSTR:	User supplied structure functions
USRPDF:	User supplied parton distribution functions

The two files in figure 6 are:

HECTOR.INP: File with the input data card contents
HECTOR.OUT: Standard output file

Figure 7 is devoted to the interplay of the subdirectories GENSTF, DIZET, and LOWLIB. The files may be characterized as follows:

CCOUPL:	Calculation of charged current Born couplings or effective couplings
DIZET:	Standard Model Electroweak Library
GCCSTF:	Calculation of generalized structure functions for charged current reactions
GENSTF:	Calculation of generalized structure functions for neutral current reactions
LOWLIB:	Files with different structure function modifications at low Q^2
MIDSTF:	Interpolation between the structure function from GENSTF and the low Q^2 modified structure function
NCOUPL:	Calculation of neutral current Born couplings or effective couplings
POLYN:	Auxiliary function for MIDSTF
PRODIS:	Parton distributions of the proton (used by DISEP)
STRFBS:	File with the Brasse/Stein modification of structure functions
USRSTR:	User supplied structure functions

The entry labelled '3' connects the present figure with figure 8.

We show in figure 8 how the actual structure functions are built from the parton distribution functions. The subdirectories STRUFC, PDFACT, USRPDF, PDFLIB are shown as boxes. Their interaction is organized by the flags ISTR, ISCH, ISSE.

HECTOR consists of four branches as may be seen in figure 6: HELIOS, TERAD, DISEP, TERADLOW. The first three branches of HECTOR which contain the different packages for the calculation of the QED corrections are shown in figure 9. The subdirectories HELIOS, TERAD, DISEP are symbolized by the big boxes. The settings of flags IMEA and IDSP define the choice of the type of corrections. HELIOS consists of one file, while inside TERAD and DISEP, the file structure is shown. The index i in the figure is a short–hand notation. For i=1, it is IORD=2 and IEPC=0, while i=2 corresponds to IORD=2 and IEPC=1.

The branch TERADLOW is attained by ILOW=1 together with IMEA=1 and IOPT=2.

Figure 10 illustrates the influence of the flags IUSR, ITV1, ITV2, IBIN, IOPT, IORD on the initialization of HECTOR. Shown are the subdirectories HECSET and HECTOR, together with some selected files (in boxes). We should comment shortly on the following of them:

FILBIN:	The selected lattice of kinematical points is filled
HECCAL:	Calls the branch of HECTOR chosen and initializes the actual calculation
HECRUN:	Runs the job
HECTER:	The common blocks of TERAD are filled
USRBIN:	Optional filling of a user supplied kinematical lattice
USRINT:	A user supplied integration over bins is performed

The entries 1 and 2 are links to figure 11, where the output organization is illustrated.

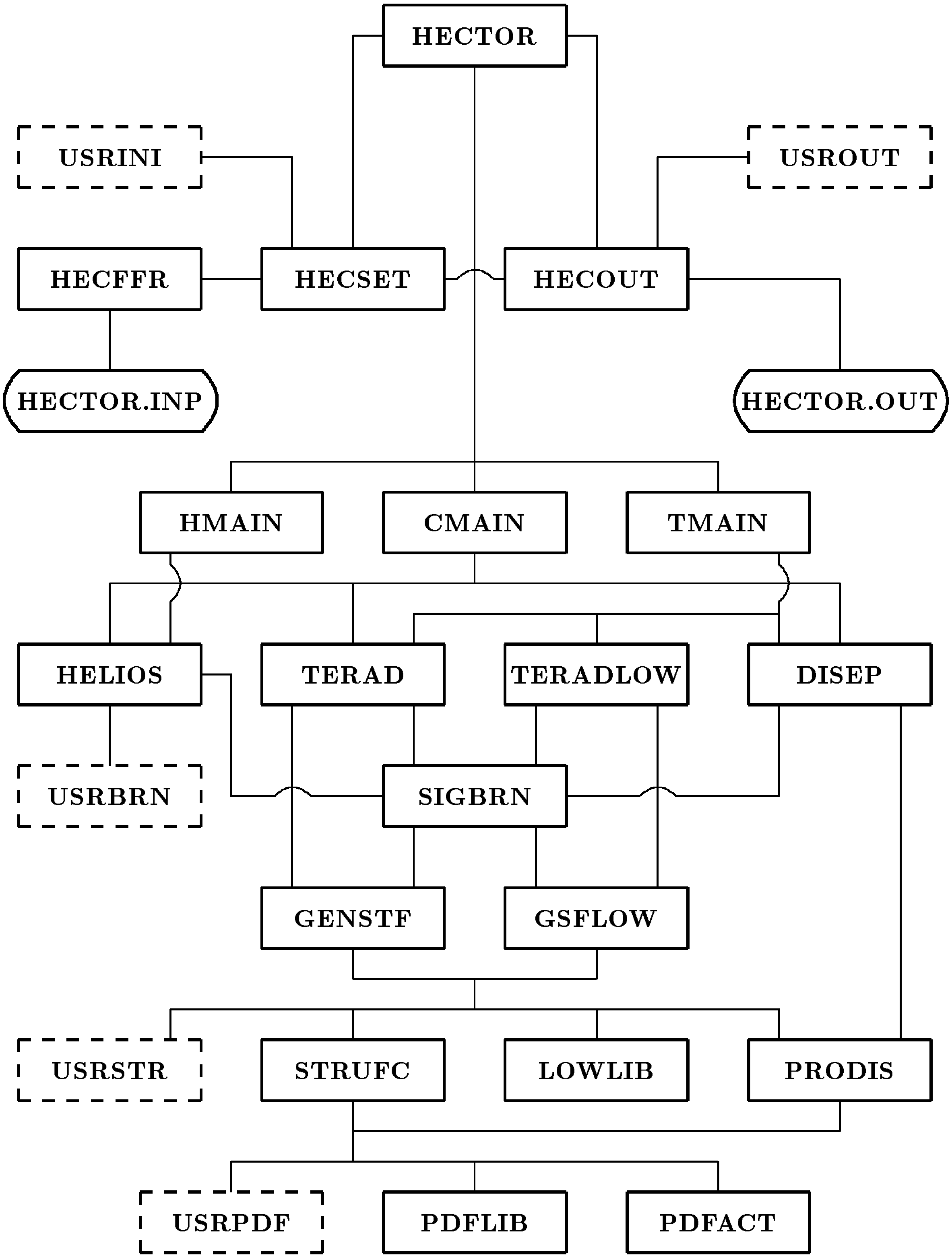

Figure 6: *Basic* `HECTOR` *flowchart. Shown are subdirectories.*

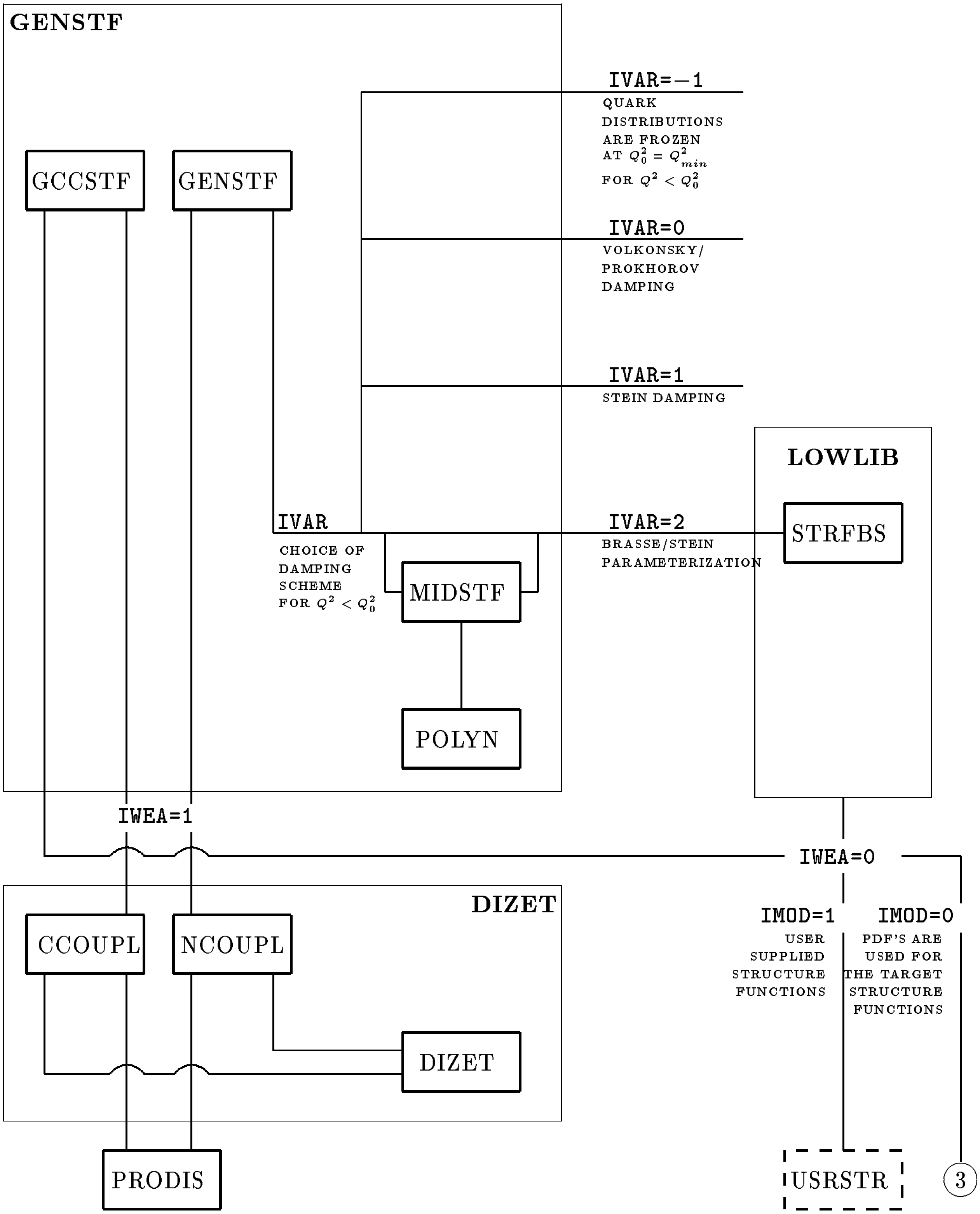

Figure 7: *Logical structure of* GENSTF

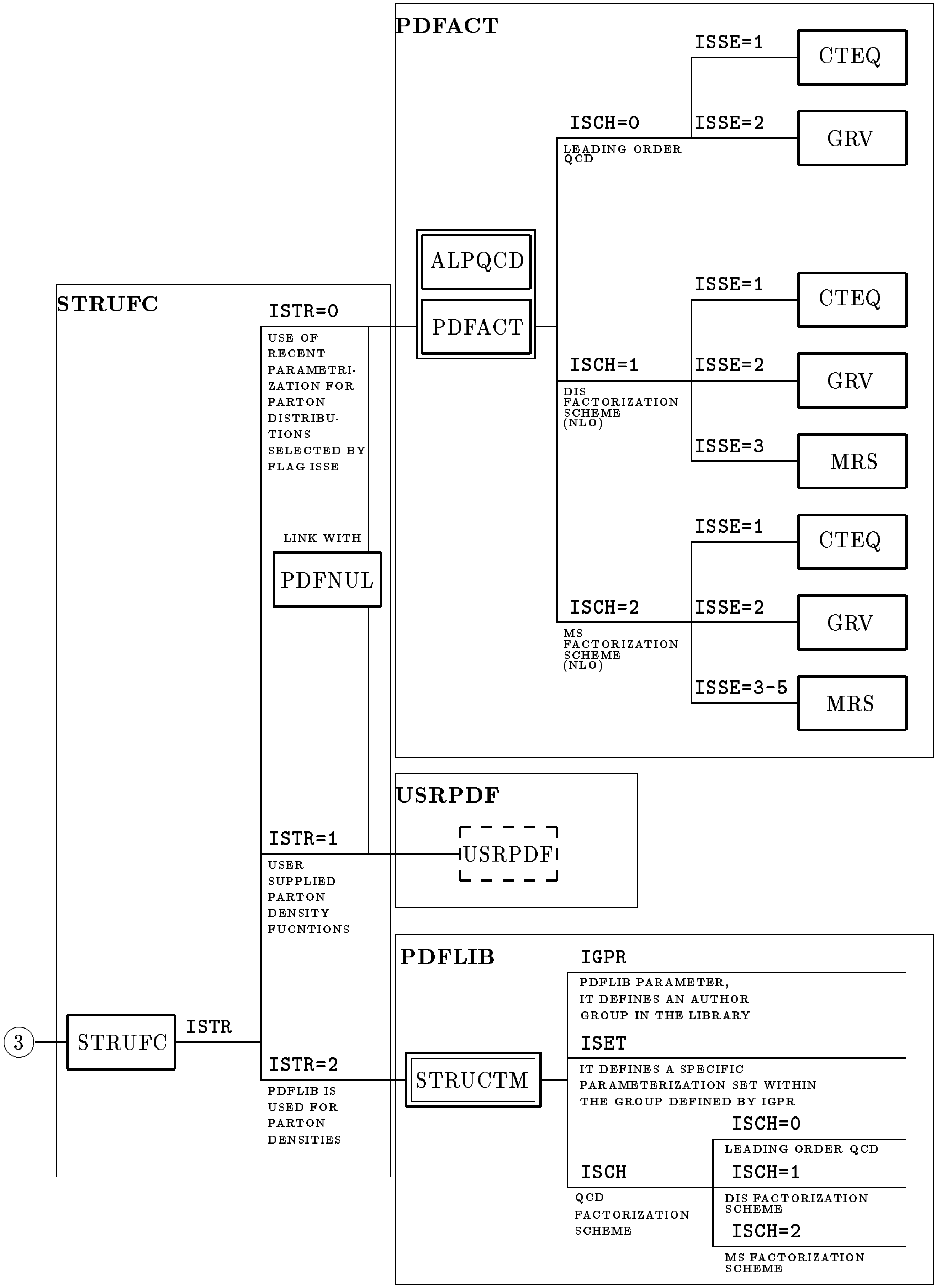

Figure 8: *Building structure functions in* HECTOR

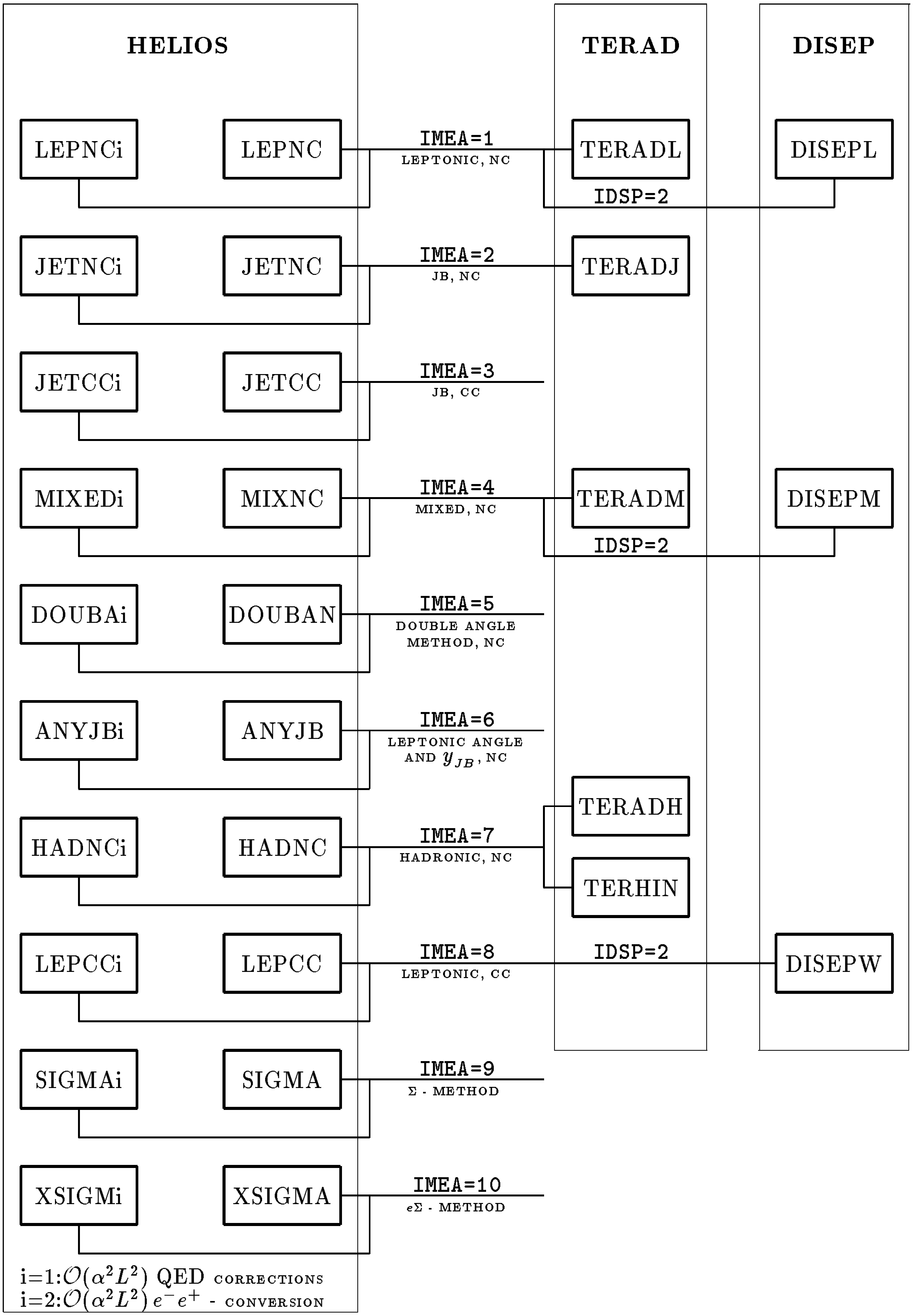

Figure 9: *The branches of* `HECTOR`. *Different treatments of QED corrections are chosen by settings of flags* `IMEA` *and* `IDSP`.

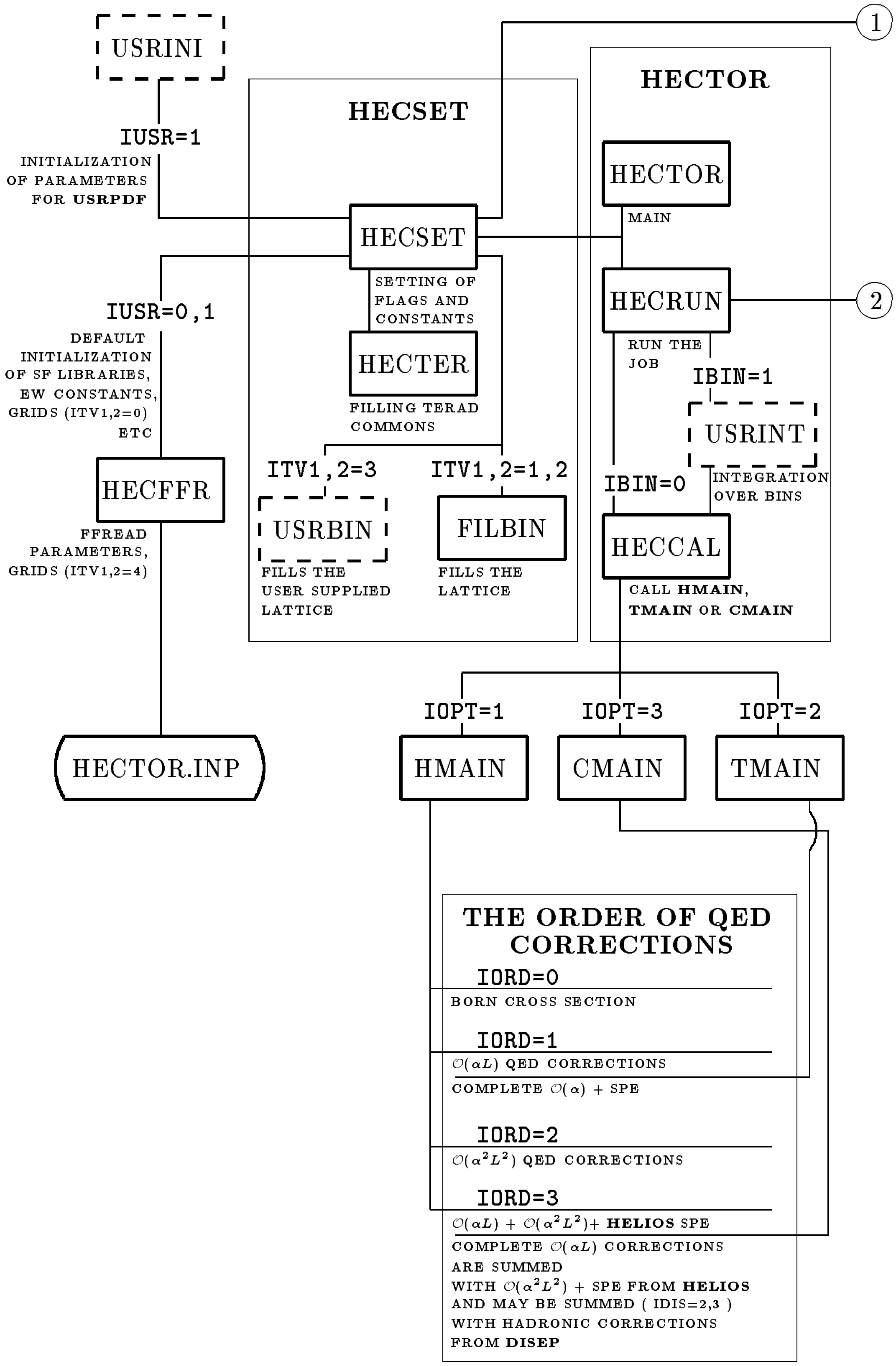

Figure 10: *Input organization and choice of QED treatments*

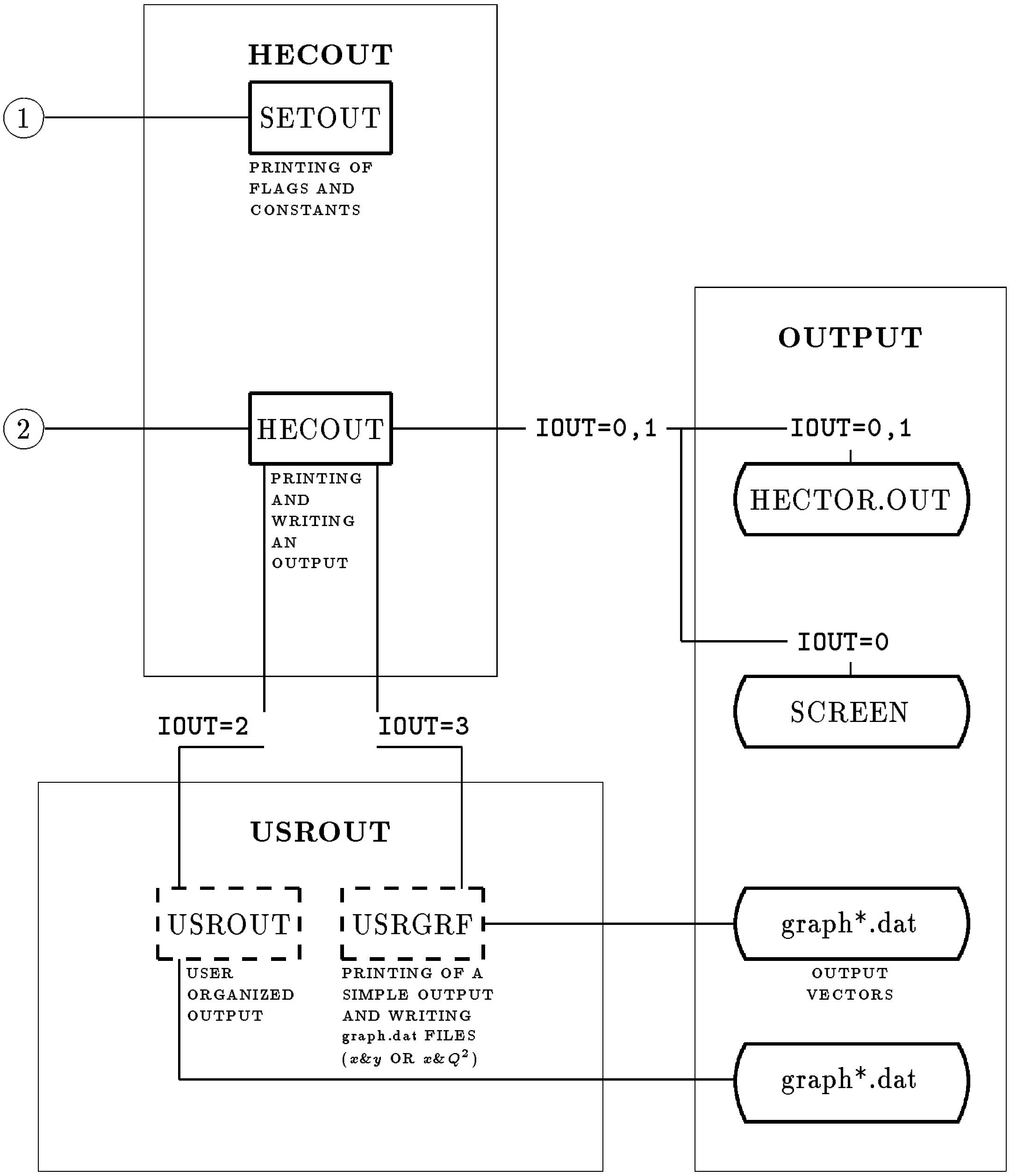

Figure 11: *The output organization of* HECTOR

4 User Options

The following set of flags and parameters may be defined by the user in the file `HECTOR.INP`. The default values of the flags, their type, names of the corresponding variables entering the program with the names of host `COMMON` blocks one may find in appendix B, table 7.

4.1 Selection of the scattering process

<u>IBEA</u>
IBEA=+1: anti-lepton beam (positron or μ^+)
IBEA=−1: lepton beam (electron or μ^-)

<u>ILEP</u>
ILEP=1: electron (or positron) scattering
ILEP=2: muon scattering

<u>ITAR</u>
ITAR=1: proton target
ITAR=2: deuteron target

<u>POLB</u>
POLB: degree of lepton beam polarization ($-1 \leq$ POLB ≤ 1)

<u>EELE</u>
EELE: lepton beam energy in GeV

<u>ETAR</u>
ETAR: energy of target beam in GeV

<u>IBRN</u>
IBRN: to be used for `IOPT=1` only
IBRN=0: Born cross section of a lepton scattering is defined by the user in the subroutine `usrbrn.f`
IBRN=1: default Born cross section

4.2 Selection of measurement type and cuts

<u>IMEA</u>
IMEA=1: lepton measurement (leptonic variables), neutral current DIS
IMEA=2: jet measurement (Jaquet-Blondel variables), neutral current DIS
IMEA=3: jet measurement (Jaquet-Blondel variables), charged current DIS (only for `IOPT=1`)
IMEA=4: mixed variables, neutral current DIS
IMEA=5: double angle method, neutral current DIS (only for `IOPT=1`)
IMEA=6: lepton angle and y_{JB} measurement, neutral current DIS (only for `IOPT=1`)
IMEA=7: hadron measurement, neutral current DIS
IMEA=8: leptonic variables, charged current DIS
IMEA=9: Σ method, neutral current DIS (only for `IOPT=1`)
IMEA=10: $e\Sigma$ method, neutral current DIS (only for `IOPT=1`)

<u>ZCUT</u>

ZCUT: cut on the reconstructed electron beam energy in GeV (only for IOPT=1); see section 2.6.1

<u>IHCU</u>

IHCU=0: no cut on the hadronic final state

IHCU=1: rejects hadronic final states with $Q_h^2 \leq$ Q2CU [GeV^2] and invariant hadronic mass (photon not included) $W_h^2 \leq$ HM2C [GeV^2]; the two desired cut values Q2CU and HM2C have to be set

<u>Q2CU</u>

Q2CU: cut on the transferred momentum squared, Q_h^2, [GeV^2]; see IHCU above

<u>HM2C</u>

HM2C: cut on the hadronic invariant mass square, [GeV^2]; see IHCU above

<u>IGCU</u>

IGCU=0: no cut on photon variables

IGCU=1: rejects all final states with a photon energy $E_\gamma \geq E_{cut}$ [GeV] and a photon angle in the interval $(\theta_{cut2}, \theta_{cut1})$ (angles in radians inside $(0, \pi)$ with respect to the electron beam); to be used only for IOPT=2, IMEA=1

<u>GCUT</u>

GCUT: cut on the photon energy in GeV; see IGCU above

<u>THC1</u>

THC1: minimal photon angle θ_{cut2} in rad; see IGCU above

<u>THC2</u>

THC2: maximal photon angle θ_{cut1} in rad; see IGCU above

4.3 Selection of the main program paths and related flags

<u>IOPT</u>

IOPT=1: HELIOS branch: QED LLA RC calculated in first and higher orders

IOPT=2: TERAD branch: complete calculation of EW and QED RC in $\mathcal{O}(\alpha)$ with soft photon exponentiation; the model independent TERAD results may be supplemented by hadronic corrections from DISEP (for IDIS=2,3); see description of IDSP and ILOW below

IOPT=3: combined calculation using both branches; here the TERAD results are supplemented by HELIOS higher order terms beginning with $\mathcal{O}(\alpha^2)$; hadronic corrections from DISEP may also be added (for IDIS=2,3); the soft photon exponentiation of TERAD is switched off

<u>IORD</u>

IORD=0: Born cross section (for IOPT=1)

IORD=1: QED corrections to $\mathcal{O}(\alpha L)$ (for IOPT=1) or to $\mathcal{O}(\alpha)$ with soft photon exponentiation (for IOPT=2)

IORD=2: QED corrections to $\mathcal{O}((\alpha L)^2)$ only (for IOPT=1)

IORD=3: QED corrections to $\mathcal{O}(\alpha L)$ and $\mathcal{O}((\alpha L)^2)$ are summed and the HELIOS soft photon exponentiation is included (for IOPT=1) *or* complete $\mathcal{O}(\alpha)$ corrections are summed with the $\mathcal{O}((\alpha L)^2)$ and the soft photon exponentiation terms from HELIOS (for IOPT=3)

IBOS

IBOS: calculation of RC to different terms in the $\mathcal{NC}$ cross section; may be used for all values of IMEA but 3,8

IBOS=1: only photon $\mathcal{NC}$ exchange contribution ($|\gamma^2|$ term)

IBOS=2: only the photon - Z boson interference contribution to the $\mathcal{NC}$ exchange ($|\gamma Z|$ term)

IBOS=3: only the Z boson $\mathcal{NC}$ exchange contribution ($|Z^2|$ term)

IBOS=4: sum of all $\mathcal{NC}$ exchange contributions

ICOR

ICOR: to be used for IOPT=1 only

ICOR=1: initial state lepton radiation

ICOR=2: final state lepton radiation

ICOR=3: Compton peak contribution (available for IMEA=1 only)

ICOR=4: initial plus final state radiation

ICOR=5: initial and final state radiation plus Compton peak contribution

ICMP

ICMP: to be used for IOPT=1 only

ICMP=0: no Compton term

ICMP=1: onefold Compton integral (2.110)

ICMP=2: twofold Compton integral (2.111)

IEPC

IEPC: to be used for IOPT=1,3 only

IEPC=0: e^+e^- conversion in $\mathcal{O}((\alpha L)^2)$ is switched off

IEPC=1: e^+e^- conversion is included

IWEA

IWEA=0: EW RC are applied via $\sin\theta_W^{\rm eff}$, the effective EW mixing angle

IWEA=1: the full set of EW form factors is calculated;
Caution! – using this flag may enlarge the CPU time considerably

IVPL

IVPL=0: $\alpha = 1/137\ldots$ is not running

IVPL=1: running α due to the complete fermionic vacuum polarization

IEXP

IEXP=0: no soft photon exponentiation

IEXP=1: soft photon exponentiation is applied; see description of IOPT

IDSP

IDSP=0: the DISEP branch in TERAD is not used

IDSP=1: together with TERAD, the DISEP branch is used (possible only for IMEA=1,4,8); see description of flag IDIS

IDSP=2: only the DISEP branch is used

IDIS

IDIS=1: DISEP leptonic corrections

IDIS=2: DISEP leptonic and lepton quark interference corrections

IDIS=3: DISEP leptonic, interference, and quarkonic corrections

IDQM

IDQM: to be used for IMEA=4, IDIS=2 only, see [13]
IDQM=1: quark masses AMQ: AMQ = xM, M the proton mass
IDQM=2: AMQ = const, no ISR LLA corrections from quarks
IDQM=3: AMQ = const, no ISR or FSR LLA correction from quarks

<u>ILOW</u>
ILOW: to be used for IMEA=1, IOPT=2 only
ILOW=0: no use of TERADLOW
ILOW=1: use of TERADLOW

4.4 Structure function parameterizations

<u>IMOD</u>
IMOD=0: use of quark distributions for the target structure functions
IMOD=1: the parameterization of structure functions is user supplied in the file **usrstr.f** (e.g. for a model independent analysis)

<u>ISTR</u>
ISTR: to be used for IMOD=0 only
ISTR=0: use of recent parameterizations for parton distributions selected by flags ISCH and ISSE
ISTR=1: parton density functions are user supplied via file **usrpdf.f**
ISTR=2: PDFLIB [44] is used for parton densities

<u>ISSE</u>
ISSE: to be used for ISTR=0
ISSE=1: parton distributions CTEQ3 [31]
ISSE=2: parton distributions GRV [26, 27] (cf. the comments in sect. (2.3.2))
ISSE=3: parton distributions MRS(A) [25]
ISSE=4: parton distributions MRS(A') [25]
ISSE=5: parton distributions MRS(G) [25]

<u>ITER</u>
ITER=0: quark distributions and α_{QED} (if running, see flag IVPL) in leptonic bremsstrahlung are artifically chosen to depend on Q^2 as defined from lepton momenta
ITER=1: they depend on Q^2 as defined from the hadronic momenta (Q^2 is the integration variable)

<u>IVAR</u>
IVAR<3: use of a damping factor for $Q^2 < Q_0^2$: $\lim_{Q^2 \to 0} q(x, Q^2) = 0$; the value of Q_0^2 depends on the method
IVAR=−1: the chosen quark distributions are not modified
IVAR=0: Volkonsky/Prokhorov damping [28]
IVAR=1: Stein damping [29]
IVAR=2: Brasse/Stein parameterization [30, 29]

<u>IGRP</u>
IGRP: to be used for ISTR=2
IGRP: a PDFLIB parameter; it defines an author group in the library (see PDFLIB write-up [44])

ISET

ISET: to be used for ISTR=2

ISET: a PDFLIB parameter which defines the specific parameterization set within the group defined by IGRP; see [44])

ISCH

ISCH: to be used for ISTR=0,2

ISCH: select the QCD factorization scheme; the flag has to be chosen in accordance with the selected parameterization in PDFACT or PDFLIB

ISCH=0: leading order QCD

ISCH=1: DIS factorization scheme

ISCH=2: $\overline{\mathrm{MS}}$ factorization scheme

4.5 Bins and grid

IBIN

IBIN=0: no integration over bins

IBIN=1: user supplied integration over bins with subroutine usrint.f

IBCO

IBCO=1: bins are defined in the variables VAR1=x and VAR2=y

IBCO=2: bins are defined in the variables VAR1=x and VAR2=Q^2

IBCO=3: bins are defined in the variables VAR1=y and VAR2=Q^2

ITV1

ITV1=0: default bins in the first variable (VAR1); see (B.32) in appendix B

ITV1=1: logarithmical grid in the first variable

ITV1=2: linear grid in the first variable

ITV1=3: the bins in the first variable are defined by the user as a set of numbers in USRBIN; see VAR1 below

ITV1=4: user supplied bins in variable 1 via FFREAD

ITV2

ITV2=0: default bins in the second variable (VAR2)

ITV2=1: logarithmical grid in the second variable

ITV2=2: linear grid in the second variable

ITV2=3: bins in the second variable are defined by the user as a set of numbers in USRBIN; see VAR2 below

ITV2=4: user supplied bins in variable 2 via FFREAD

NVA1

NVA1: number of bins (≤ 100) in the first variable, to be used for ITV1=1,2

NVA2

NVA2: number of bins (≤ 100) in the second variable, to be used for ITV2=1,2

V1MN

V1MN: minimal value of the first variable (V1MN > 0.0), to be used for ITV1=1,2

V1MX

`V1MX:` maximal value of the first variable (`V1MX` < 1.0), to be used for `ITV1`=1,2

`V2MN`

`V2MN:` minimal value of the second variable (`V2MN` > 0.0), to be used for `ITV2`=1,2

`V2MX`

`V2MX:` maximal value of the second variable (`V2MX`: $y < 1.0$ or $Q^2 < s$), to be used for `ITV2`=1,2

`VAR1`

`VAR1:` user supplied bins in the first variable, a set ($N \leq 100$) of `REAL` numbers from the interval $0.0 < x < 1.0$ for `IBCO`=1,2, or $0.0 < y < 1.0$ for `IBCO`=3; to be used for `ITV1`=4

`VAR2`

`VAR2:` user supplied bins in the second variable, a set ($N \leq 100$) of `REAL` numbers from the interval $0.0 < y < 1.0$ for `IBCO`=1, or $Q^2_{min} \leq Q^2 < s$ for `IBCO`=2,3, (Q^2_{min} depends on the structure functions used, usually Q^2_{min}=4 or 5 GeV^2); to be used for `ITV2`=4

4.6 User initialization

`IUSR`

`IUSR`=0: default initialization of libraries, electroweak constants, etc.

`IUSR`=1: some additional initialization may be done using the external subroutine `usrini.f` where the user may initialize her/his set of wanted quantities

4.7 Choice of an output

`IOUT`

`IOUT`=0,1: output is written into file `HECTOR.OUT` and on the screen

`IOUT`=2: output is created by user subroutine `usrout.f`

`IOUT`=3: a simple output for creation of figures

After reading the data cards supplied by the user the program checks their compatibility. The following flags have highest priority: `IOPT`, `IMOD`, `ITV1`, and `ITV2`. If there are flags being in conflict with them they will be ignored. In other cases of incompatibility the program will stop and write a message into the file `HECTOR.OUT`.

4.8 User subroutines

The user may organize her/his own interface for using the code with help of several subroutines, which can be rewritten by the user. Simple examples can be found inside the correspondent files.

<u>`usrini.f`</u>: is called if `IUSR`=1.

```
CALL USRINI
```

In this subroutine the user may initialize parameters for later use. They can be transferred via `COMMON /HECUSR/`.

<u>`usrdpf.f`</u>: is called if `ISTR`=1.

```
CALL USRPDF(X,Q,UV,DV,US,DS,SS,CS,BS,TS,GLU)
```

In this subroutine the user may put a set of parton density functions.

`INPUT: X,Q`, with `Q`=$\sqrt{Q^2}$.

`OUTPUT: UV,DV,US,DS,SS,CS,BS,TS,GLU`

Note that all density functions should be multiplied by x, i.e. `UV` $= xu_v(x, Q^2)$. If the chosen density functions are calculated up to the next to leading order one has also to implement QCD radiative corrections, in dependence on the factorization scheme used.

<u>`usrstf.f`</u>: is called if `IMOD`=1.

```
CALL USRSTR(X,Q,FL,GL,HL,F2,G2,H2,XG3,XH3,WL,W2,XW3)
```

In this subroutine the user may put a set of target structure functions.

`INPUT: X,Q`, with `Q`=$\sqrt{Q^2}$.

`OUTPUT: FL,GL,HL,F2,G2,H2,XG3,XH3,WL,W2,XW3`

Note that in this case one may not necessarily refer to the parton picture but use a model independent parameterization of the structure functions instead.

<u>`usrbin.f`</u>: is called if `ITV1`=3 and/or `ITV2`=3.

```
CALL USRBIN
```

In this subroutine the user may define a set of points in two variables, where the cross sections and radiative correction are calculated. Note that the internal definitions of `usrbin.f` have higher priority than the definitions in `HECTOR.INP`. An example is given.

__`usrint.f`__: is called if `IBIN`=1.

```
CALL USRINT
```

In this subroutine the user may organize an integration over bins. A simple example is illustrated by the default routine.

__`usrbrn.f`__: is called if `IBRN`=0.

```
CALL USRBRN(X,Y,S,SIGO)
```

This subroutine can only be used in the `HELIOS` branch of the code (`IOPT`=1).

`INPUT: X,Y,S`

`OUTPUT: SIGO`, the Born cross section $d^2\sigma/dxdy$ in [nb].

The radiative corrections to the process defined are then calculated according to the flags defined in `HECTOR.INP`.

__`usrout.f`__: is called if `IOUT`=2.

```
CALL USROUT
```

In this subroutine the user may organize her/his own output. A simple example, which creates `OUTPUT VECTORS` to be used e.g. in graphics programs, is given.

__`usrgrf.f`__: is called if `IOUT`=3.

```
CALL USRGRF
```

This is another example of the user supplied `OUTPUT` routine creating `OUTPUT VECTORS` of a different format.

5 Examples of numerical output of HECTOR

In the initialization phase, the user has to decide whether user supplied input shall be used in addition. If not, IUSR=0 has to be chosen (see section 4.6). The setting of all the electroweak parameters is done in the file **hecset.f**. The unexperienced user can refer to the default flags. The beam parameters (particle type, charge, polarization, energy) are selected by the parameters given in section 4.1. Furthermore, one has to decide in which variable set the QED corrections shall be calculated (flag **IMEA**) and whether (possible) cut parameters shall be set. Then, the type of corrections to be applied has to be defined, including a choice of branch(es) (**HELIOS, TERAD, DISEP**, or a combination of them) to be used. This is done by the flags **IOPT**, **IORD** etc. being described in section 4.3. The structure function parameterizations are chosen by the flags of section 4.4. For **IBIN**=0, the output is calculated for the pre–defined grid of kinematical points and no integration over bins is performed. The flags and parameters of section 4.5 have to be set.

Two example files for **HECTOR.INP**, **HECTOR.INP.SRT** and **HECTOR.INP.STD** are available. The sample outputs (appendices C, D) correspond to these input files (renamed as **HECTOR.INP**). In addition, we show in tables 3–6 cross section values for varying input parameters and in figures 12–14 radiative corrections in percent for three selected types of variables. These examples are foreseen to act as benchmarks for typical variations in the use of **HECTOR**.

ISSE	ISCH	σ_B [nb]	$\delta_{\rm ini}^{(1)}$ [%]
1	0	0.2246E+06	13.01
2	0	0.2441E+06	12.94
1	1	0.2225E+06	12.65
3	1	0.2679E+06	11.76
1	2	0.2345E+06	12.82
2	2	0.2702E+06	12.13
3	2	0.2695E+06	11.84
4	2	0.2410E+06	13.56
5	2	0.2415E+06	13.55

Table 3: *Born cross sections for unpolarized e^+p scattering with NLO QCD corrections for different sets of parton parameterizations. Parameters: $x = 10^{-3}, y = 0.1$, $E_e = 26.7$ GeV, $E_p = 820.0$ GeV, and* **IVPL** *= 1 (i.e. running α). The low Q^2 damping corresponds to* **IVAR** *= 0.*

IVAR	σ_B [nb]	$\delta_{\rm ini}^{(1)}$ [%]
–1	0.4026E+08	9.452
0	0.3816E+08	9.222
1	0.3616E+08	9.197
2	0.7445E+07	32.53

Table 4: *Born cross sections for e^+p scattering with different parton distribution parameterizations at low Q^2. Parameters: $x = 10^{-4}, y = 0.1$, $E_e = 26.7$ GeV, $E_p = 820.0$ GeV,* **ISSE** *= 1,* **ISCH** *= 0,* **IVPL** *= 1.*

IOPT	IDSP	IWEA	σ_B [nb]	$\delta^{(1)}_{\rm all}$ [%]
3	0	0	0.1260E-02	-3.980
3	0	1	0.1278E-02	-5.130
2	2	0	0.1260E-02	-5.129
2	2	1	0.1278E-02	-6.268

Table 5: *Born cross sections for e^+p scattering for different values of flag* `IWEA`*. Parameters:* $x = 0.5, y = 0.5$, $E_e = 26.7$ GeV, $E_p = 820.0$ GeV, `ISSE` $= 1$, `ISCH` $= 0$, `IVAR` $= 0$, `IVPL` $= 1$.

IMEA	ICOR	IDSP	IOPT	σ_B [nb]	$\delta^{(1)}$ [%]	$\delta^{(2)}$ [%]	$\delta^{\rm all}$ [%]	Comments
1	1	0	1	0.2246E+06	13.01	−.4092	12.60	
2	1	0	1	0.2246E+06	3.928	.3942	4.322	
3	1	0	1	0.1066E+01	−.0837	.4420	.3583	
4	1	0	1	0.2246E+06	10.22	.6968	10.92	
5	1	0	1	0.2246E+06	5.512	.2490	5.761	$E_{cut} = 35$ GeV
6	1	0	1	0.2246E+06	6.182	.2334	6.416	$E_{cut} = 35$ GeV
7	1	0	1	0.2246E+06	4.469	.4151	4.884	
8	1	0	1	0.1066E+01	3.172	−.1664	3.006	
9	1	0	1	0.2246E+06	4.150	.5021	4.652	
10	1	0	1	0.2246E+06	9.328	2.492	11.82	
1	2	0	1	0.2246E+06	4.643			
4	2	0	1	0.2246E+06	−2.687			
9	2	0	1	0.2246E+06	−2.873			
10	2	0	1	0.2246E+06	−3.040			
1	3	0	1	0.2246E+06	.02006			`ICMP`=1
				0.2246E+06	.02753			`ICMP`=2
1	1	0	2,3	0.2246E+06	17.82		17.41	
2	1	0	2,3	0.2246E+06	3.748		4.133	
4	1	0	2,3	0.2246E+06	7.195		7.783	
7	1	0	2,3	0.2246E+06	4.337		4.736	
1	1	1	2,3	0.2246E+06	17.82		17.41	
1	1	2	2,3	0.2246E+06	17.95		17.41	
4	1	1	2,3	0.2246E+06	7.189		7.783	
4	1	2	2,3	0.2246E+06	7.189		7.783	
8	1	1	2,3	0.1066E+01	8.100		7.933	`IDIS`=2
				0.1066E+01	8.070		7.903	`IDIS`=1

Table 6: *Cross sections and radiative corrections for e^+p scattering as functions of flags* `IMEA`*,* `ICOR`*,* `IDSP`*,* `IOPT`*. Parameters:* $x = 10^{-3}, y = 0.1$, $E_e = 26.7$ GeV, $E_p = 820.0$ GeV, *and* `IVPL` $= 1$ *(i.e. running α). The parton density parameterization* `CTEQ3LO` *is chosen together with the low Q^2 damping factor corresponding to* `IVAR` $= 0$*. Longitudinal structure functions are disregarded. For the $\mathcal{O}(\alpha^2)$ corrections e^+e^- beam conversion terms were added. For* `IOPT` $= 2$ *(3), the values of $\delta^{(i)}$ are given in columns 6(8). Note the comments in [8] for the branches selected by* `IMEA` $= 5, 6$.

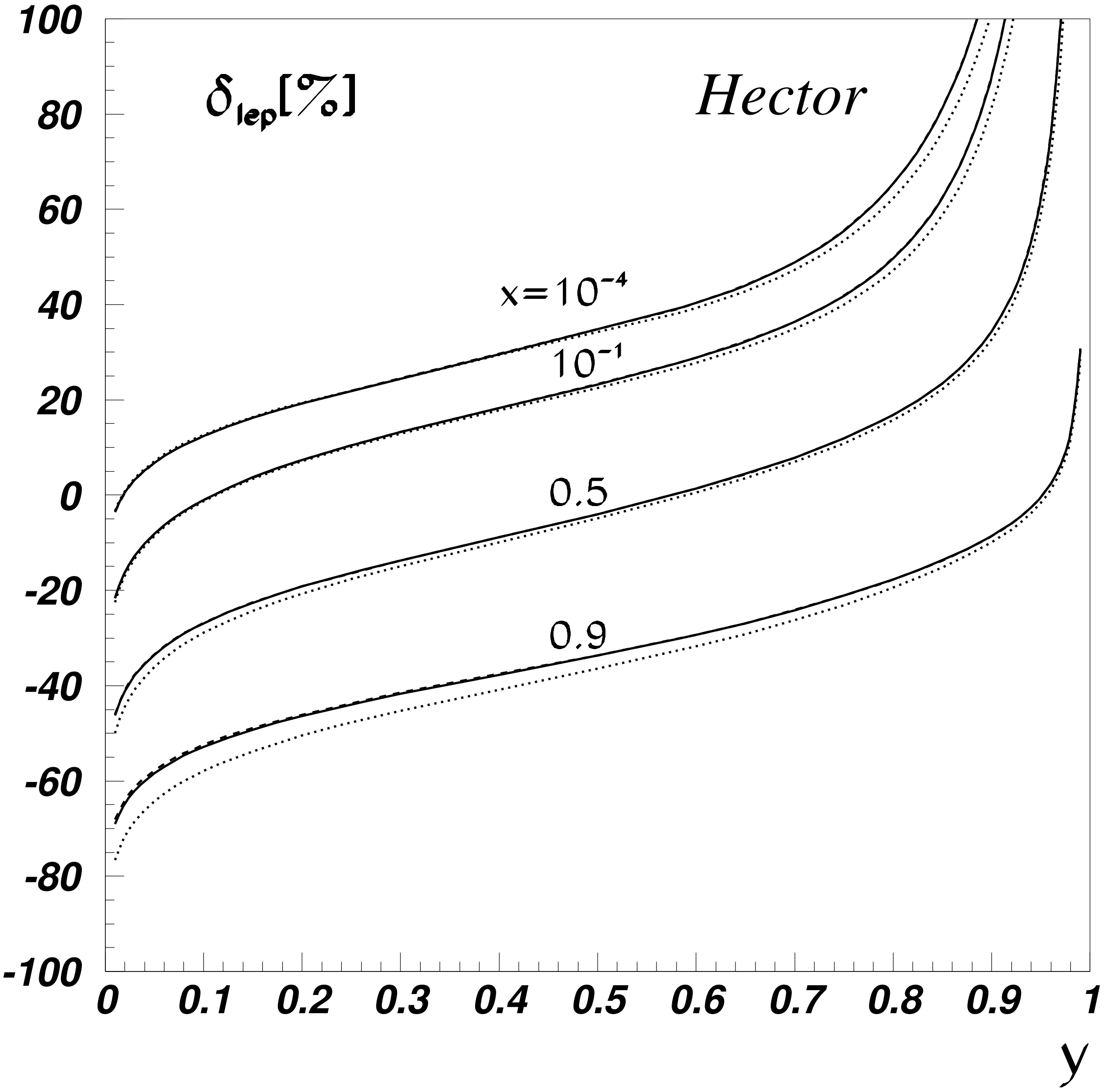

Figure 12: *Radiative corrections in leptonic variables for the default settings in percent. Dotted lines:* $\mathcal{O}(\alpha)$*, dashed lines:* $\mathcal{O}(\alpha^2)$*, solid lines: in addition soft photon exponentiation.*

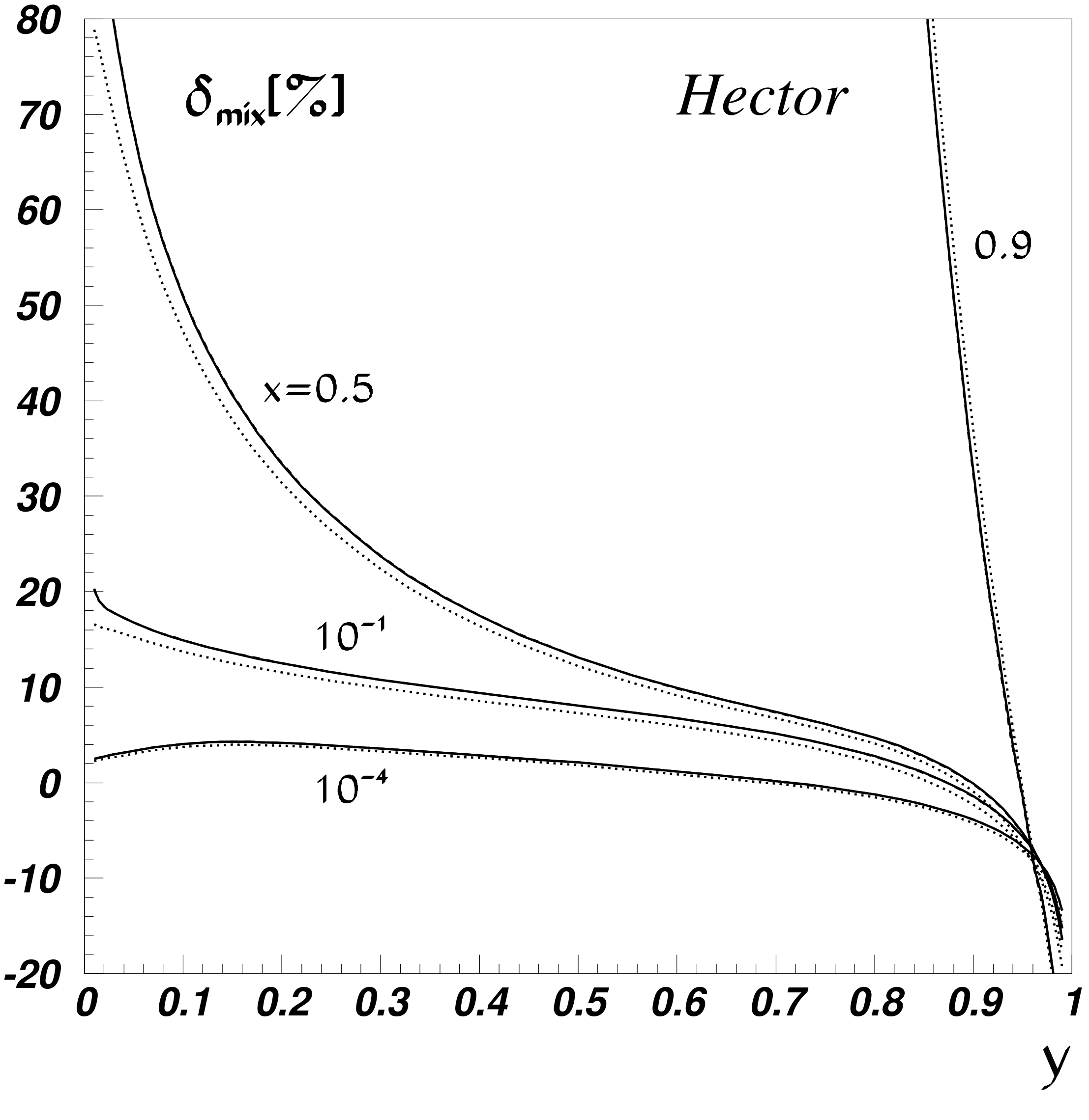

Figure 13: *Radiative corrections in mixed variables for the default settings in percent. Dotted lines: $\mathcal{O}(\alpha)$, dashed lines: $\mathcal{O}(\alpha^2)$, solid lines: in addition soft photon exponentiation.*

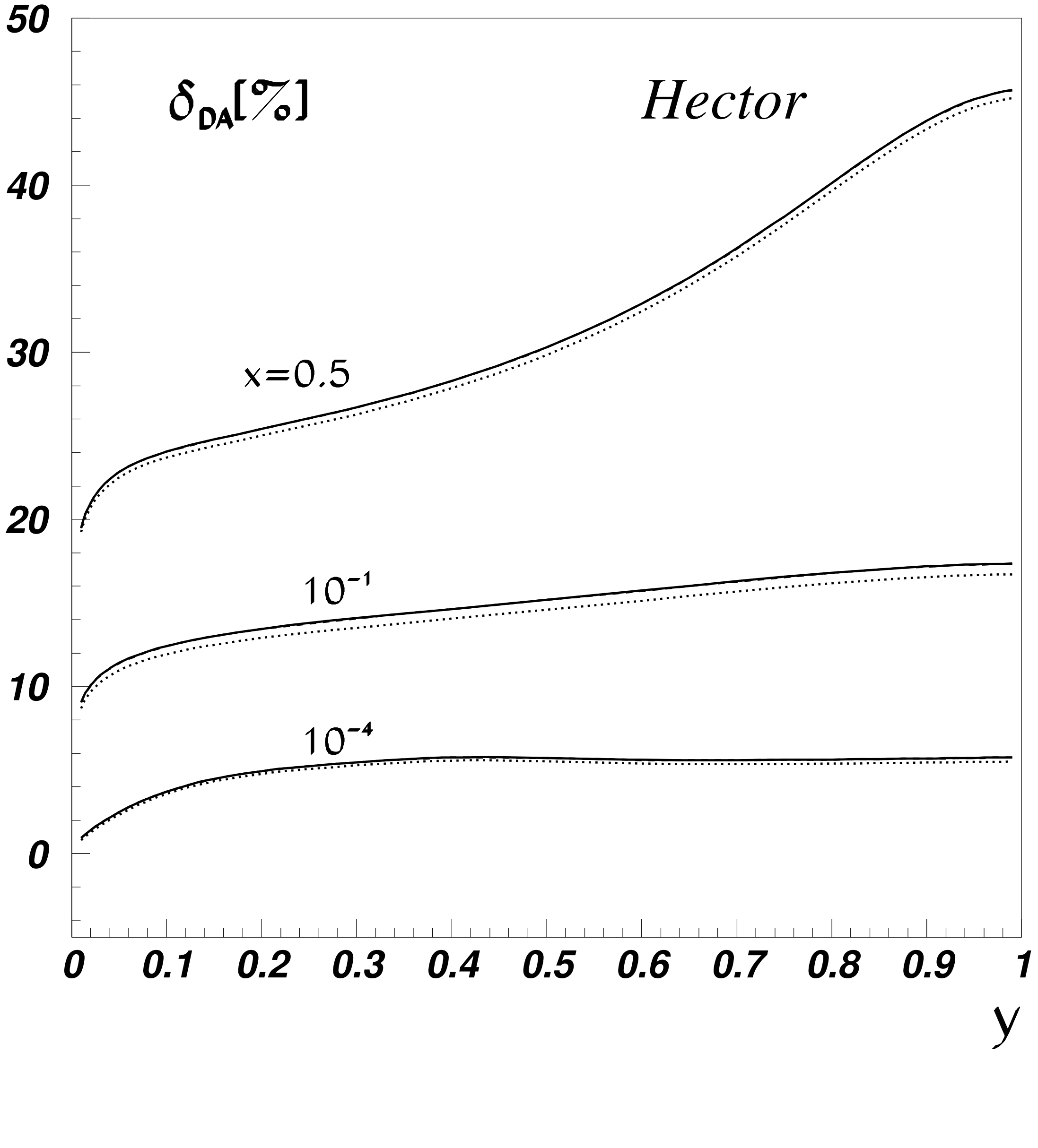

Figure 14: *Radiative corrections in double angle variables for the default settings in percent. Dotted lines: $\mathcal{O}(\alpha)$, dashed lines: $\mathcal{O}(\alpha^2)$, solid lines: in addition soft photon exponentiation.*

6 Output

The code writes a file HECTOR.OUT by default (IOUT=0). The user can organize her/his own output in the user file usrout.f, which is called for IOUT=2. Option IOUT=3 pipelines output to the user file usrgrf.f. Examples for these subroutines and some instructions on their use are given in the files usrout.f and usrgrf.f. Examples of the program output may be found in appendices C, D.

The program writes the following messages into the file HECTOR.OUT:

- program Logo labeled with the version number;
- the data cards which have been read in the FFREAD format;
- in case of an *incompatible* flag definition the program writes the values of the corresponding flags into the file HECTOR.OUT and terminates thereafter;
- version number of the library PDFLIB (if activated by the user);
- an information about the chosen program branch;
- type of process and variables selected;
- target type;
- information about the projectile particle: its charge, type and polarization;
- type the structure functions used;
- choice of the low Q^2 modification used;
- input parameters and calculated electroweak parameters;
- energy characteristics of the colliding particles;
- ZCUT cut parameter (for IOPT=1);
- choice of bin variables VAR1 and VAR2;
- type of grid in each variable;
- number of bins in each variable;
- label of the chosen set of parton densities (for ISTR=0).

The results of the calculation are presented in six columns: values of the first variable VAR1, values of the second variable VAR2, the s values in GeV^2, Born cross section values in nbarn, corrected cross section values in nbarn, correction values in %.

Acknowledgement We would like to thank Paul Söding for his constant support during the performance of the project. A.A., D.B., and L.K. would like to thank DESY–Zeuthen for the warm hospitality extended to them and for financial support. A.A. thanks the Heisenberg–Landau foundation for financial support. The project was founded in part by the EC-Network 'Capital Humain et Mobilite' under contract CHRX–CT92–0004. We would like to thank Dick Roberts for a version of the MRSA distribution in the DIS scheme.

References

[1] M. Grant and J. Hazel, *Who's Who in Classical Mythology?*, (Hodder and Stoughton, London, 1979).

[2] G. Schwab, Sagen des klassischen Altertums (Schmidt & Günther, Leipzig, 1944) p. 300.

[3] J. Blümlein, Fortran program `HELIOS 1.00`, in: Proceedings of the Workshop on Physics at HERA, W. Buchmüller and G. Ingelman (eds.), Hamburg (1991), Vol. **3**, p. 1272.

[4] A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, Fortran program `TERAD91`, in: Proceedings of the Workshop on Physics at HERA, W. Buchmüller and G. Ingelman (eds.), Hamburg (1991), Vol. **3**, p. 1285.

[5] J. Blümlein, *Z. Physik* **C47** (1990) 89.

[6] J. Blümlein, *Phys. Letters* **B 271** (1991) 267.

[7] J. Blümlein, G. Levman and H. Spiesberger, in: E. Berger (ed.), Proceedings of the Workshop on Research Directions of the Decade, Snowmass 1990 (World Scientific, Singapore, 1992), p. 554; *J. Phys.* **G19** (1993) 1695.

[8] J. Blümlein, *Z. Physik* **C65** (1995) 293.

[9] D. Bardin, C. Burdik, P. Christova and T. Riemann, contribution to the 1987 Workshop on Physics at HERA, JINR Dubna preprint E2-87-595 (1987); *Z. Physik* **C42** (1989) 679; Fortran program `DISEPNC`, which is a branch of the Fortran program `TERAD91` [4].

[10] D. Bardin, C. Burdik, P. Christova and T. Riemann, *Z. Physik* **C44** (1989) 149; Fortran program `DISEPCC`, which is a branch of the Fortran program `TERAD91` [4].

[11] D. Bardin, A. Akhundov, L. Kalinovskaya and T. Riemann, in: J. Blümlein and T. Riemann (eds.), Proceedings of the Zeuthen Workshop on Elementary Particle Theory – Deep Inelastic Scattering, Teupitz, April 1992, *Nucl. Phys.* **B** (Proc. Suppl.) **29A** (1992) 209;
A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, preprint DESY 94–115 (1994), [hep-ph/9407266]; see also references quoted therein.

[12] A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, *Phys. Letters* **301B** (1993) 447.

[13] D. Bardin, P. Christova, L. Kalinovskaya and T. Riemann, *Phys. Letters* **B357** (1995) 456.

[14] A. Akhundov, D. Bardin, M. Bilenky, P. Christova, S. Riemann, T. Riemann, M. Sachwitz and H. Vogt, Fortran program `DIZET 4.04` (Aug 21, 1991), which is a branch of the Fortran program `TERAD91` [4]; version 2 was described in: D. Bardin, M. Bilenky, T. Riemann, M. Sachwitz and H. Vogt, *Comput. Phys. Commun.* **59** (1990) 303; see also [16] for an update of higher order virtual corrections.

[15] C. Callan and D. Gross, *Phys. Rev. Letters* **22** (1969) 156.

[16] D. Bardin, G. Passarino and W. Hollik (eds.), Reports of the working group on precision calculations for the Z resonance, CERN 95–03 (March 1995).

[17] W. Hollik, *Fortschritte d. Physik* **38** (1990) 165.

[18] F. Jegerlehner, in: P. Langacker and M. Cvetic (eds.), Proc. of the 1990 TASI in Elem. Particle Physics, (World Scientific, Singapore, 1991), p. 476.

[19] F. Jegerlehner, Fortran subroutine **hadr5**; see also: S. Eidelman and F. Jegerlehner, *Z. Physik* **C67** (1995) 585.

[20] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, *Z. Physik.* **C42** (1989) 697.

[21] A. Djouadi and C. Verzegnassi, *Phys. Letters* **B195** (1987) 265;
A. Djouadi, *Nuovo Cim.* **100A** (1988) 357.

[22] D. Bardin and A. Chizhov, in: JINR Dubna E2–89–525 (1989) p. 42.

[23] A. Akhundov, D. Bardin and T. Riemann, *Nucl. Phys.* **B276** (1986) 1.

[24] D. Bardin et al., preprint CERN-TH.6443/92 [hep-ph/9412201].

[25] A.D. Martin, W.J. Stirling and R.G. Roberts, *Phys. Rev.* **D50** (1994) 6734; *Phys. Letters* **B354** (1995) 155.

[26] M. Glück, E. Reya and A. Vogt, *Z. Physik* **C67** (1995) 433.

[27] M. Glück, E. Reya and A. Vogt, *Z. Physik* **C53** (1992) 127.

[28] N. Volkonsky and L. Prokhorov, *Sov. J. Exp. Theor. Phys. Letters* **21** (1975) 389.

[29] S. Stein et al., *Phys. Rev.* **D12** (1975) 1884.

[30] F. W. Brasse et al., *Nucl. Phys.* **B110** (1976) 413.

[31] H.L. Lai, J. Botts, J. Huston, J.G. Morfin, J.F. Owen, J.W. Qiu, W.K. Tung and H. Weerts, *Phys. Rev.* **D51** (1995) 4763.

[32] J. Blümlein and M. Klein, *Nucl. Instr. Meth.* **A329** (1993) 112.

[33] F. Jaquet and A. Blondel, in: U. Amaldi (ed.), Proceedings of the study of an *ep* facility for Europe, DESY 79/48 (1979), p. 391.

[34] J. Blümlein and M. Klein, in: E. Berger (ed.), Proceedings of the Workshop on Research Directions of the Decade, Snowmass 1990 (World Scientific, Singapore, 1992), p. 549.

[35] U. Bassler and G. Bernardi, preprint DESY 94–231 (1994), [hep-ex/9412004].

[36] S. Bentvelsen, J. Engelen and P. Kooijman, in: W. Buchmüller and G. Ingelman (eds.), Proceedings of the Workshop on Physics at HERA, Oct. 29–30, 1991, Hamburg (DESY, Hamburg, 1992), Vol. 1, p. 23.

[37] A. De Rujula, R. Petronzio and A. Savoy–Navarro, *Nucl. Phys.* **B154** (1979) 394.

[38] J. Kripfganz and H. Perlt, *Z. Physik* **C41** (1988) 319.

[39] H. Spiesberger, Univ.Bielefeld preprint BI-TP.94/60 (1994) [hep-ph/9412286].

[40] T. Kinoshita, *J. Math. Phys.* **B271** (1962) 267;
T.D. Lee and M. Nauenberg, *Phys. Rev.* **133** (1964) B1549.

[41] V. Gribov and L. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 451, 675.

[42] J. Blümlein, A. Leike and T. Riemann, in: G. Jarlskog and D. Rein (eds.), Proceedings of the Aachen ECFA Workshop on Large Hadron Colliders, CERN 90–10, Vol.II, p. 1010.

[43] J. Blümlein, G.J. van Oldenborgh and R. Rückl, *Nucl. Phys.* **B395** (1993) 35.

[44] H. Plothow-Besch, PDFLIB: Nucleon, Pion and Photon Parton Density Functions and α_s Calculations, version 4.13 (1993), CERN Program Library entry W5051, Item 1-3-34.

A Definition of scaling variables in presence of hard photon emission

In this appendix, we collect the definitions of all the sets of kinematical variables, which may be used in HECTOR. Some motivations for the specific choices may be found in [32]-[35].

Leptonic variables:

The scaling variables are defined from leptonic variables:

$$
\begin{aligned}
y_l &= \frac{p_1(k_1-k_2)}{p_1 k_1}, && \text{(A.1)}\\
Q_l^2 &= (k_1-k_2)^2 = -2k_1k_2, && \text{(A.2)}\\
x_l &= \frac{Q_l^2}{S y_l}. && \text{(A.3)}
\end{aligned}
$$

Hadronic variables:

The scaling variables are defined from hadronic variables using the 4–vector of the current jet:

$$
\begin{aligned}
y_h &= \frac{p_1(p_2-p_1)}{p_1 k_1}, && \text{(A.4)}\\
Q_h^2 &= (p_2-p_1)^2, && \text{(A.5)}\\
x_h &= \frac{Q_h^2}{S y_h}. && \text{(A.6)}
\end{aligned}
$$

Jaquet-Blondel variables [33]:

Again, the scaling variables are defined from hadronic variables. Using the transverse hadronic momentum in the laboratory system, $\vec{p}_2^{\perp}$, instead of p_2 and the energy flow in z direction in the definition of the scaling variables[5] :

$$
\begin{aligned}
y_{\rm JB} &= \frac{\Sigma}{2E_e}, && \text{(A.7)}\\
Q_{\rm JB}^2 &= \frac{(\vec{p}_{2\perp})^2}{1-y_{\rm JB}}, && \text{(A.8)}\\
x_{\rm JB} &= \frac{Q_{\rm JB}^2}{S y_{\rm JB}}. && \text{(A.9)}
\end{aligned}
$$

with

$$
\Sigma = \sum_h (E_h - p_{h,z}), \tag{A.10}
$$

[5]Note that in the complete $\mathcal{O}(\alpha)$ calculations for Jaquet-Blondel and mixed variables in refs. [13] the *approximate* relation $y_{\rm JB} \approx y_h$ was used which holds iff all masses in the final state can be neglected with respect to the particle momenta.

and $E_h, p_{h,x}, p_{h,y}, p_{h,z}$ being the four-momentum components of the hadronic final state.

Mixed variables [34]:

The scaling variables are defined from both leptonic and Jaquet-Blondel variables:

$$
\begin{aligned}
Q^2_m &= Q^2_l, && \text{(A.11)}\\
y_m &= y_{\text{JB}}, && \text{(A.12)}\\
x_m &= \frac{Q^2_l}{S y_{\text{JB}}}. && \text{(A.13)}
\end{aligned}
$$

For mixed variables, the allowed values of x are not necessarily restricted to be smaller than one; see for details appendix B.3.1 of [11].

Σ method [35]

The scaling variables are defined from both leptonic and hadronic variables as being measured in the laboratory system:

$$
\begin{aligned}
y_{\Sigma} &= \frac{\Sigma}{\Sigma + E'_l(1-\cos\theta_l)}, && \text{(A.14)}\\
Q^2_{\Sigma} &= \frac{(\vec{k}_{2\perp})^2}{1-y_{\Sigma}}, && \text{(A.15)}\\
x_{\Sigma} &= \frac{Q^2_{\Sigma}}{S y_{\Sigma}}, && \text{(A.16)}
\end{aligned}
$$

The integration boundary $z_0^{\Sigma,f}$ in the case of final state radiation is the solution of the equation

$$x\left[1-y(1-z_0)\right]^2 = z_0^3, \qquad \text{(A.17)}$$

$$z_0^{\Sigma,f} = E_1^{1/3} + E_2 E_1^{-1/3} + \frac{1}{3}xy^2, \qquad \text{(A.18)}$$

with

$$
\begin{aligned}
E_1 &= x(E_1^a + E_1^b), && \text{(A.19)}\\
E_1^a &= \frac{1}{2} - y\left(1-\frac{1}{2}y-\frac{1}{27}x^2y^5\right) + \frac{1}{3}xy^3(1-y), && \text{(A.20)}\\
E_1^b &= \frac{1}{6}\sqrt{9(1-4y+y^4)+2y^2\left[27-18y-\frac{2}{3}xy^4+2xy\left(\frac{1}{3}-y+y^2\right)\right]}, && \text{(A.21)}\\
E_2 &= \frac{2}{3}xy\left(1-y+\frac{1}{6}xy^3\right). && \text{(A.22)}
\end{aligned}
$$

$e\Sigma$ method [35]:

The $e\Sigma$ method mixes variables from the set of leptonic variables and of the Σ method:

$$\begin{aligned} Q^2_{e\Sigma} &= Q^2_l, &\text{(A.23)}\\ x_{e\Sigma} &= x_{\Sigma}, &\text{(A.24)}\\ y_{e\Sigma} &= \frac{Q^2_{e\Sigma}}{Sx_{e\Sigma}} = \frac{Q^2_l}{Sx_{\Sigma}}. &\text{(A.25)}\end{aligned}$$

The integration boundary $z_0^{\Sigma,f}$ is the same as in the Σ method.

Double angle method [36]:

The scaling variables are defined from both leptonic and hadronic scattering angles as being measured in the laboratory system.

The definitions of scaling variables in the double angle method are:

$$\begin{aligned} Q^2_{\mathrm{DA}} &= \frac{4E_l^2\cos^2\frac{\theta_l}{2}}{\sin^2\frac{\theta_l}{2} + \sin\frac{\theta_l}{2}\cos\frac{\theta_l}{2}\tan\frac{\theta_h}{2}}, &\text{(A.26)}\\ y_{\mathrm{DA}} &= 1 - \frac{\sin\frac{\theta_l}{2}}{\sin\frac{\theta_l}{2} + \cos\frac{\theta_l}{2}\tan\frac{\theta_h}{2}}, &\text{(A.27)}\end{aligned}$$

$$x_{\mathrm{DA}} = \frac{Q^2_{\mathrm{DA}}}{Sy_{\mathrm{DA}}}. \qquad \text{(A.28)}$$

θy method [8]:

A quite similar opportunity is to express the cross sections in terms of θ_e in the laboratory system and y_{JB}. The kinematical variables are:

$$\begin{aligned} Q^2_{\theta\mathrm{y}} &= 4E_e^2(1-y_{\mathrm{JB}})\frac{1+\cos\theta_l}{1-\cos\theta_l}, &\text{(A.29)}\\ y_{\theta\mathrm{y}} &= y_{\mathrm{JB}}, &\text{(A.30)}\\ x_{\theta\mathrm{y}} &= \frac{Q^2_{\theta\mathrm{y}}}{Sy_{\theta\mathrm{y}}}. &\text{(A.31)}\end{aligned}$$

B Table of user options

	FFREAD flag	type	parameter	common	default	references
1	EELE	R	EEL	HECPAR	30.0	4.1
2	ETAR	R	EPR	HECPAR	820.0	4.1
3	GCUT	R	ECUT	HECGMC	0.0	4.2
4	HM2C	R	AMF2CT	HECHDC	0.0	4.2
5	IBCO	I	IBCO	HECGRD	1	4.5
6	IBEA	I	IBEAM	HECOPT	−1	4.1
7	IBIN	I	IBIN	HECBIN	0	4.5
8	IBOS	I	ITERM	HECCOR	4	4.3
9	IBRN	I	IUBRN	HECBRN	0	4.3
10	ICMP	I	ICMPT	HECCOR	1	4.3
11	ICOR	I	ICOR	HECCOR	5	4.3
12	IDIS	I	IDIS	HECIDS	1	4.3
13	IDQM	I	IDQM	HECIDS	1	4.3
14	IDSP	I	IDSP	HECCOR	0	4.3
15	IEPC	I	IEPC	HECCOR	0	4.3
16	IEXP	I	IEXP	HECDPI	0	4.3
17	IGCU	I	IGCUT	HECGMC	0	4.2
18	IGRP	I	NGROUP	HECPDF	3	4.4
19	IHCU	I	IHCUT	HECIHC	0	4.2
20	ILEP	I	ILEPTO	HECOPT	1	4.1
21	ILOW	I	ILOW	HECLOW	0	4.3
22	IMEA	I	IMEAS	HECOPT	1	4.2
23	IMOD	I	MODEL	HECFLG	0	4.4
24	IOPT	I	IOPT	HECOPT	1	4.3
25	IORD	I	IORD	HECCOR	1	4.3
26	IOUT	I	IOUT	HECOUP	0	4.7
27	ISCH	I	ISCHM	HECQCD	2	4.4
28	ISET	I	NSET	HECPDF	41	4.4
29	ISSE	I	ISSET	HECSTF	1	4.4
30	ISTR	I	ISTR	HECSTF	0	4.4
31	ITAR	I	ITARG	HECOPT	1	4.1
32	ITER	I	ITERAD	HECCOR	1	4.4
33	ITV1	I	ITV1	HECGRD	0	4.5

Table 7: *User defined flags in the file* HECTOR.INP

	FFREAD flag	type	parameter	common	default	references
34	ITV2	I	ITV2	HECGRD	2	4.5
35	IUSR	I	IUSR	HECUSR	0	4.6
36	IVAR	I	IVAR	HECFLG	0	4.4
37	IVPL	I	IVPOL	HECVPL	1	4.3
38	IWEA	I	IWEAK	HECFLG	0	4.3
39	NVA1	I	NV1	HECBIN	6	4.5
40	NVA2	I	NV2	HECBIN	20	4.5
41	POLB	R	POL	HECPAR	0.0	4.1
42	THC1	R	THCUT1	HECGMC	0.0	4.2
43	THC2	R	THCUT2	HECGMC	0.0	4.2
44	VAR1	R	V1(100)	HECBIN	—	4.5
45	VAR2	R	V2(100)	HECBIN	—	4.5
46	V1MN	R	V1MI	HECGRD	1.0E–04	4.5
47	V1MX	R	V1MA	HECGRD	1.0	4.5
48	V2MN	R	V2MI	HECGRD	1.0E–02	4.5
49	V2MX	R	V2MA	HECGRD	1.0	4.5
50	Q2CU	R	TCUT	HECHDC	0.0	4.2
51	ZCUT	R	EECU	HECPAR	35.0	4.2

Table 7 (continued): *The flags to be set by the user in the file* `HECTOR.INP`.

The default bins in x are:

$$x = 10^{-4},\ 10^{-3},\ 10^{-2},\ 10^{-1},\ 0.5,\ 0.9\ . \qquad \text{(B.32)}$$

The default values of y are defined in table 7. They are located at 20 equidistant points in the interval from 0.01 to 1.

C Example file HECTOR.OUT

The following sample file HECTOR.OUT corresponds to the first row of Table 3.

```
          ------- HECTOR v1.00 -------

1           USER'S DIRECTIVES TO RUN THIS JOB
            ---------------------------------


 ***** DATA CARD CONTENT     LIST
 ***** DATA CARD CONTENT     C.... SETTING OF FLAGS AND PARAMETERS
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... FLAGS
 ***** DATA CARD CONTENT     IOPT    1
 ***** DATA CARD CONTENT     IMEA    1
 ***** DATA CARD CONTENT     IBEA    1
 ***** DATA CARD CONTENT     ILEP    1
 ***** DATA CARD CONTENT     ITAR    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IORD    1
 ***** DATA CARD CONTENT     IBOS    4
 ***** DATA CARD CONTENT     ICOR    1
 ***** DATA CARD CONTENT     ICMP    1
 ***** DATA CARD CONTENT     IEPC    0
 ***** DATA CARD CONTENT     IBRN    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IDSP    0
 ***** DATA CARD CONTENT     IDIS    1
 ***** DATA CARD CONTENT     IDQM    1
 ***** DATA CARD CONTENT     ILOW    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IMOD    0
 ***** DATA CARD CONTENT     ISTR    0
 ***** DATA CARD CONTENT     ISSE    1
 ***** DATA CARD CONTENT     ITER    1
 ***** DATA CARD CONTENT     IVAR    0
 ***** DATA CARD CONTENT     IGRP    3
 ***** DATA CARD CONTENT     ISET    17
 ***** DATA CARD CONTENT     ISCH    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IWEA    0
 ***** DATA CARD CONTENT     IVPL    1
 ***** DATA CARD CONTENT     IEXP    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IHCU    0
```

```
 ***** DATA CARD CONTENT     Q2CU    0.0
 ***** DATA CARD CONTENT     HM2C    0.0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IGCU    0
 ***** DATA CARD CONTENT     GCUT    0.0
 ***** DATA CARD CONTENT     THC1    0.0
 ***** DATA CARD CONTENT     THC2    0.0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... PARAMETERS
 ***** DATA CARD CONTENT     POLB    0.0
 ***** DATA CARD CONTENT     EELE    26.70
 ***** DATA CARD CONTENT     ETAR    820.0
 ***** DATA CARD CONTENT     ZCUT     0.00
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... LATTICE
 ***** DATA CARD CONTENT     IBIN    0
 ***** DATA CARD CONTENT     IBCO    1
 ***** DATA CARD CONTENT     ITV1    4
 ***** DATA CARD CONTENT     ITV2    4
 ***** DATA CARD CONTENT     NVA1    1
 ***** DATA CARD CONTENT     NVA2    1
 ***** DATA CARD CONTENT     V1MN    1.E-4
 ***** DATA CARD CONTENT     V1MX    1.0
 ***** DATA CARD CONTENT     V2MN    0.01
 ***** DATA CARD CONTENT     V2MX    0.5
 ***** DATA CARD CONTENT     VAR1    0.001
 ***** DATA CARD CONTENT     VAR2    0.1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IOUT    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IUSR    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     STOP
BINS:
VAR1(   1)=   .1000E-02
VAR2(   1)=   .1000E+00
 ***** LLA & NTLA BRANCH
 ***** O(ALPHA*L) CORRECTIONS IN LLA
 ***** LEPTON MEASUREMENT, NEUTRAL CURRENT
 ***** CHARGED ANTI-LEPTON SCATTERING
 ***** PROTON TARGET
 ***** LEPTON BEAM POLARIZATION= .0
 ***** L-BEAM: E
 ***** PARTON DESCRIPTION FOR STRUCTURE FUNCTIONS IS USED
 ***** CTEQ3 PDF PARAMETERIZATION, LO
 ***** LOW Q2 MODIFICATION: Q2 < Q2MIN: VOLKONSKY/PROKHOROV DAMPING
 ***** Q2MIN=   4.000  GeV**2
```

```
***** INPUT ELECTOROWEAK PARAMETERS USDED:
***** Z-BOSON     MASS AMZ (GEV) =    91.175
***** W-BOSON     MASS AMW (GEV) =    80.399
***** HIGSS BOSON MASS AMH (GEV) =   300.000
***** TOP QUARK   MASS AMT (GEV) =   180.000
***** CALCULATED EW PARAMETERS:
***** SIN(THETA_W)   =    .232000
***** DELTA_R        =    .036948
***** BEAM KINEMATICS *****
***** LEPTON BEAM ENERGY (GEV) = 26.70000076293945
***** PROTON BEAM ENERGY (GEV) = 820.0
***** BINNING IN X & Y
***** USER SUPPLIED BINS IN VARIABLE 1
***** USER SUPPLIED BINS IN VARIABLE 2
***** NUMBER OF BINS IN V1: NVA1= 1
***** NUMBER OF BINS IN V2: NVA2= 1
CTEQ3(LO )
    VAR1       VAR2        S         BORN     SIG_CORR     DELTA
  .100E-02   .100E+00   .876E+05   .225E+06   .254E+06   .130E+02
```

D Test run output

```
       ------- HECTOR v1.00 -------

1            USER'S DIRECTIVES TO RUN THIS JOB
             ----------------------------------


 ***** DATA CARD CONTENT     LIST
 ***** DATA CARD CONTENT     C.... SETTING OF FLAGS AND PARAMETERS
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... FLAGS
 ***** DATA CARD CONTENT     IOPT    3
 ***** DATA CARD CONTENT     IMEA    1
 ***** DATA CARD CONTENT     IBEA    1
 ***** DATA CARD CONTENT     ILEP    1
 ***** DATA CARD CONTENT     ITAR    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IORD    3
 ***** DATA CARD CONTENT     IBOS    4
 ***** DATA CARD CONTENT     ICOR    5
 ***** DATA CARD CONTENT     ICMP    1
 ***** DATA CARD CONTENT     IEPC    0
 ***** DATA CARD CONTENT     IBRN    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IDSP    0
 ***** DATA CARD CONTENT     IDIS    1
 ***** DATA CARD CONTENT     IDQM    1
 ***** DATA CARD CONTENT     ILOW    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IMOD    0
 ***** DATA CARD CONTENT     ISTR    0
 ***** DATA CARD CONTENT     ISSE    1
 ***** DATA CARD CONTENT     ITER    1
 ***** DATA CARD CONTENT     IVAR    0
 ***** DATA CARD CONTENT     IGRP    3
 ***** DATA CARD CONTENT     ISET    17
 ***** DATA CARD CONTENT     ISCH    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IWEA    0
 ***** DATA CARD CONTENT     IVPL    1
 ***** DATA CARD CONTENT     IEXP    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IHCU    0
 ***** DATA CARD CONTENT     Q2CU    0.0
 ***** DATA CARD CONTENT     HM2C    0.0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IGCU    0
```

```
 ***** DATA CARD CONTENT     GCUT    0.0
 ***** DATA CARD CONTENT     THC1    0.0
 ***** DATA CARD CONTENT     THC2    0.0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... PARAMETERS
 ***** DATA CARD CONTENT     POLB    0.0
 ***** DATA CARD CONTENT     EELE    26.70
 ***** DATA CARD CONTENT     ETAR    820.0
 ***** DATA CARD CONTENT     ZCUT     0.00
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     C.... LATTICE
 ***** DATA CARD CONTENT     IBIN    0
 ***** DATA CARD CONTENT     IBCO    1
 ***** DATA CARD CONTENT     ITV1    4
 ***** DATA CARD CONTENT     ITV2    4
 ***** DATA CARD CONTENT     NVA1    1
 ***** DATA CARD CONTENT     NVA2    1
 ***** DATA CARD CONTENT     V1MN    1.E-4
 ***** DATA CARD CONTENT     V1MX    1.0
 ***** DATA CARD CONTENT     V2MN    0.01
 ***** DATA CARD CONTENT     V2MX    0.5
 ***** DATA CARD CONTENT     VAR1    0.0001  0.01  0.1  0.5  0.90
 ***** DATA CARD CONTENT     VAR2    0.01    0.1   0.5  0.9  0.99
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IOUT    1
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     IUSR    0
 ***** DATA CARD CONTENT
 ***** DATA CARD CONTENT     STOP
BINS:
VAR1(   1)=  0.1000E-03
VAR1(   2)=  0.1000E-01
VAR1(   3)=  0.1000E+00
VAR1(   4)=  0.5000E+00
VAR1(   5)=  0.9000E+00
VAR2(   1)=  0.1000E-01
VAR2(   2)=  0.1000E+00
VAR2(   3)=  0.5000E+00
VAR2(   4)=  0.9000E+00
VAR2(   5)=  0.9900E+00
 ***** O(ALPHA) COMPLETE CORRECTIONS SUPPLEMENTED
       BY LLA TERMS IN HIGHER ORDERS
 ***** LEPTON MEASUREMENT, NEUTRAL CURRENT
 ***** CHARGED ANTI-LEPTON SCATTERING
 ***** PROTON TARGET
 ***** LEPTON BEAM POLARIZATION=  0.0000000000000000E+00
 ***** L-BEAM: E
```

```
***** PARTON DESCRIPTION FOR STRUCTURE FUNCTIONS IS USED
***** CTEQ3 PDF PARAMETERIZATION, LO
***** LOW Q2 MODIFICATION: Q2 < Q2MIN: VOLKONSKY/PROKHOROV DAMPING
***** Q2MIN=   4.000  GeV**2
***** INPUT ELECTOROWEAK PARAMETERS USDED:
***** Z-BOSON     MASS AMZ (GEV) =    91.175
***** W-BOSON     MASS AMW (GEV) =    80.399
***** HIGSS BOSON MASS AMH (GEV) =   300.000
***** TOP QUARK   MASS AMT (GEV) =   180.000
***** CALCULATED EW PARAMETERS:
***** SIN(THETA_W)   =   0.232000
***** DELTA_R        =   0.036948
***** BEAM KINEMATICS *****
***** LEPTON BEAM ENERGY (GEV) =   26.70000076293945
***** PROTON BEAM ENERGY (GEV) =   820.0000000000000
***** BINNING IN X & Y
***** USER SUPPLIED BINS IN VARIABLE 1
***** USER SUPPLIED BINS IN VARIABLE 2
***** NUMBER OF BINS IN V1: NVA1=          5
***** NUMBER OF BINS IN V2: NVA2=          5
CTEQ3(LO )
```

VAR1	VAR2	S	BORN	SIG_CORR	DELTA
0.100E-03	0.100E-01	0.876E+05	0.113E+10	0.109E+10	-0.353E+01
0.100E-03	0.100E+00	0.876E+05	0.382E+08	0.429E+08	0.124E+02
0.100E-03	0.500E+00	0.876E+05	0.113E+07	0.153E+07	0.348E+02
0.100E-03	0.900E+00	0.876E+05	0.324E+06	0.684E+06	0.111E+03
0.100E-03	0.990E+00	0.876E+05	0.271E+06	0.176E+07	0.549E+03
0.100E-01	0.100E-01	0.876E+05	0.137E+06	0.134E+06	-0.233E+01
0.100E-01	0.100E+00	0.876E+05	0.175E+04	0.200E+04	0.139E+02
0.100E-01	0.500E+00	0.876E+05	0.573E+02	0.785E+02	0.369E+02
0.100E-01	0.900E+00	0.876E+05	0.150E+02	0.343E+02	0.129E+03
0.100E-01	0.990E+00	0.876E+05	0.123E+02	0.229E+03	0.175E+04
0.100E+00	0.100E-01	0.876E+05	0.120E+04	0.941E+03	-0.216E+02
0.100E+00	0.100E+00	0.876E+05	0.110E+02	0.109E+02	-0.971E+00
0.100E+00	0.500E+00	0.876E+05	0.283E+00	0.348E+00	0.232E+02
0.100E+00	0.900E+00	0.876E+05	0.611E-01	0.115E+00	0.878E+02
0.100E+00	0.990E+00	0.876E+05	0.494E-01	0.594E+00	0.110E+04
0.500E+00	0.100E-01	0.876E+05	0.956E+01	0.514E+01	-0.463E+02
0.500E+00	0.100E+00	0.876E+05	0.717E-01	0.524E-01	-0.269E+02
0.500E+00	0.500E+00	0.876E+05	0.126E-02	0.121E-02	-0.398E+01
0.500E+00	0.900E+00	0.876E+05	0.149E-03	0.200E-03	0.344E+02
0.500E+00	0.990E+00	0.876E+05	0.112E-03	0.471E-03	0.319E+03
0.900E+00	0.100E-01	0.876E+05	0.661E-02	0.204E-02	-0.691E+02
0.900E+00	0.100E+00	0.876E+05	0.387E-04	0.183E-04	-0.528E+02
0.900E+00	0.500E+00	0.876E+05	0.546E-06	0.362E-06	-0.337E+02
0.900E+00	0.900E+00	0.876E+05	0.540E-07	0.493E-07	-0.857E+01
0.900E+00	0.990E+00	0.876E+05	0.400E-07	0.523E-07	0.307E+02