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Future measurements of α_s and xg from scaling violations at HERA

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Abstract

Results are presented of a study of the experimental and theoretical accuracy one may achieve at HERA in measuring the strong coupling constant α_s and the gluon distribution from scaling violations of F_2 structure functions.

Introduction

Accurate measurements of F_2 structure functions in deep inelastic scattering provide one of the cleanest tests of perturbative QCD. In the first few years of experimentation at HERA the available kinematic range was extended to low values of $x \simeq 10^{-4}$ and large $Q^2 \simeq 5000$ GeV^2. In the next 8 years of operation HERA might deliver integrated luminosities of 0.5–1 fb^{-1}. The extended kinematic coverage and increased luminosity allows for detailed measurements of the scaling violations in F_2 and hence of the strong coupling constant α_s and the gluon distribution $xg(x)$ at low x.

In this report I will summarise the results of studies done during the workshop 'Future Physics at Hera' [1, 2, 3] to estimate the experimental and theoretical errors on $\alpha_s(M_Z^2)$ obtained from a QCD analysis of future F_2 structure function data. Also shown is the precision one may reach in the determination of the gluon distribution.

Comparison of NLO evolution codes

In this section I present results of a study on how well the various implementations of perturbative QCD are numerically and conceptually under control [1].

The evolution equations for the parton distributions $f(x, Q^2)$ in the proton are given by

$$\frac{\partial f(x,Q^2)}{\partial \ln Q^2} = \left[a_s(Q^2)P_0(x) + a_s^2(Q^2)P_1(x) + O(a_s^3)\right] \otimes f(x,Q^2) \tag{1}$$

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where we write $a_s(Q^2) \equiv \alpha_s(Q^2)/4\pi$ and where P_0 and P_1 are the leading order (LO) and NLO splitting functions respectively.

Two methods are widely used to solve Eq. (1). In the first approach ('x–space') the parton distributions are numerically evolved on a grid in x and Q^2. This method is conceptually simple but the numerical accuracy depends on the number of gridpoints in x and Q^2 which is limited by the amount of CPU time one can afford to spend in the computations.

In the second approach ('N–space') the Mellin transform of Eq. (1) is taken so that the convolution integrals become simple products. The resulting ordinary differential equations can be solved analytically. The result is then transformed back to x–space. This method is mathematically more involved but accuracies of $\sim 10^{-5}$ are readily achieved [4].

Detailed comparisons were made of two x–space programs used by the ZEUS (B) [5] and H1 (PZ) [6] collaborations and two N–space progams which we label (V) [7] and (R) [8] respectively. With these four programs identical sets of input parton distributions ($10^{-5} < x < 1$) were evolved in NLO from the input scale $Q^2 = 4$ GeV^2 up to $Q^2 = 10^4$ GeV^2.

Fig. 1 shows a comparison of the results. One notices the very good agreement between the N–space programs (V) and (R). The two x–space programs (B) and (PZ) agree to within 0.05% with the N–space programs over a wide kinematic range. The agreement is slighly worse ($\sim 1\%$) at very high x where the parton distributions vanish. We remark that the agreement between the N–space and x–space programs can in principle be further improved by increasing the number of gridpoints in the latter.

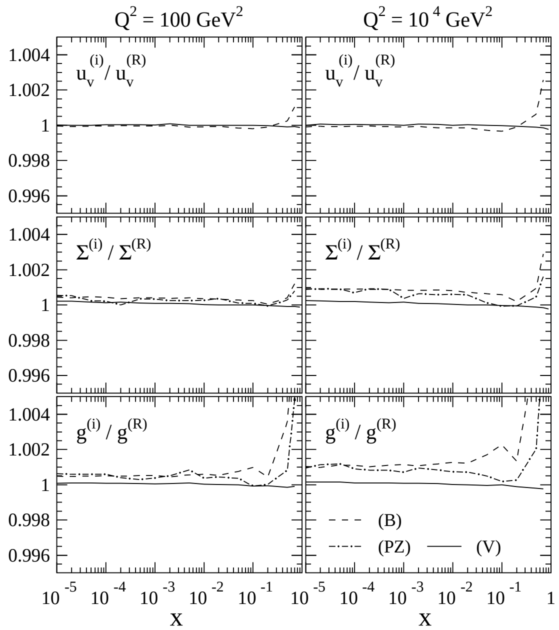

Figure 1: The relative difference between the up-valence (u_v), singlet (Σ) and gluon (g) densities as obtained from evolving identical input at $Q^2 = 4$ GeV^2 in NLO with the evolution programs i = (B, PZ, V) and (R).

Experimental errors on α_s

To investigate the experimental error on $\alpha_s(M_Z^2)$ QCD fits were performed to simulated HERA F_2 datasets listed in Table . The data cover a kinematic range $1.5 \times 10^{-5} < x < 0.7$ and $0.5 < Q^2 < 5 \times 10^4$ GeV^2.

Seven independent sources of systematic error were taken into account (see [2] for details) giving a total systematic error of $\sim$ 1–5% over most of the kinematic range which is a factor of 2–5 better than presently achieved. Residual systematic effects were represented by a point to point uncorrelated systematic error of 1%.

The following model was fitted to the simulated data:

$$F_i(p,s) = F_i^{QCD}(p) \;\; (1 - \sum_l s_l \Delta_{li}^{syst}) \tag{2}$$

where $F_i^{QCD}(p)$ is the QCD prediction for F_2, Δ_{li}^{syst} is the (relative) systematic error on datapoint (i) stemming from source (l) and s_l are the systematic parameters. It is assumed that these parameters are uncorrelated and gaussian distributed with zero mean and unit variance.

The parameters (p) in Eq. (2) represent $\alpha_s(M_Z^2)$ and those parameters describing the parton distributions at the input scale $Q_0^2 = 4$ GeV^2. The gluon distribution (xg), the quark singlet distribution $(x\Sigma)$ and the difference of the up and down quark distributions $(x\Delta_{ud})$ were parametrised as

$$xh(x, Q_0^2) = A_h x^{B_h} (1-x)^{C_h} P(x) \tag{3}$$

with $P(x) = 1$ for xg and $x\Delta_{ud}$ and $P(x) = 1 + D\sqrt{x} + Ex$ for $x\Sigma$. In the studies presented below two types of fit were considered: (i) leave the parameters (p) and s_l free in the fit and (ii) fix the systematic parameters s_l to zero. In the latter fits the systematic errors $(\Delta s_l = 1)$ are propagated to the covariance matrix of the fitted parameters (p) using the technique described in [11]. Since we are only interested in the errors the data were replaced by the model so that the fits immediately converged.

The results for the total error on α_s are given in Table . A fit of the proton datasets I and II with a Q^2 cut of of 3 GeV^2 yields $\Delta\alpha_s(M_z^2) = 0.006$ (0.012) depending on whether the systematic parameters are fitted or fixed (fit 1 in Table). The error on α_s is much improved when pertubative QCD is assumed to be valid at lower values of Q^2 and the cut is lowered from 3 to 1 GeV^2 (fit 2). Doubling the luminosity of the high Q^2 sample has no effect which illustrates the fact that the error on α_s is dominated by the experimental systematic errors (fit 4). A modest improvement in the α_s error is obtained when lower energy proton data are included (fit 6).

Dataset	nucleon	E_e (GeV)	E_N (GeV)	L (pb^{-1})	Q^2_{min} (GeV^2)	Q^2_{max} (GeV^2)
I	proton	27.6	820	10	0.5	100
II	proton	27.6	820	500	100	50000
III	proton	27.6	400	200	100	20000
IV	proton	15.0	820	10	0.5	100
V	deuteron	27.6	410	10	0.5	100
VI	deuteron	27.6	410	50	100	20000

Table 1: Summary of simulated data sets for this study.

Fit	datasets	Q^2_{cut} (GeV^2)	L(set II) (pb^{-1})	syst. fitted	syst. fixed
1	I, II	3	500	0.006	0.012
2	I, II	1	500	0.002	0.006
3	I, II	8	500	0.007	0.012
4	I, II	3	1000	0.006	0.012
5	I, II	3	10	0.010	0.015
6	I, II, III, IV	3	500	0.005	
7	I, II, V, VI	3	500	0.004	0.009

Table 2: The error $\Delta\alpha_s(M_Z^2)$ for several fits to Hera data alone.

In the fits 1–6 $x\Delta_{ud}(x, Q_0^2)$ was kept fixed to the nominal input value. The error on this distribution was taken from the QCD analysis of ref. [9] and contributes 0.004 to the error on $\alpha_s(M_Z^2)$ (not included in Table). This error is eliminated if HERA deuteron data are included: the difference of F_2^p and F_2^d constrains $x\Delta_{ud}$ which can thus be left free in the fit. It is seen from Table that the error on α_s is reduced even though the number of fit parameters has increased (fit 7).

To investigate if HERA can improve the error on α_s from fixed target data [10] fits were performed including SLAC [12], BCDMS [13] and NMC [14] proton and deuteron F_2 data. To remove higher twist effects a cut $W^2 > 10$ GeV^2 was imposed. A fit to SLAC and BCDMS data alone reproduced the result of ref. [10]: $\Delta\alpha_s(M_Z^2) = 0.003$.

Including the fixed target data the error on α_s is much less sensitive to the Q^2 cuts imposed and the luminosity of the high Q^2 HERA sample. Depending on the cuts and the available luminosity the fits yield $\Delta\alpha_s(M_Z^2) = 0.0015 - 0.0020$ when all systematic parameters are left free and $\Delta\alpha_s(M_Z^2) = 0.0025 - 0.0035$ when they are kept fixed.

Of course, contrary to an analysis of high x fixed target data alone, the QCD fits to HERA structure functions result in a joint determination of both α_s and xg. For instance a fit to the datasets I and II (fit 1) with all systematic parameters left free results in a determination of the gluon distribution with an accuracy of about 3% at $x = 10^{-4}$ and $Q^2 = 20$ GeV^2, as illustrated in Fig. 2.

Theoretical errors on α_s

The following possible sources of theoretical uncertainty were investigated [3, 4]:

- The effect arising from different representations of α_s. The scale dependence of the strong coupling constant reads

$$\frac{\partial a_s(Q^2)}{\partial \ln Q^2} = -\beta_0 a_s^2(Q^2) - \beta_1 a_s^3(Q^2) + O(a_s^4) \tag{4}$$

  This equation can easily be solved numerically given a_s at some input scale Q_0^2. The following approximate solution in terms of the QCD scale parameter Λ is widely used:

$$a_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{\ln\ln(Q^2/\Lambda^2)}{\ln^2(Q^2/\Lambda^2)} + O(\ln^{-3}(Q^2/\Lambda^2)) \tag{5}$$

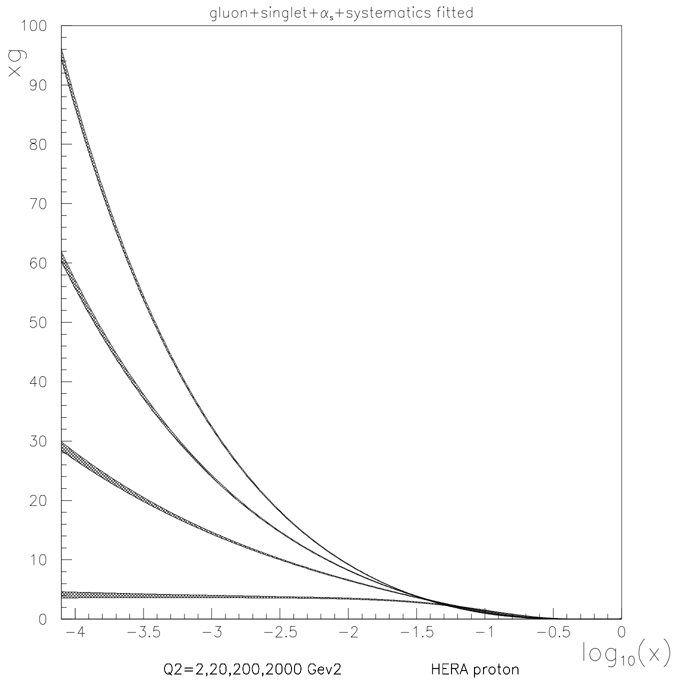

Figure 2: The gluon distribution versus x for different values of Q^2 obtained from a fit to simulated high energy HERA proton data. The bands show the total error.

Either using Eq. (4) or Eq. (5) causes a shift in $\alpha_s(M_Z^2)$ of 0.001 or less.

- The offsets originating from the different prescriptions of the NLO evolution. In the $N-$space approach the evolution equations and their analytic solutions are usually expanded as a power series in a_s and terms $O(a_s^3)$ discarded. The various truncation prescriptions are extensively studied in ref. [4]. It turns out that the effect on the Q^2 evolution of e.g. F_2 is surprisingly large: differences of up to 6% show up at low $x \simeq 10^{-4}$ and are caused by terms in NNLO and beyond. The corresponding shift in $\alpha_s(M_Z^2)$ is estimated to be about 0.003.

- Renormalisation and factorisation scale uncertainties. In Eq. (1) and Eq. (4) the renormalisation scale (R^2) and the mass factorisation scale (M^2) are both assumed to be equal to the momentum transfer Q^2. The expressions for the case that R^2 and M^2 are chosen to be unequal can be found in [4]. Table gives the shifts in $\alpha_s(M_Z^2)$ when these scales are varied independently in the range $Q^2/4$ to $4Q^2$ for different values of a Q^2 cut made on the data. It is seen that the scale dependence is by far the largest contribution to the theoretical uncertainty in α_s and that in particular the mass factorisation scale dependence increases strongly with a decreasing Q^2 cut.

Summary

An experimental accuracy of $\Delta\alpha_s(M_Z^2) = 0.001$–$0.002$ might be in reach provided the following conditions are satisfied: (i) F_2 measurements become available in the full HERA kinematic

Cut (GeV^2)	$M^2 = Q^2/4$	$M^2 = 4Q^2$	$R^2 = Q^2/4$	$R^2 = 4Q^2$
4	–	−0.0120	−0.0076	+0.0059
20	+0.0067	−0.0044	−0.0067	+0.0049
50	+0.0032	−0.0029	−0.0061	+0.0042

Table 3: The theoretical shifts on $\alpha_s(M_Z^2)$ from scale variations.

range with systematic and statistical errors of a few percent only; (ii) The dependence of the systematic errors on x and Q^2 is sufficiently well known so that their effects can be absorbed in the QCD analysis and (iii) The HERA data can be reliably combined with fixed target F_2 data.

The various prescriptions of the NLO evolution, which differ by terms of NNLO and beyond, cause a theoretical error in $\alpha_s(M_Z^2)$ of about 0.003. If the renormalisation scale (R) and the mass factorisation scale (M) are varied independently in the range $Q^2/4$ to $4Q^2$ a theoretical uncertainty on $\alpha_s(M_Z^2)$ of about ± 0.005 (R) and ± 0.003 (M) is estimated, provided a Q^2 cut of 50 GeV^2 is applied. These errors increase when the Q^2 cut is lowered. The theoretical uncertainties are expected to be reduced substantially once the NNLO splitting functions become available.

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