

# Determination of the strong coupling constant $\alpha_s(m_Z)$ using H1 jet cross section measurements

Daniel Britzger

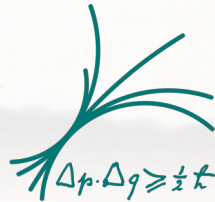
for the H1 Collaboration and NNLOJET

Eur.Phys.J.C 77 (2017), 791 [arXiv:1709.07251]

Eur.Phys.J.C 77 (2017), 215 [arXiv:1611.03421]

$\alpha_s$  workshop Trento, Italy

12.02.2019



Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)



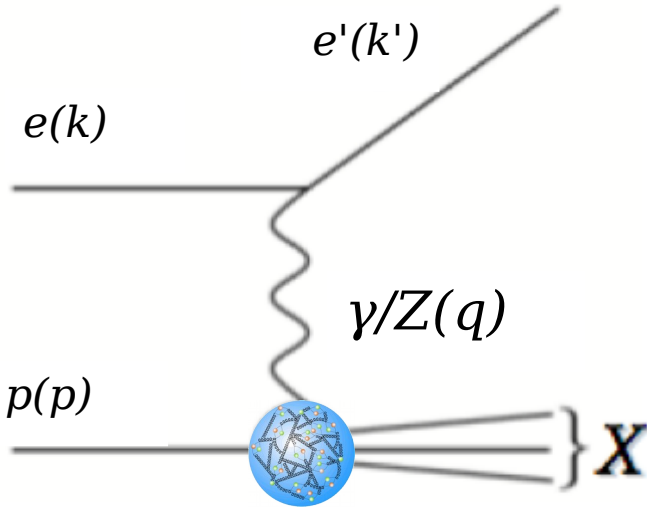
MAX-PLANCK-GESELLSCHAFT



# Deep-inelastic $ep$ scattering at HERA

## Neutral current scattering (NC)

$$ep \rightarrow e'X$$



Photon virtuality

$$Q^2 = -q^2 = -(k - k')^2$$

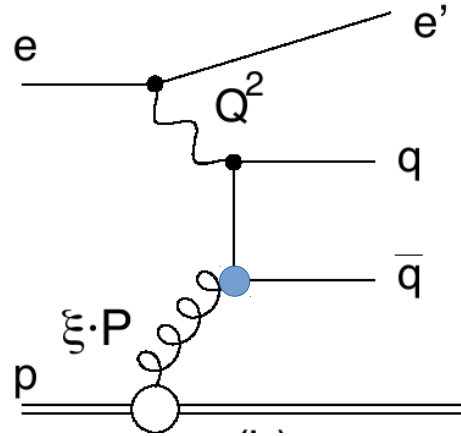
## HERA $ep$ collider in Hamburg



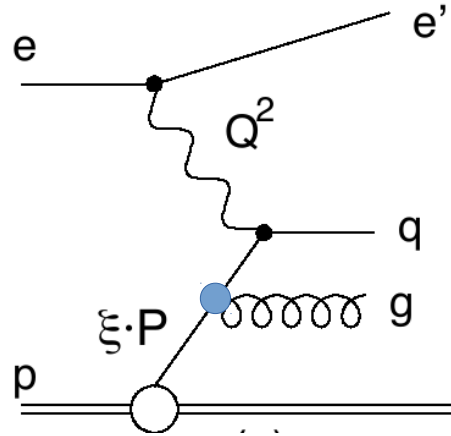
## Data taking periods

- HERA I: 1994 – 2000
- HERA II: 2003 – 2007
- $\sqrt{s} = 300$  or  $319$  GeV

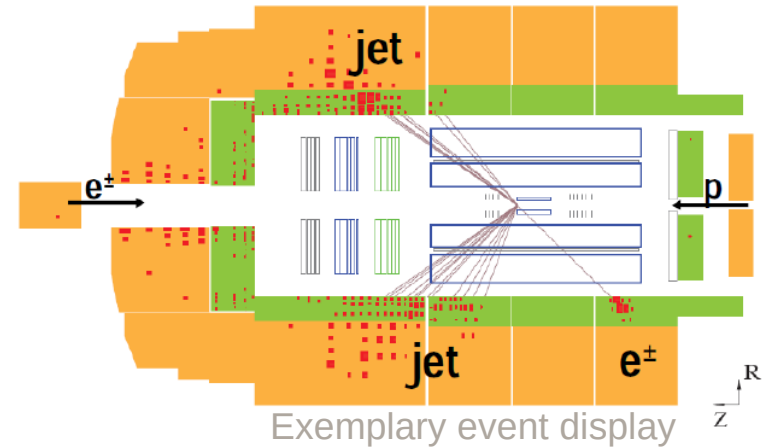
# Jet production in DIS



Boson-gluon fusion



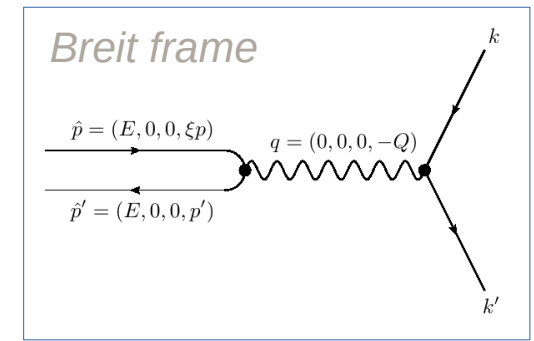
QCD Compton



## Jets in DIS measured in Breit frame

- $ep \rightarrow 2\text{jets}$
- Virtual boson collides 'head-on' with parton from proton
- Boson-gluon fusion dominant process  
QCD compton important only for high- $p_T$  jets (high- $x$ )

Jet measurement sensitive to  $\alpha_s$  and gluon density



# H1 Experiment at HERA

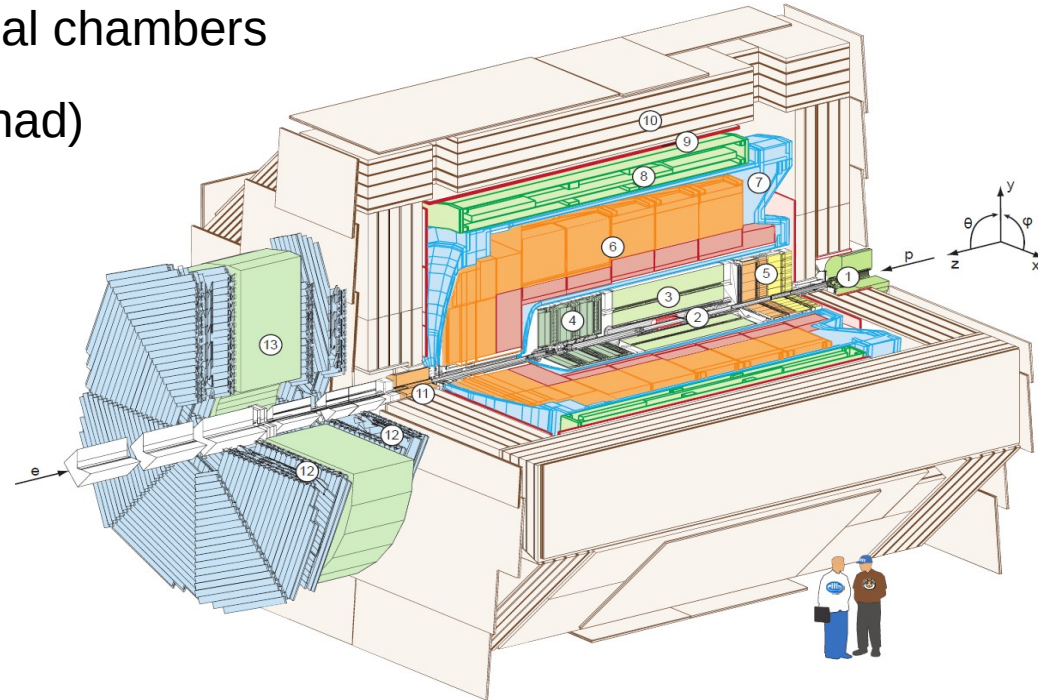
- Precise Trackers
  - Silicon tracker; jet chambers; proportional chambers
- Calorimeters
  - Liquid Argon sampling calorimeter (em/had)
  - Scintillating fiber calorimeter

## **Jet energy scale calibration**

- **Overconstrained system** in NC DIS: Jet calibration using NC DIS events
- Track and calorimeter information exploited ('**particle flow**')
  - Important for  $E_{\text{track}} < \sim 25 \text{ GeV}$
- **Neural network** (cluster classification) based in-situ jet **calibration for data and MC**

## **High experimental precision**

- Electron measurement: 0.5 – 1% scale uncertainty
- Jet energy scale: 1%



Drawing of the H1 experiment

# Inclusive jet cross sections

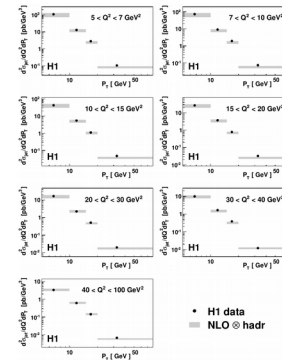
## Inclusive jet cross sections

- $d\sigma/dQ^2 dP_{T,jet}$
- 300 GeV, HERA-I & HERA-II
- low- $Q^2$  ( $<100 \text{ GeV}^2$ ) and high- $Q^2$  ( $>150 \text{ GeV}^2$ ) regions

## Consistency

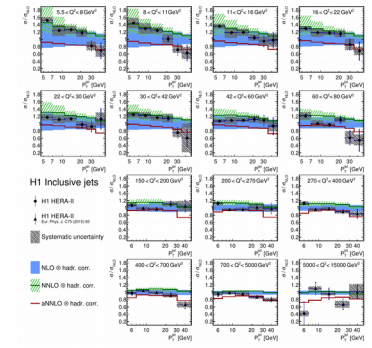
- kt-algorithm,  $R=1$
- $-1.0 < \eta < 2.5$
- $P_T$  ranges from 4.5 to 50 GeV

## HERA-I low- $Q^2$



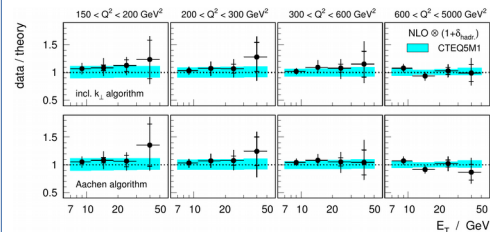
*Eur.Phys.J.C67 (2010) 1*

## HERA-II low- $Q^2$



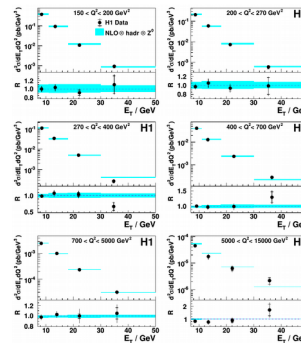
*Eur.Phys.J. C77 (2017) 215*

## 300 GeV high- $Q^2$



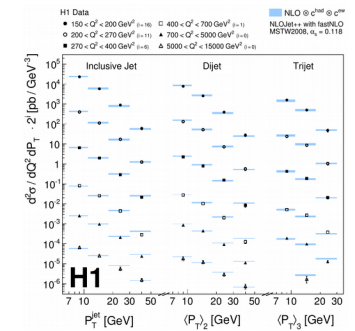
*Eur.Phys.J.C19 (2001) 289*

## HERA-I high- $Q^2$



*Phys.Lett.B653 (2007) 134*

## HERA-II high- $Q^2$



*Eur.Phys.J.C75 (2015) 2*  
*arXiv:1611.03421*

# Inclusive jet cross sections

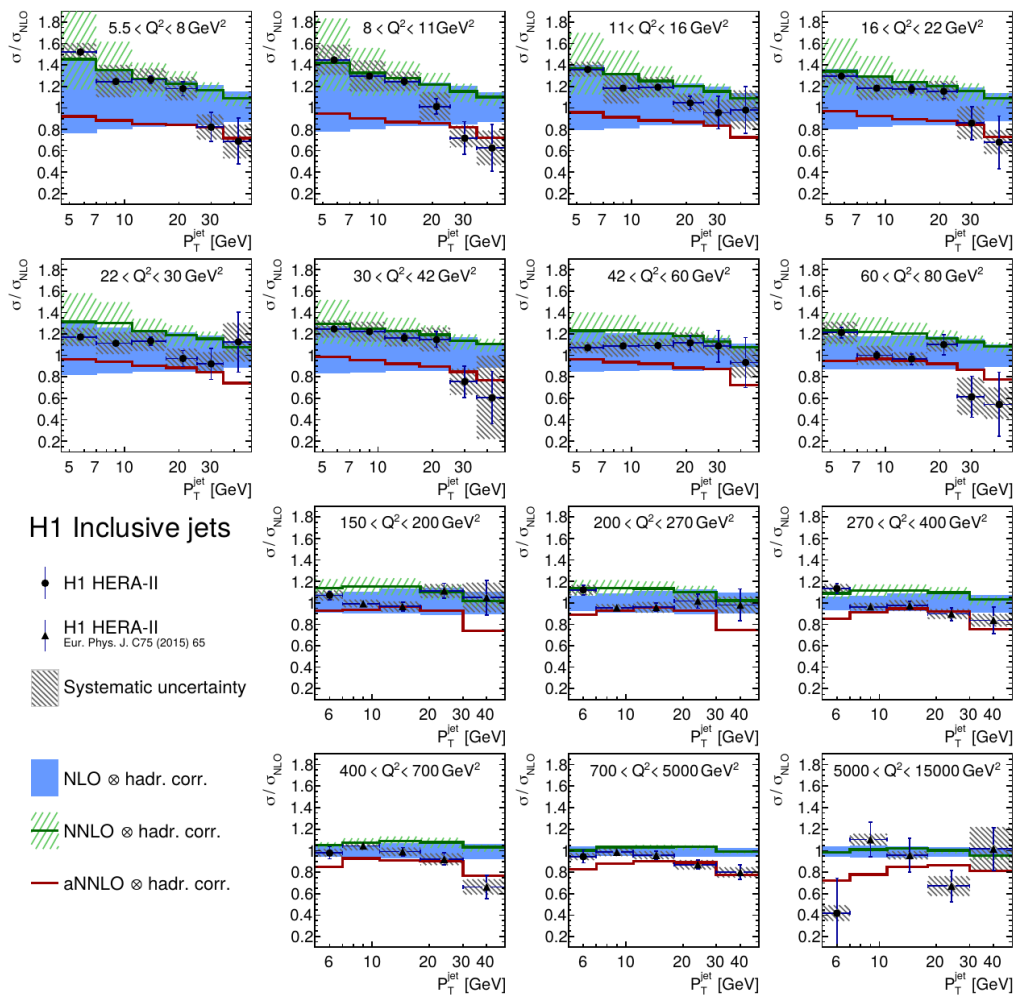
Inclus

- do
- 30
- low
- high

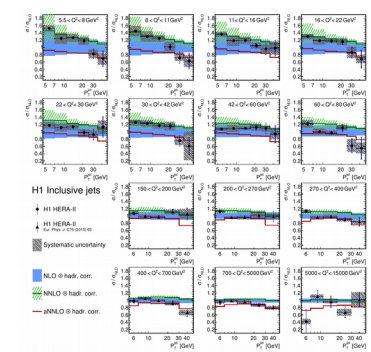
Cons

- kt
- -1
- $P_T$

## HERA-II low- and high- $Q^2$ inclusive jets

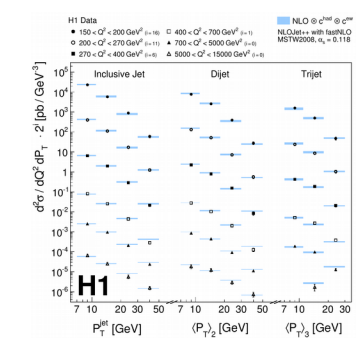


## HERA-II low- $Q^2$



Eur.Phys.J. C77 (2017) 215

## HERA-II high- $Q^2$



Eur.Phys.J.C75 (2015) 2  
arXiv:1611.03421

# Dijet cross section

## Dijet definitions

- $\langle p_T \rangle$  greater than 5, 7 or 8.5 GeV
- $P_T$  jet greater 4, 5 or 7 GeV
- Asymmetric cuts on  $p_{T,jet1}$  and  $p_{T,jet2}$
- $M_{12}$  cut for two data sets

## Dijet cross sections

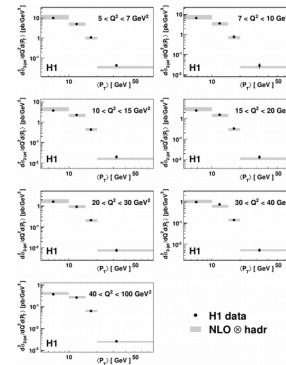
- $d\sigma/dQ^2 d\langle p_T \rangle$
- 300 GeV, HERA-I & HERA-II
- low- $Q^2$  and high- $Q^2$

## Earlier studies

All inclusive jet and dijet data have been employed for  $\alpha_s$  extractions previously

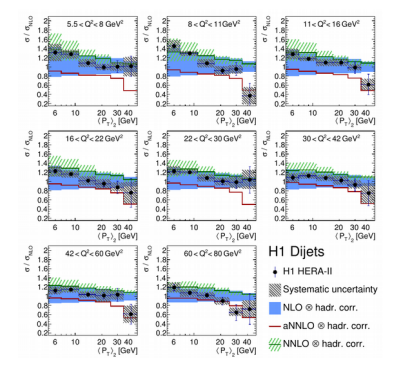
- > Data and uncertainties well-understood
- > NNLO theory is new

## HERA-I low- $Q^2$



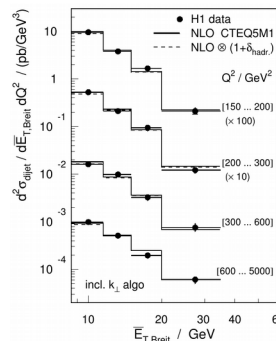
*Eur.Phys.J.C67 (2010) 1*

## HERA-II low- $Q^2$



*Eur.Phys.J. C77 (2017) 215*

## 300 GeV high- $Q^2$

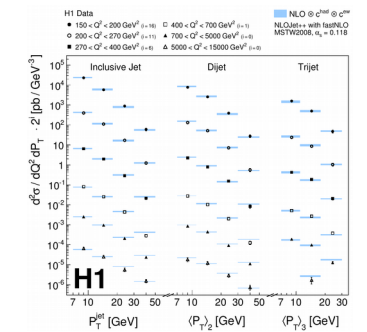


*Eur.Phys.J.C19 (2001) 289*

## HERA-I high- $Q^2$

Dijet cross sections not statistically independent from HERA-II analysis  
*Eur.Phys.J.C65 (2010) 363*

## HERA-II high- $Q^2$



*Eur.Phys.J.C75 (2015) 2*

$\alpha_s$  determination in **NLO**  
from HERA-II data  
→ **Highest experimental precision**



# $\alpha_s(M_Z)$ from HERA-II jet data at NLO

## H1 HERA-II low- and high- $Q^2$ data

- Low- $Q^2$  jets (Eur.Phys.J.C 77 (2017) 21)
- high- $Q^2$  jets (Eur.Phys.J.C75 (2015) 2)

## All normalised jet cross sections

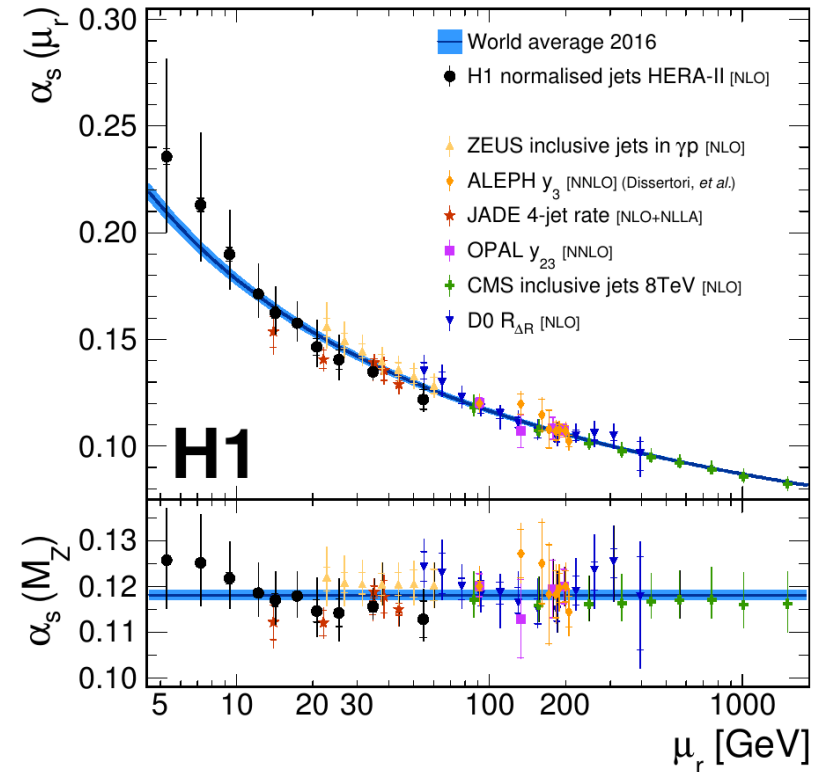
- Normalised inclusive jet
- Normalised dijets
- Normalised three-jets
- Correlations of uncertainties are known
- Fit  $\alpha_s(M_Z)$  in  $\chi^2$ -minimization procedure

## Results at NLO

- fit to all HERA-II data points

$$\alpha_s(M_Z) = 0.1173 \boxed{(4)_{\text{exp}}} (3)_{\text{PDF}} (7)_{\text{PDF}(\alpha_s)} (11)_{\text{PDFset}} (6)_{\text{had}} \left( \begin{smallmatrix} +51 \\ -43 \end{smallmatrix} \right)_{\text{scale}}$$

- Very high experimental precision
- $\alpha_s$  determination fully limited by NLO scale uncertainties

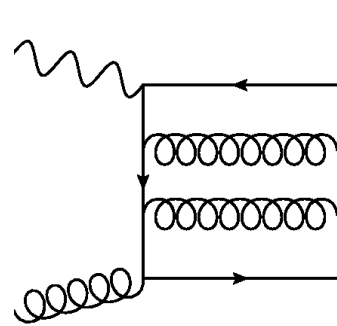


Eur.Phys.J.C 77 (2017), 215  
[arXiv:1611.03421]

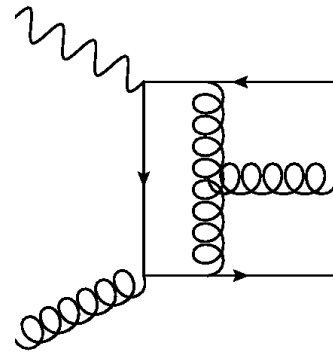
$\alpha_s$  determination in NNLO  
from all H1 inclusive jet and dijet  
cross section data

# DIS jet production in NNLO

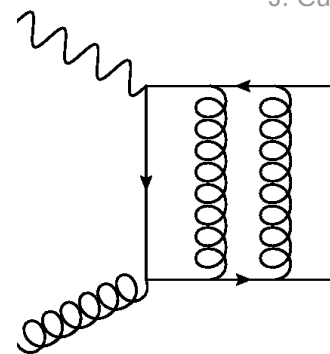
J. Currie, et al. [RPL 117 (2016) 042001]  
 J. Currie, et al. [JHEP 1707 (2017) 018]



Double-real



Real-virtual



Double-virtual

$$d\sigma_{NNLO}^{RR,S} \approx \underbrace{X(\{p_X\})}_{\text{antenna}} \overbrace{d\Phi_3(\{p_X\})}^{\text{Antenna PS}} \times \underbrace{|\mathcal{M}(\{\tilde{p}_m\})|^2}_{\text{reduced ME}} \overbrace{d\Phi_m(\{\tilde{p}_m\})}^{\text{reduced PS}} \times \underbrace{\mathcal{J}(\{\tilde{p}_m\})}_{\text{jet function}}$$

## A bit of history

- 1973 asymptotic freedom of QCD  
 [PRL 30(1973) 1343 & 1346]
- 1993 NLO studies of DIS jet cross sections  
 [Phys. Rev. D49 (1994) 3291]
- 2016 NNLO corrections for DIS jets  
 [Phys. Rev. Lett. 117 (2016) 042001], [arXiv:1703.05977]

## Antenna subtraction

- Cancellation of IR divergences with local subtraction terms
- Move IR divergences across different phase space multiplicities

# $\alpha_s$ -fit methodology

## $\alpha_s$ determined in $\chi^2$ -minimisation

- $\alpha_s(m_Z)$  is a free parameter to theory prediction  $\sigma_i$

$$\chi^2 = \sum_{i,j} \log \frac{S_i}{\sigma_i} (V_{\text{exp}} + V_{\text{had}} + V_{\text{PDF}})_{ij}^{-1} \log \frac{S_j}{\sigma_j}$$

## NNLO theory is sensitive to $\alpha_s(m_Z)$

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}_{i,k}^{(n)}(x, \mu_R, \mu_F) \cdot C_{\text{had}}$$

- $\alpha_s$  dependence of PDF is accounted for by using PDF at  $\mu_{F,0} = 20\text{GeV}$  and applying DGLAP
- Important for reliable uncertainty estimates!

## Separate fits are performed to

- All inclusive jet data sets (137 data points)
- All dijet data sets (103 data points)
- All H1 jet data taken together (denoted as 'H1 jets')

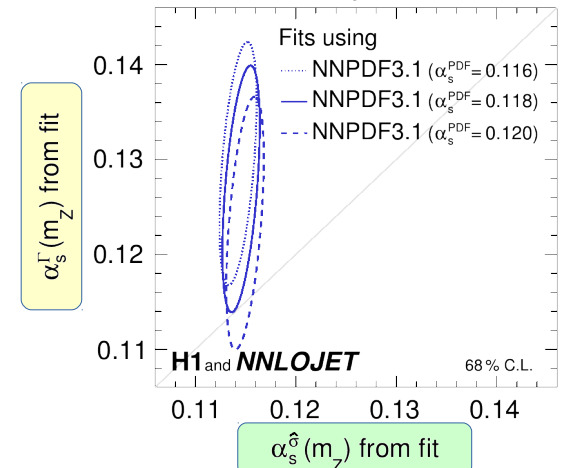
$S_i$  H1 jet data  
 $\sigma_i$  NNLO theory  
 $V$  covariance matrices

### Hard ME's

$$\hat{\sigma}_{i,k}^{(n)} = \alpha_s^n(\mu_R) \tilde{\sigma}_{i,k}^{(n)}(x, \mu_R, \mu_F)$$

$$\text{PDFs } \frac{\partial f}{\partial \alpha_s} = \frac{P \otimes f}{\beta}$$

H1 jets



# Inclusive jets

## $\alpha_s$ in NNLO from individual data sets

- All fits with good  $\chi^2$
- Data sets found to be consistent
- Consistency between low- and high- $Q^2$

## Fit to all inclusive jets data in NNLO

$$0.1132 (10)_{\text{exp}} (5)_{\text{had}} (4)_{\text{PDF}} (4)_{\text{PDF}_{\alpha_s}} (2)_{\text{PDF}_{\text{set}}} (40)_{\text{scale}}$$

- $\chi^2/\text{ndf} = 134/133$
- High experimental precision
- Scale uncertainty is largest (theory) error

## Fit with $\mu > 28\text{GeV}$

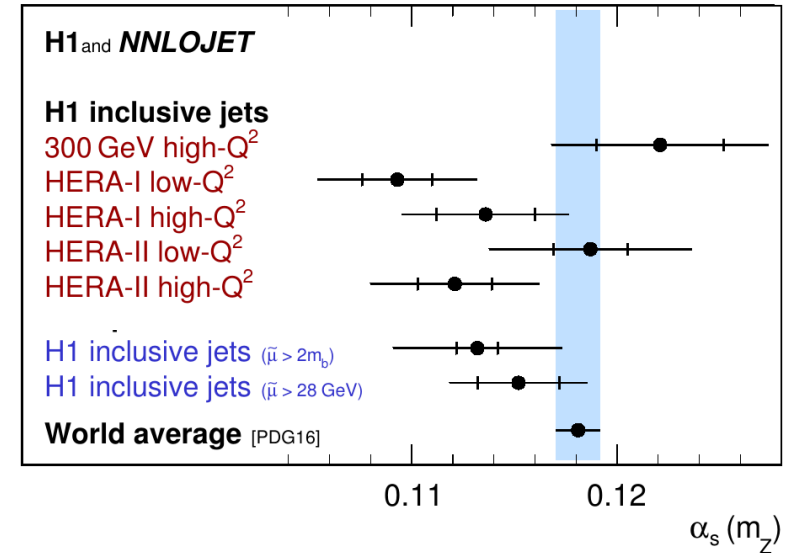
( $\mu$  is a characteristic scale assigned to any data point)

$$0.1152 (20)_{\text{exp}} (6)_{\text{had}} (2)_{\text{PDF}} (2)_{\text{PDF}_{\alpha_s}} (3)_{\text{PDF}_{\text{set}}} (26)_{\text{scale}}$$

- Reduced scale, but increased exp. uncertainty
- No significant dependence on  $\mu$  cut

$$\alpha_s(m_Z) = 0.1152 (20)_{\text{exp}} (27)_{\text{th}}$$

$\alpha_s$  results from H1 jet data in NNLO



See backup slides for a summary of all numerical values

# Dijets

## *Fits to individual dijet data sets*

- All data sets with good  $\chi^2$
- Reasonable consistency of data sets found

## *Fit to all dijet data in NNLO*

- $\chi^2/\text{ndf} = 93.9/102$ :  
consistency of data sets

$0.1148 (11)_{\text{exp}} (6)_{\text{had}} (5)_{\text{PDF}} (4)_{\text{PDF}_{\alpha_s}} (4)_{\text{PDF}_{\text{set}}} (40)_{\text{scale}}$

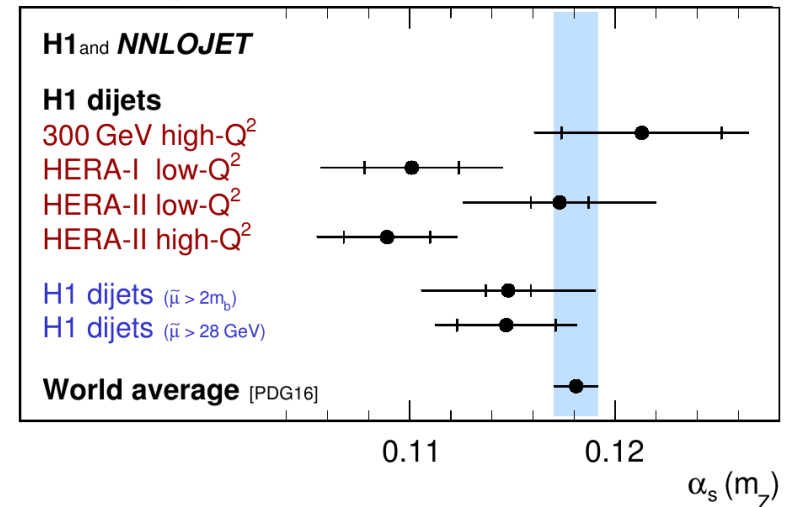
- Value consistent with inclusive jets

## *Fits to all dijets with $\mu > 28\text{GeV}$*

$0.1147 (24)_{\text{exp}} (5)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}_{\alpha_s}} (3)_{\text{PDF}_{\text{set}}} (24)_{\text{scale}}$

- Reduced scale, but increased exp. uncertainty

$\alpha_s$  results from H1 jet data in NNLO



$$\alpha_s(m_Z) = 0.1147 (24)_{\text{exp}} (25)_{\text{th}}$$

# Uncertainty budget

$$0.1152 (20)_{\text{exp}} (6)_{\text{had}} (2)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (26)_{\text{scale}}$$

## ***Experimental uncertainty (exp)***

- All exp. uncertainties (incl. all correlations)

## ***hadronisation uncertainty (had)***

- Propagation of hadronisation uncertainties as published with the data:  
commonly: difference between two MC generators (Django,Rapgap,Sherpa)

## ***PDF uncertainties***

- 'PDF' uncertainty
- $\text{PDF}\alpha_s$  uncertainty
- PDFset uncertainty
- $\text{PDF}\mu_0$  uncertainty

Next slides...

## ***Scale uncertainty***

- Scale factors: 0.5 and 2

# PDF related uncertainties

## 'PDF' uncertainty:

Uncertainties of NNPDF3.1 propagated to the result

## PDFset uncertainty

1/2 max. difference between various PDF sets

## PDF $\alpha_s$ uncertainty

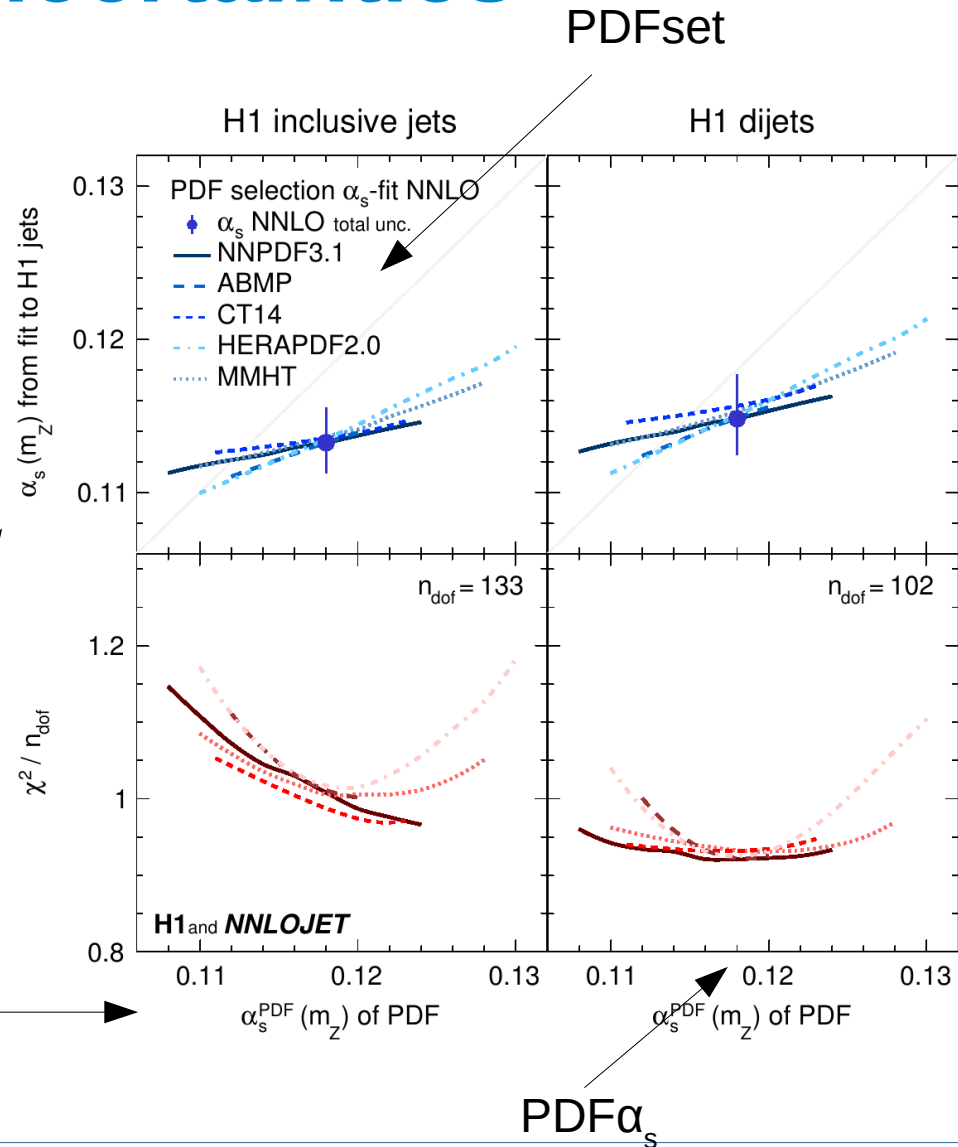
Variation of  $\alpha_s$ -input to PDF by  $\pm 0.002$

## PDF $\mu_{F,0}$ uncertainty

negligible if  $\mu_{F,0}$  varied between 10 to 90GeV

Result of our  $\alpha_s$  fit

'Input' value of  $\alpha_s(m_Z)$  to the PDF determination





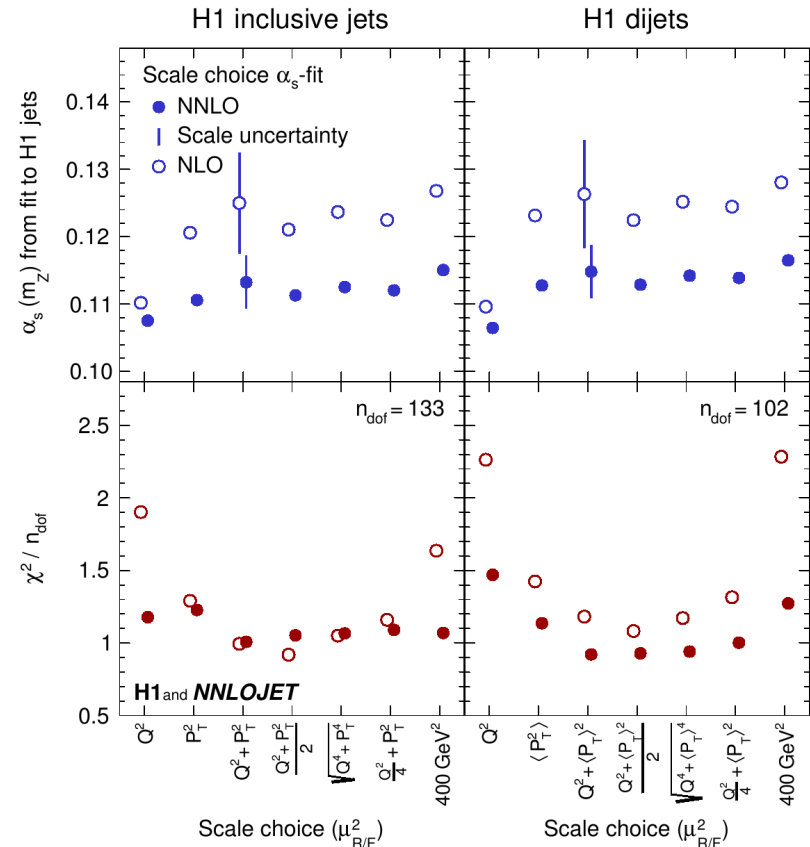
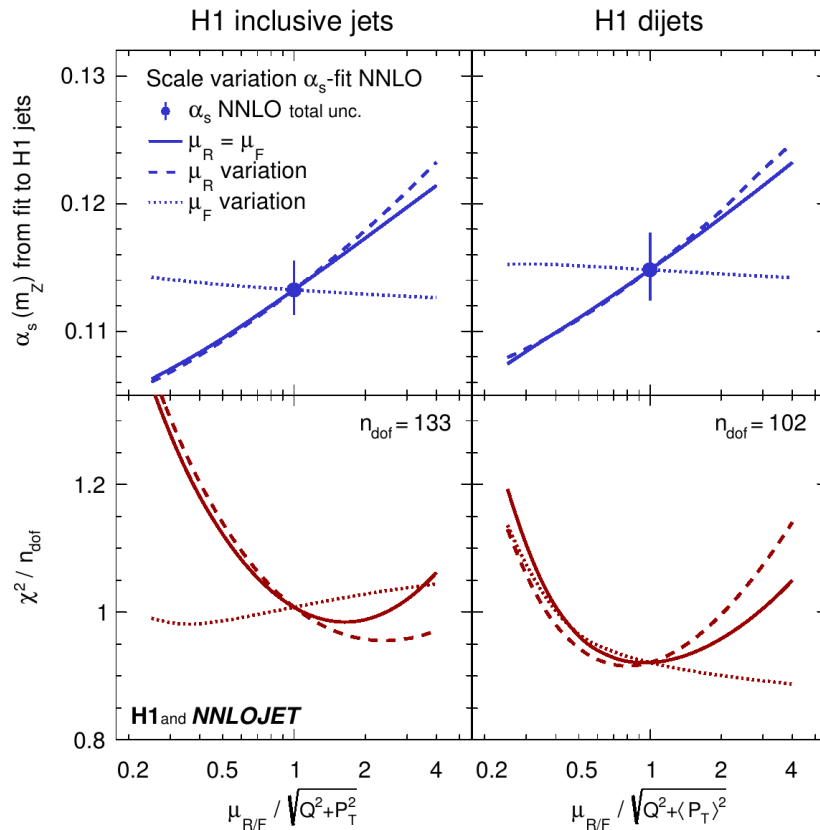
# Scale uncertainty

## Scale variations

- $\mu_R$  variation dominates
- Large scale factor cause large  $\chi^2$  values

## Scale choices

- Scale uncertainty also covers different scale choices
- $\mu = 20\text{GeV}$ : fixed scale!  
 → no running, no DGLAP → consistent results



# H1jets

## fit to inclusive jet and dijet cross sections

# H1 jets

## Fit to inclusive jet and dijet together

- Stat. and experimental correlations are known
- $\chi^2/\text{ndf} = 0.98$  for 200 data points
- > Inclusive jet and dijet data are consistent

$$0.1143 (9)_{\text{exp}} (6)_{\text{had}} (5)_{\text{PDF}} (5)_{\text{PDF}\alpha_s} (4)_{\text{PDFset}} (42)_{\text{scale}}$$

## H1 jets with $\mu > 28\text{GeV}$

- 91 data points

$$0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (27)_{\text{scale}}$$

- Moderate exp. precision (due to  $\mu > 28\text{GeV}$ )
- Scale uncertainty dominates
- PDF uncertainties negligible

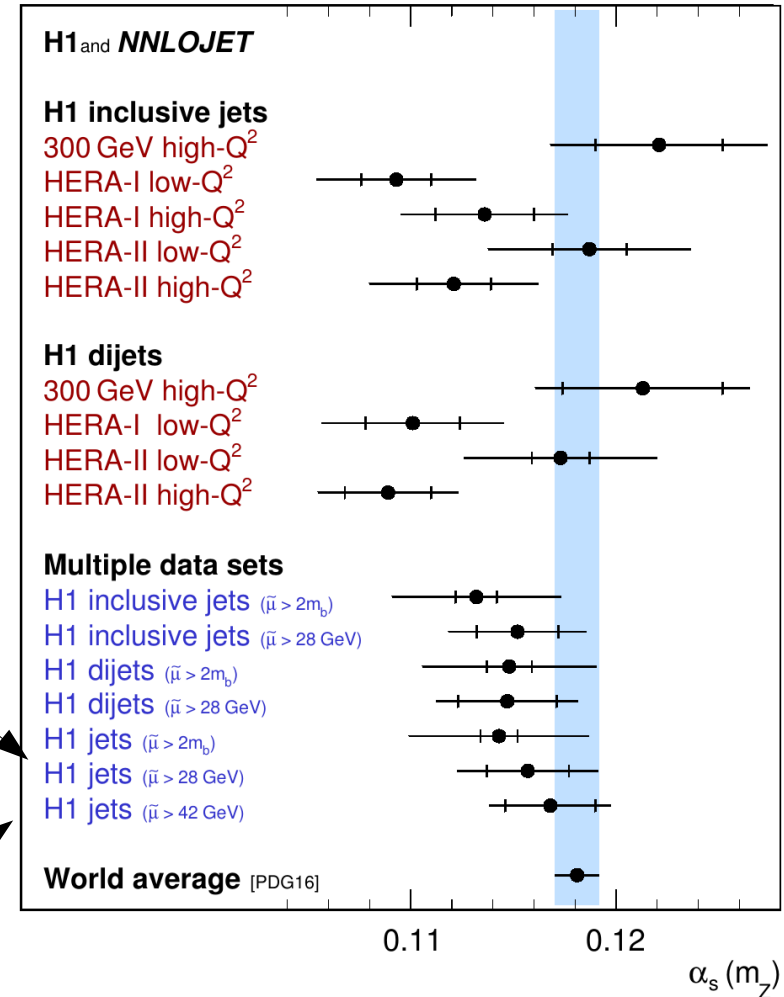
## Smallest theo. uncertainty for: $\mu > 42\text{GeV}$

$$0.1168 (22)_{\text{exp}} (7)_{\text{had}} (2)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (5)_{\text{PDFset}} (17)_{\text{scale}}$$

## Main result with: $\mu > 28\text{GeV}$

$$\alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (29)_{\text{th}}$$

$\alpha_s$  results from H1 jet data in NNLO



# Study of total uncertainty

## Scale uncertainties at various scales $\mu$

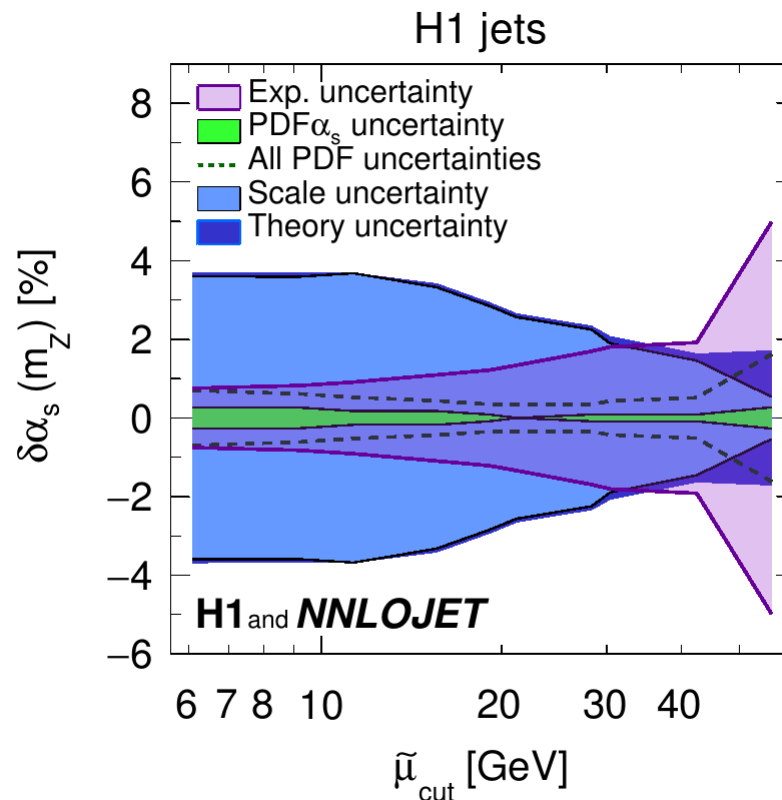
- At low- $\mu$ : large scale uncertainties...
- ... but also high sensitivity to  $\alpha_s(m_Z)$

## Fits imposing a cut on scale $\mu$

- Repeat  $\alpha_s$  fits:  
successively cut away data below  $\mu_{\text{cut}}$

## Cut on $\mu$

- Scale uncertainty decreases with  $\mu_{\text{cut}}$
- Exp. uncertainty increases with  $\mu_{\text{cut}}$



# Comparison of NNLO predictions with data

## All H1 jet cross section data compared to NNLO predictions

- Inclusive jets
- Dijets

## Overall good agreement

- NNLO describes all data very well
- Also justified by good  $\chi^2$  values of the fits

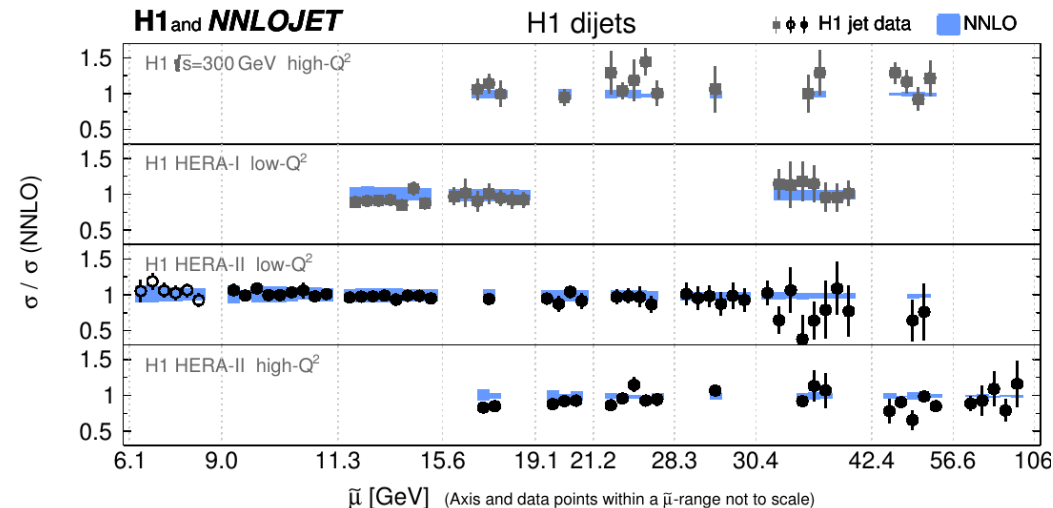
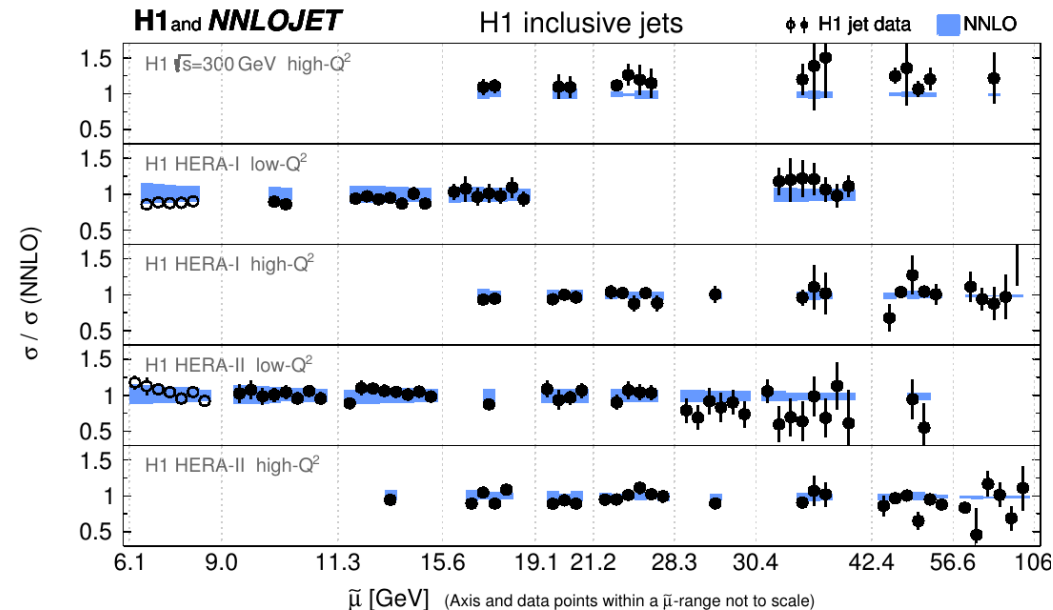
## Data points displayed vs. $\mu$

- apply grouping/binning

→ use for scale-dependent studies

Reminder: our scale choice

$$\mu_R^2 = \mu_F^2 = Q^2 + P_T^2$$



# Scale dependence

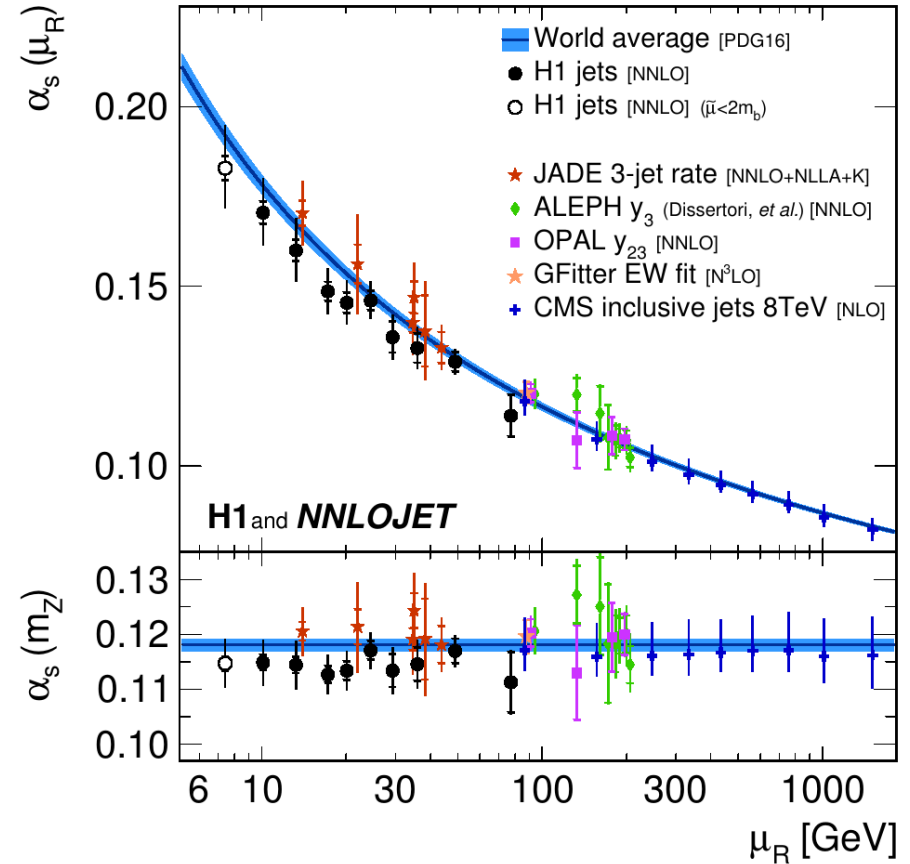
- Perform fits to groups of data points at similar scale
- Assume running to be valid within the limited range covered by interval

## H1 jets

- Good consistency with other data
- First determination using jet data in NNLO

## Most precise determination of $\alpha_s(\mu_R)$ in range between 7 and 90 GeV

- Measurement bridges the gap between low-scale  $\alpha_s$  determinations and LEP/LHC determinations



# Alternative $\alpha_s$ fitting approach

'PDF+ $\alpha_s$ -fit'  
H1PDF2017

# Alternative $\alpha_s$ fit approach: 'PDF+ $\alpha_s$ -fit'

## Perform H1 alone PDF fit: **H1PDF2017**

- Use (all) H1 inclusive DIS data ( $Q^2 > 10 \text{ GeV}^2$ )
  - Use (all) H1 **normalised** jet cross section data
- > 1529 data points

## Normalised jet cross sections

- Jet cross sections normalised to inclusive DIS
- Correlations of jets and inclusive DIS cancel

## PDFs are parameterised as

$$xf(x)|_{\mu_0} = f_A x^{f_B} (1-x)^{f_C} (1 + f_D x + f_E x^2)$$

- Similar to HERAPDF/H1PDF2012

Mind: all PDFs are commonly determined predominantly from (H1) inclusive DIS data

Cross section:  $\sim \text{PDF} \otimes \sigma$

$$\sigma_i = \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}_{i,k}(x, \mu_R, \mu_F) \cdot C_{\text{had},i}$$

## Inclusive NC & CC DIS

Data set [ref.]	Lepton type	$\sqrt{s}$ [GeV]	$Q^2$ range [GeV <sup>2</sup> ]	NC cross sections	CC cross sections	Lepton beam polarisation
Combined low- $Q^2$ [64]	$e^+$	301,319	(0.5) 12 – 150	✓	–	–
Combined low- $E_p$ [64]	$e^+$	225,252	(1.5) 12 – 90	✓	–	–
94 – 97 [61]	$e^+$	301	150 – 30 000	✓	✓	–
98 – 99 [62,63]	$e^-$	319	150 – 30 000	✓	✓	–
99 – 00 [63]	$e^+$	319	150 – 30 000	✓	✓	–
HERA-II [65]	$e^+$	319	120 – 30 000	✓	✓	✓
HERA-II [65]	$e^-$	319	120 – 50 000	✓	✓	✓

## Normalised jets

Data set [ref.]	$Q^2$ domain	Inclusive jets	Dijets	Normalised inclusive jets	Normalised dijets	Stat. corr. between samples
300 GeV [17]	high- $Q^2$	✓	✓	–	–	–
HERA-I [23]	low- $Q^2$	✓	✓	–	–	–
HERA-I [21]	high- $Q^2$	✓	–	✓	–	–
HERA-II [15]	low- $Q^2$	✓	✓	✓	✓	✓
HERA-II [15,24]	high- $Q^2$	✓	✓	✓	✓	✓



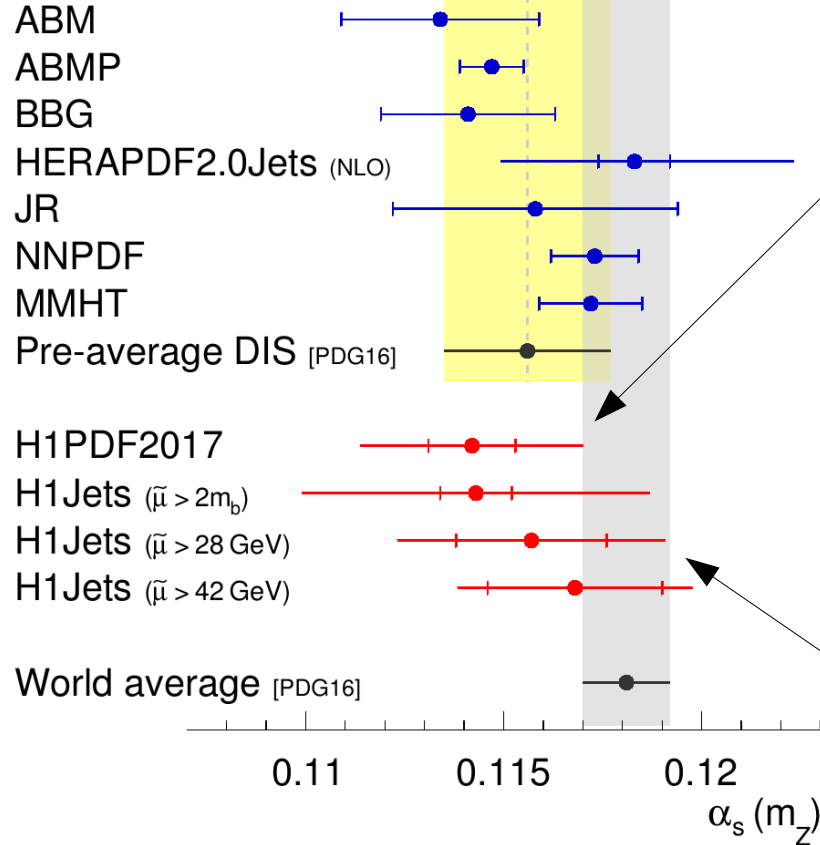
# Results

Inner errors: exp. uncertainty  
Outer errors: total uncertainty

$\alpha_s$  determinations in NNLO

H1 and NNLOJET

$\alpha_s$  determined in PDF+ $\alpha_s$ -fit



$$\alpha_s(m_Z) = 0.1142 (11)_{\text{exp,had,PDF}} (2)_{\text{mod}} (2)_{\text{par}} (26)_{\text{scale}}$$

- $\chi^2/\text{ndf} \sim 1.01$
- High experimental precision
- Scale uncertainty dominates:  
determined from simultaneous variation of  
all scales involved in calculation

## Discussion / comparison

- Result consistent with our other determination

Our two main results are fairly distinct:

- PDF+ $\alpha_s$ -fit mostly sensitive to jets at lower scale
- H1jets:  $\mu > 28$  GeV

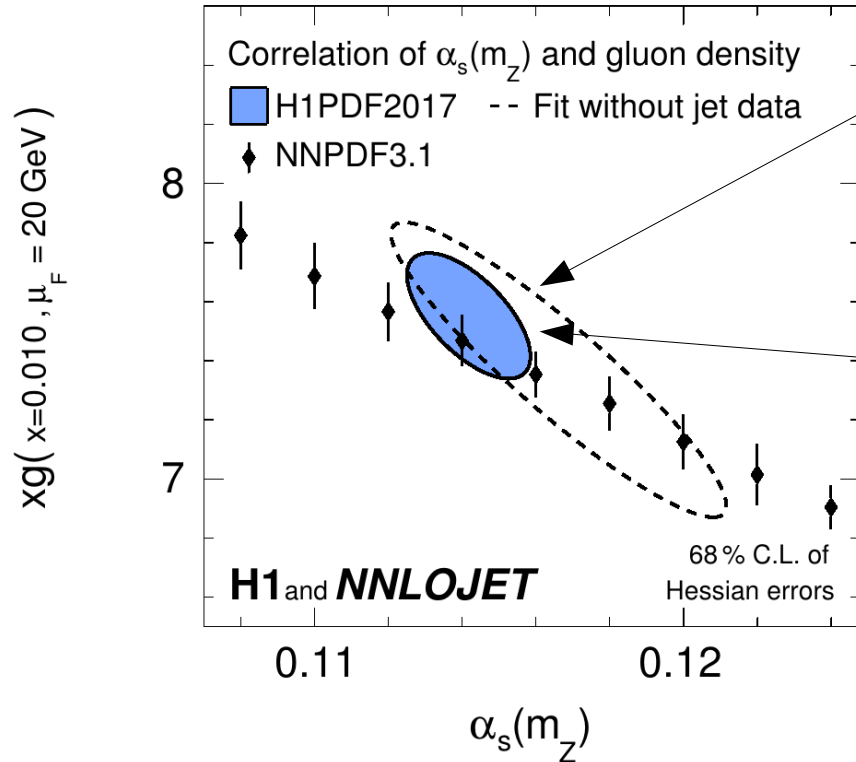
$$\alpha_s(m_Z) = 0.1142 (28)_{\text{tot}}$$

# $\alpha_s$ and the gluon-PDF

## *H1PDF2017: PDF+ $\alpha_s$ -fit*

- Simultaneous determination of the gluon and  $\alpha_s$

### *Correlation of $\alpha_s$ and $g$*



## *Fit to inclusive DIS data alone*

- no jet data
- Large correlation:  
 $\alpha_s$  and gluon **cannot** be determined simultaneously from inclusive DIS data alone

## *Including jet data*

- determination of  $\alpha_s$  and gluon feasible

## *Comparison with NNPDF3.1*

- Error ellipses with similar correlation as individual NNPDF3.1 fits
- Uncertainty of gluon in H1PDF2017 somewhat competitive to NNPDF3.1...
- .... but  $\alpha_s$  is a free parameter !

# PDF+ $\alpha_s$ -fit – H1PDF2017 [NNLO]

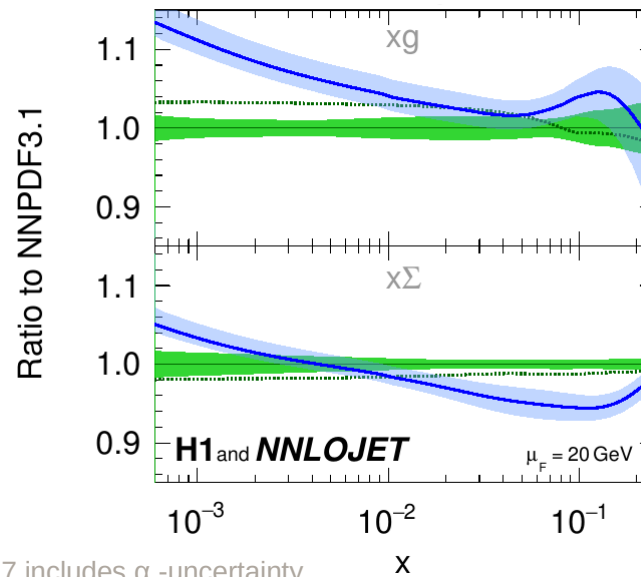
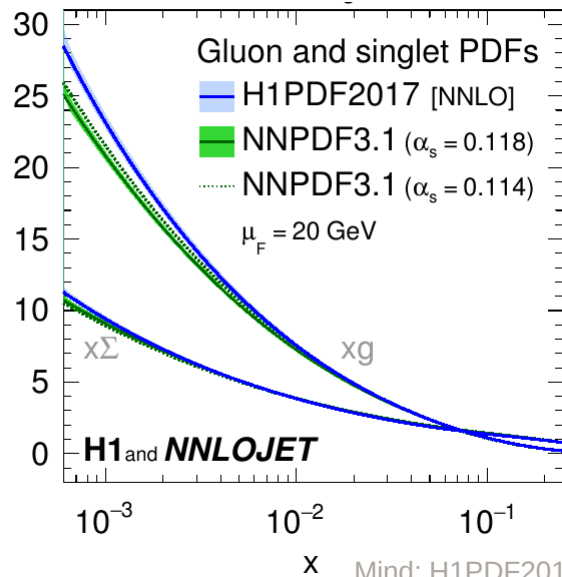
## Result for PDFs

- Set of PDFs determined with high precision
- Precision is competitive with global PDF fitters  
...despite  $\alpha_s$  is a free parameter to the fit:

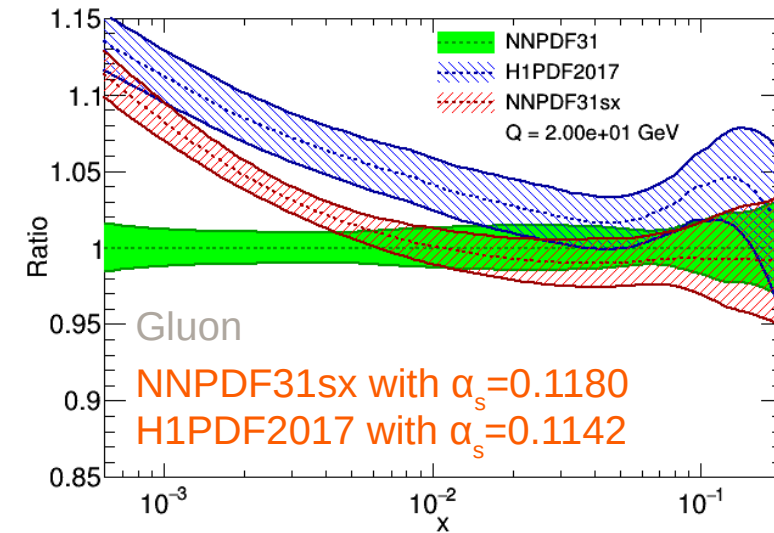
## H1PDF2017

- Gluon at lower x-values tends to be higher (than e.g. NNPDF3.1)
- Gluon very similar to NNPDF3.1sx, which includes low-x resummation (no low- $Q^2$  data included in our H1 fit)

Comparison of H1PDF2017 and NNPDF3.1



Comparison with NNPDF3.1sx

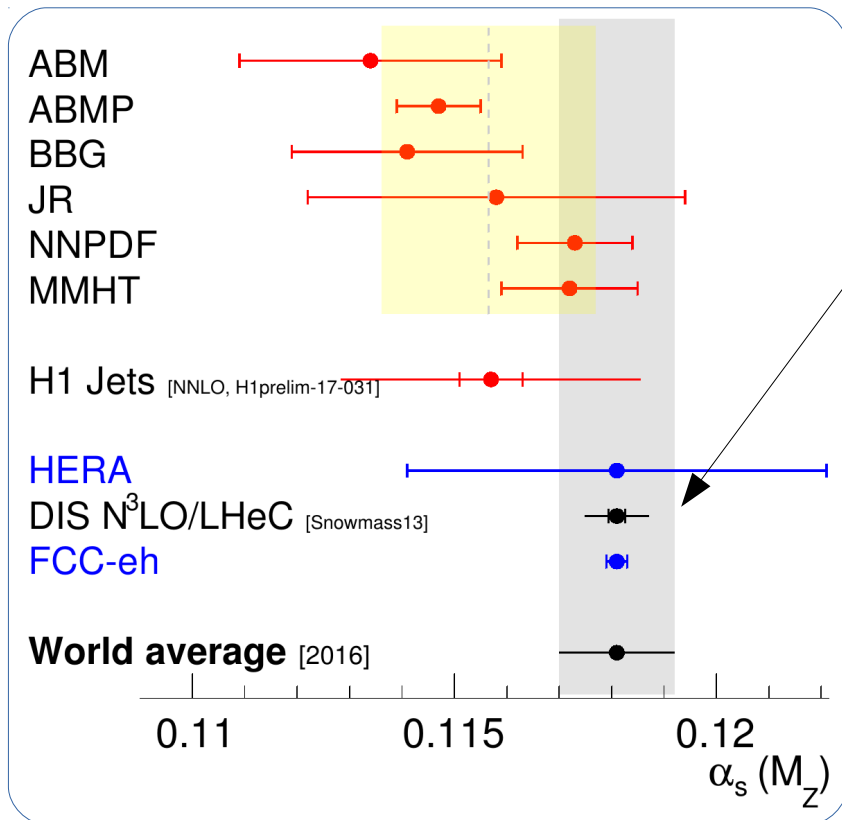


Apfelweb. Thanks to S. Carraza

# Possible future improvements

# Possible future improvements in DIS

Inner errors: exp. only  
Outer errors: exp+theo.



DB, M. Klein, FCC Week '17 Berlin



## New experiments: LHeC, FCC-eh

- incl. DIS:
    - LHeC: 0.1-0.2% (exp) + 0.4%(N3LO)
    - FCC-eh: 0.1% (exp)
  - jets, jet shapes, ... O(‰)
- LHeC Study Group [arXiv:1206.2913]  
See also: M. Klein in 'Memory of G. Altarelli', Appendix 1 [arXiv:1802.04317]

## Old experiments (HERA)

- H1&ZEUS combination: w.i.p.
- Jets in photoproduction
- Jet shapes
  - Analysis of HERA-II data (H1,ZEUS)
  - new jet-shape observables
- $F_2$  measurement at high-x-low- $Q^2$

## Theory: present

- Incl. DIS: N3LO
- Event shapes: N3LO (J Currie et al, JHEP 1805 (2018) 209)
- Event shapes: N3LL (Kang et al. PoS DIS2015 (2015) 142)
- NNLO+PS (Hoeche et al. Phys.Rev. D98 (2018), 114013)

## Theory: future

- N3LO DGLAP (4-loop) (A Vogt et al., PoS LL2018 (2018) 050)
- three-jets in NNLO
- Jets: NNLO + approx.

# Summary

## **All H1 jet data confronted with NNLO predictions**

- NNLO provides improved description w.r.t. NLO
- Quantitative comparison of all data
- NNLO predictions studied in great detail

## **NNLO used for determination of $\alpha_s(m_Z)$**

- $\alpha_s$ -fit  $\alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (27)_{\text{scale}}$
- $\alpha_s$ +PDF-fit  $\alpha_s(m_Z) = 0.1142 (11)_{\text{exp,had,PDF}} (2)_{\text{mod}} (2)_{\text{par}} (26)_{\text{scale}}$
- High experimental and theoretical precision

## **NNLO predictions for jets are used for $\alpha_s$ (and PDF) fits for the first time**

- Successful determination of gluon-density and  $\alpha_s(m_Z)$  simultaneously
- Competitive precision for  $\alpha_s(m_Z)$  and PDFs
- H1PDF2017 available at LHAPDF

## **Fruitful collaboration of theoreticians and experimentalists (H1 & NNLOJET)**



# Scale dependence of NNLO cross sections

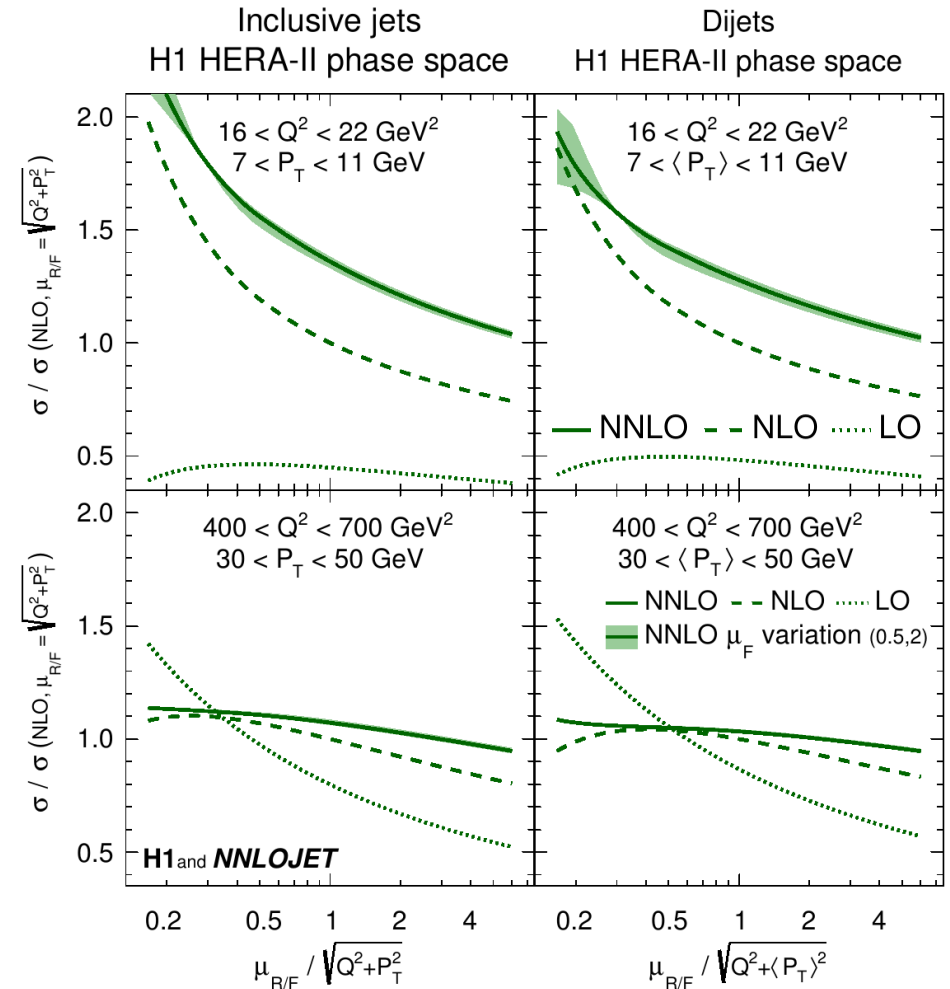
## Simultaneous variation of $\mu_R$ and $\mu_F$

### At lower scales

- Significant NNLO k-factors
- NNLO with reduced scale dependence
- Inclusive jets with higher scale dependence than dijets

### At higher scales

- NNLO with reduced scale dependence
- $\mu_F$  dependence very small





# $\alpha_s(m_Z)$ dependence of cross sections

**Jet cross sections directly sensitive to  $\alpha_s$**

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}_{i,k}^{(n)}(x, \mu_R, \mu_F) \cdot C_{\text{had}}$$

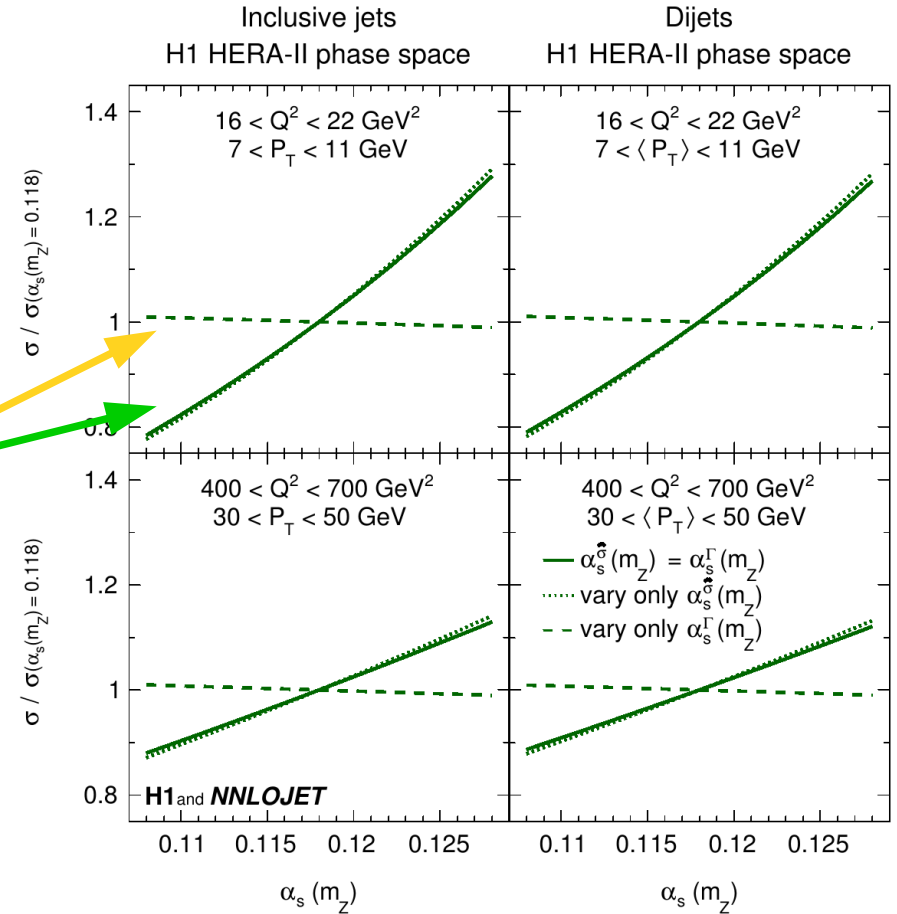
**Two  $\alpha_s$ -dependencies**

PDFs  $\frac{\partial f}{\partial \alpha_s} = \frac{\mathcal{P} \otimes f}{\beta}$

Hard ME's

$$\hat{\sigma}_{i,k}^{(n)} = \alpha_s^n(\mu_R) \tilde{\sigma}_{i,k}^{(n)}(x, \mu_R, \mu_F)$$

- Predominant  $\alpha_s$ -sensitivity from ME's
- PDF's with almost negligible sensitivity



# $\alpha_s$ dependencies separately fitted

## Fits to

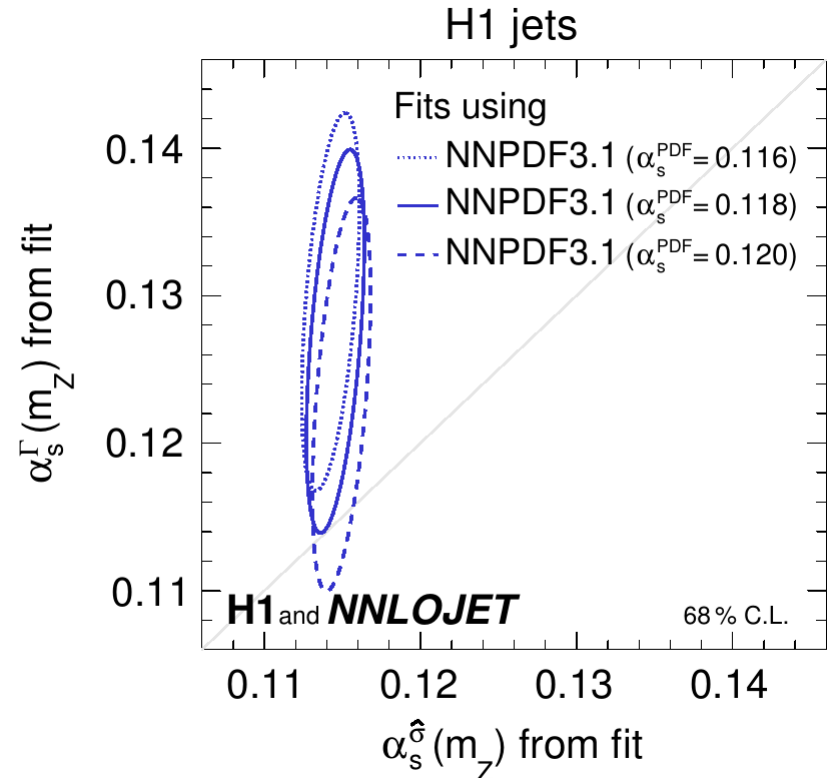
- Inclusive jet and dijet data fitted together
- Fits performed for different PDFs

## Fits with two free $\alpha_s$ parameters

$$\sigma_i = f(\alpha_s^f(m_Z)) \otimes \hat{\sigma}_k(\alpha_s^{\hat{\sigma}}(m_Z)) \cdot C_{\text{had}}$$

## Results

- Most sensitivity arises from **matrix elements**
- Best-fit  $\alpha_s$ -values in **PDF's** and **ME's** are consistent
- Anti-correlation between  $\alpha_s^{\text{PDF}}(m_Z)$  and  $\alpha_s^{\Gamma}(m_Z)$



# Summary of all $\alpha_s$ results

$\alpha_s(m_Z)$  values from H1 jet cross sections

Data	$\tilde{\mu}_{cut}$	$\alpha_s(m_Z)$ with uncertainties	th	tot	$\chi^2/n_{dof}$
<b>Inclusive jets</b>					
300 GeV high- $Q^2$	$2m_b$	0.1221 (31) <sub>exp</sub> (22) <sub>had</sub> (5) <sub>PDF</sub> (3) <sub>PDF<math>\alpha_s</math></sub> (4) <sub>PDFset</sub> (36) <sub>scale</sub>	(43) <sub>th</sub>	(53) <sub>tot</sub>	6.5/15
HERA-I low- $Q^2$	$2m_b$	0.1093 (17) <sub>exp</sub> (8) <sub>had</sub> (5) <sub>PDF</sub> (5) <sub>PDF<math>\alpha_s</math></sub> (7) <sub>PDFset</sub> (33) <sub>scale</sub>	(35) <sub>th</sub>	(39) <sub>tot</sub>	17.5/22
HERA-I high- $Q^2$	$2m_b$	0.1136 (24) <sub>exp</sub> (9) <sub>had</sub> (6) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (4) <sub>PDFset</sub> (31) <sub>scale</sub>	(33) <sub>th</sub>	(41) <sub>tot</sub>	14.7/23
HERA-II low- $Q^2$	$2m_b$	0.1187 (18) <sub>exp</sub> (8) <sub>had</sub> (4) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (45) <sub>scale</sub>	(46) <sub>th</sub>	(50) <sub>tot</sub>	29.6/40
HERA-II high- $Q^2$	$2m_b$	0.1121 (18) <sub>exp</sub> (9) <sub>had</sub> (5) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (2) <sub>PDFset</sub> (35) <sub>scale</sub>	(37) <sub>th</sub>	(41) <sub>tot</sub>	42.5/29
<b>Dijets</b>					
300 GeV high- $Q^2$	$2m_b$	0.1213 (39) <sub>exp</sub> (17) <sub>had</sub> (5) <sub>PDF</sub> (2) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (31) <sub>scale</sub>	(35) <sub>th</sub>	(52) <sub>tot</sub>	13.6/15
HERA-I low- $Q^2$	$2m_b$	0.1101 (23) <sub>exp</sub> (8) <sub>had</sub> (5) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (5) <sub>PDFset</sub> (36) <sub>scale</sub>	(38) <sub>th</sub>	(45) <sub>tot</sub>	10.4/20
HERA-II low- $Q^2$	$2m_b$	0.1173 (14) <sub>exp</sub> (9) <sub>had</sub> (5) <sub>PDF</sub> (5) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (44) <sub>scale</sub>	(45) <sub>th</sub>	(47) <sub>tot</sub>	17.4/41
HERA-II high- $Q^2$	$2m_b$	0.1089 (21) <sub>exp</sub> (7) <sub>had</sub> (5) <sub>PDF</sub> (3) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (25) <sub>scale</sub>	(27) <sub>th</sub>	(34) <sub>tot</sub>	28.0/23
H1 inclusive jets	$2m_b$	0.1132 (10) <sub>exp</sub> (5) <sub>had</sub> (4) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (2) <sub>PDFset</sub> (40) <sub>scale</sub>	(40) <sub>th</sub>	(42) <sub>tot</sub>	134.0/133
H1 inclusive jets	28 GeV	0.1152 (20) <sub>exp</sub> (6) <sub>had</sub> (2) <sub>PDF</sub> (2) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (26) <sub>scale</sub>	(27) <sub>th</sub>	(33) <sub>tot</sub>	44.1/60
H1 dijets	$2m_b$	0.1148 (11) <sub>exp</sub> (6) <sub>had</sub> (5) <sub>PDF</sub> (4) <sub>PDF<math>\alpha_s</math></sub> (4) <sub>PDFset</sub> (40) <sub>scale</sub>	(41) <sub>th</sub>	(42) <sub>tot</sub>	93.9/102
H1 dijets	28 GeV	0.1147 (24) <sub>exp</sub> (5) <sub>had</sub> (3) <sub>PDF</sub> (2) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (24) <sub>scale</sub>	(25) <sub>th</sub>	(35) <sub>tot</sub>	30.8/43
H1 jets	$2m_b$	0.1143 (9) <sub>exp</sub> (6) <sub>had</sub> (5) <sub>PDF</sub> (5) <sub>PDF<math>\alpha_s</math></sub> (4) <sub>PDFset</sub> (42) <sub>scale</sub>	(43) <sub>th</sub>	(44) <sub>tot</sub>	195.0/199
H1 jets	28 GeV	0.1157 (20) <sub>exp</sub> (6) <sub>had</sub> (3) <sub>PDF</sub> (2) <sub>PDF<math>\alpha_s</math></sub> (3) <sub>PDFset</sub> (27) <sub>scale</sub>	(28) <sub>th</sub>	(34) <sub>tot</sub>	63.2/90
H1 jets	42 GeV	0.1168 (22) <sub>exp</sub> (7) <sub>had</sub> (2) <sub>PDF</sub> (2) <sub>PDF<math>\alpha_s</math></sub> (5) <sub>PDFset</sub> (17) <sub>scale</sub>	(20) <sub>th</sub>	(30) <sub>tot</sub>	37.6/40
H1PDF2017 [NNLO]	$2m_b$	0.1142 (11) <sub>exp,NP,PDF</sub> (2) <sub>mod</sub> (2) <sub>par</sub> (26) <sub>scale</sub>		(28) <sub>tot</sub>	1539.7/1516

### Running of the strong coupling

$\mu_R$ [GeV]	Inclusive jets		Dijets		H1 jets	
	$\alpha_s(m_Z)$	$\alpha_s(\mu_R)$	$\alpha_s(m_Z)$	$\alpha_s(\mu_R)$	$\alpha_s(m_Z)$	$\alpha_s(\mu_R)$
7.4	0.1148 (13) (42)	0.1830 (34) (114)	0.1182 (28) (41)	0.1923 (77) (116)	0.1147 (13) (43)	0.1829 (34) (114)
10.1	0.1136 (17) (36)	0.1678 (39) (81)	0.1169 (14) (42)	0.1751 (34) (99)	0.1148 (14) (40)	0.1705 (31) (91)
13.3	0.1147 (15) (43)	0.1605 (30) (88)	0.1131 (18) (38)	0.1573 (36) (76)	0.1144 (15) (42)	0.1600 (30) (86)
17.2	0.1130 (15) (33)	0.1492 (26) (59)	0.1104 (19) (30)	0.1445 (33) (53)	0.1127 (15) (33)	0.1486 (27) (59)
20.1	0.1136 (17) (33)	0.1457 (29) (56)	0.1116 (22) (31)	0.1425 (36) (52)	0.1134 (17) (33)	0.1454 (29) (55)
24.5	0.1173 (17) (30)	0.1463 (26) (48)	0.1147 (23) (24)	0.1423 (36) (38)	0.1171 (17) (29)	0.1460 (27) (46)
29.3	0.1084 (36) (29)	0.1287 (51) (41)	0.1163 (34) (34)	0.1401 (50) (50)	0.1134 (30) (32)	0.1358 (44) (46)
36.0	0.1153 (32) (37)	0.1338 (43) (50)	0.1135 (37) (29)	0.1314 (50) (39)	0.1146 (30) (33)	0.1328 (41) (44)
49.0	0.1170 (22) (20)	0.1290 (27) (25)	0.1127 (31) (15)	0.1238 (37) (18)	0.1169 (23) (19)	0.1290 (28) (24)
77.5	0.1111 (55) (19)	0.1137 (58) (20)	0.1074 (84) (19)	0.1099 (88) (20)	0.1113 (55) (19)	0.1139 (58) (20)

Table 5: Values of the strong coupling constant  $\alpha_s(\mu_R)$  and at the  $Z$ -boson mass,  $\alpha_s(m_Z)$ , obtained from fits to groups of data points with comparable values of  $\mu_R$ . The first (second) uncertainty of each point corresponds to the experimental (theory) uncertainty. The theory uncertainties include PDF related uncertainties and the dominating scale uncertainty.

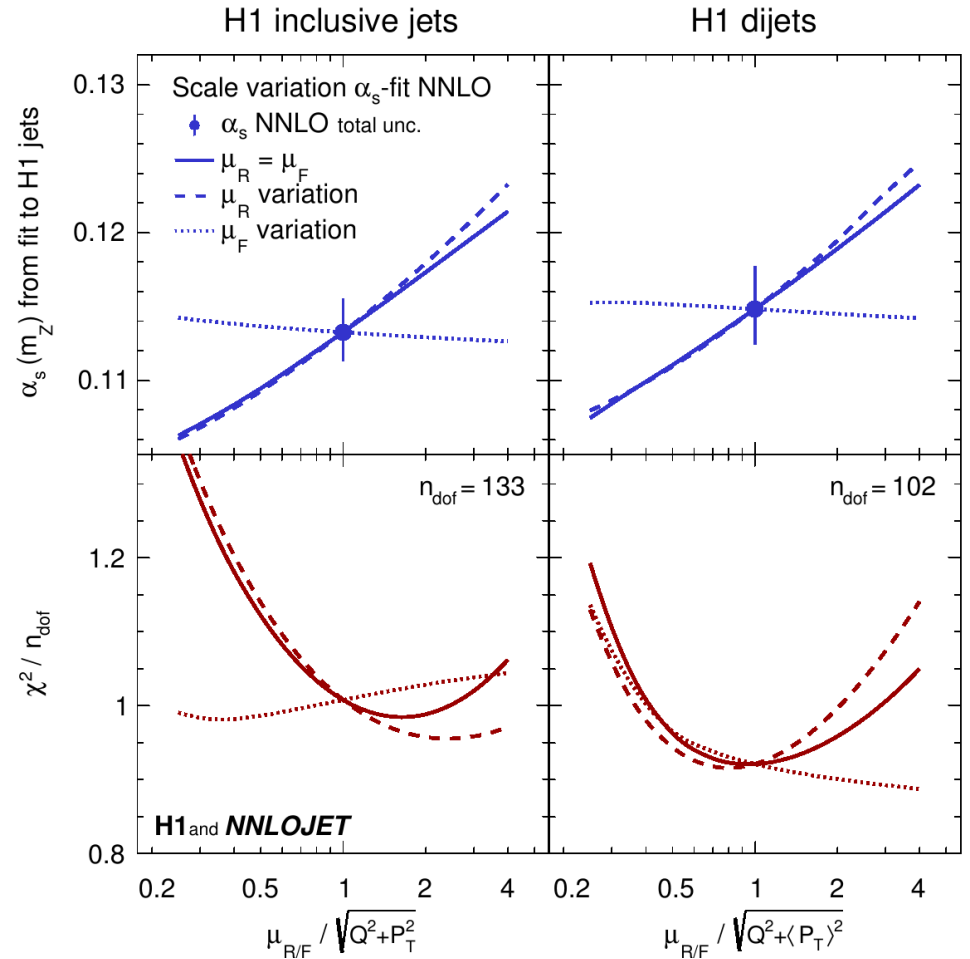
# Scale dependence of $\alpha_s$ fit

## $\alpha_s$ results as a function of scale factors

- Smooth results for studied scale variations
- $\mu_R$  variation with more impact than  $\mu_F$

## $\chi^2$ values

- somewhat a 'technical parameter'  
-> not intended to be a parabolas
- $\chi^2$  values increase for large scale factors  
-> large scale factors disvafoered



# Scale choice for $\alpha_s$ fit

## Study scales calculated from $Q^2$ and $p_T$

' $p_T$ ' refers to:  $p_{T}^{\text{jet}}$  or  $\langle p_T \rangle$

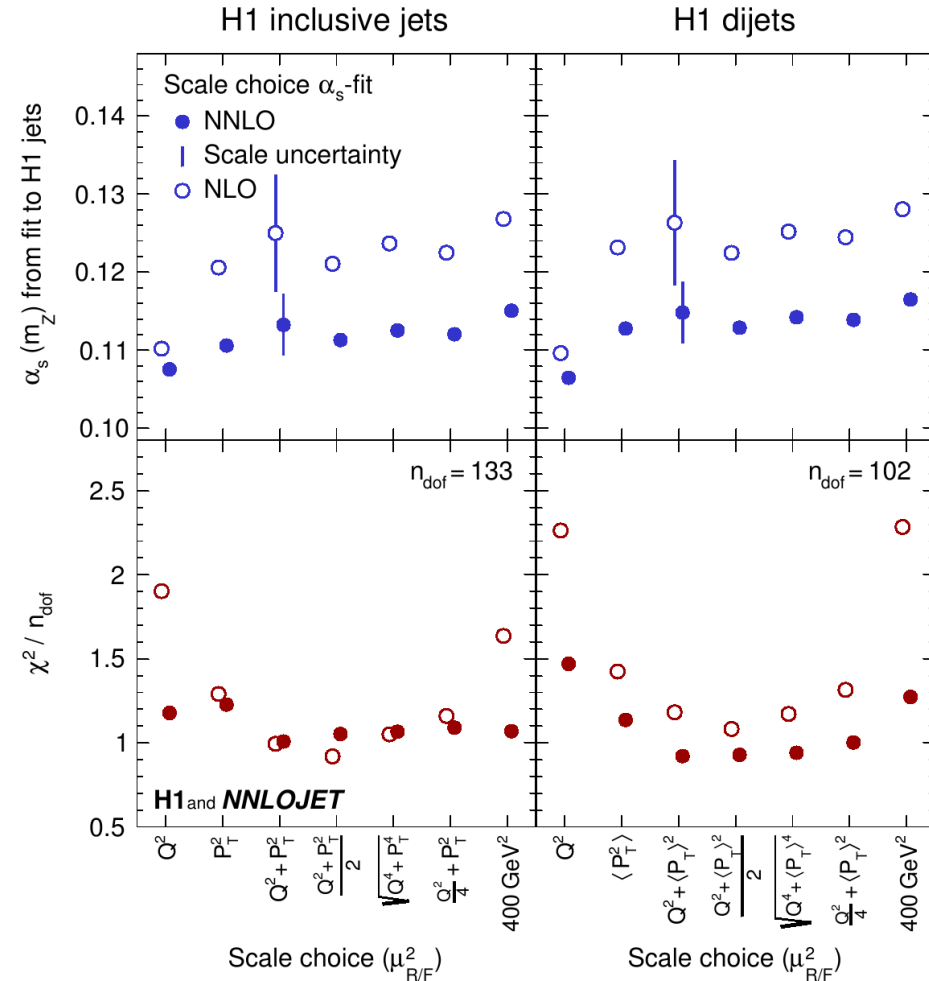
## $\alpha_s$ results and $\chi^2$ values

- Spread of results covered by scale uncertainty
- $\chi^2$  values are similar for different choices  
-> NNLO with small 'scale dependence'

## NLO matrix elements

- Large scale uncertainty
- Relevant dependence of result on scale choice
- Mainly larger  $\chi^2$  values than NNLO
- Larger fluctuation of  $\chi^2$  values than NNLO

NNLO with reduced scale dependence



# Dependence on the PDF

**PDF is an external input to NNLO calculation**

**PDF fitting groups differ**

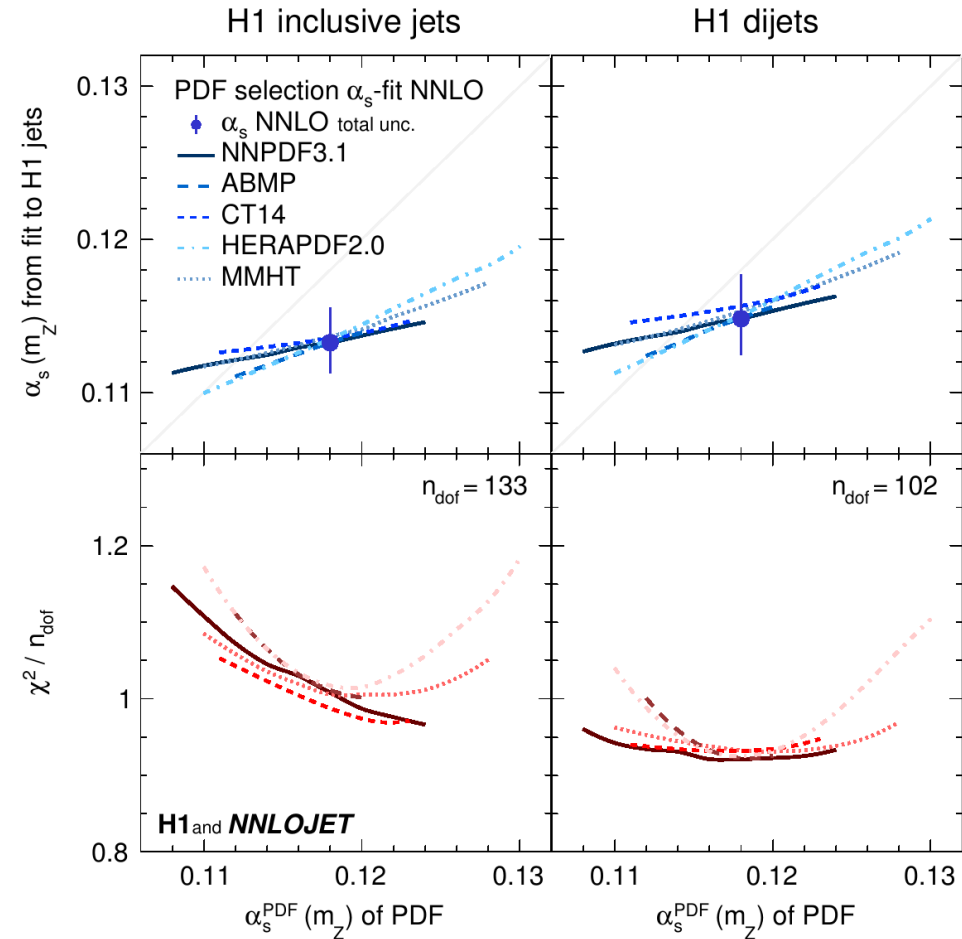
- choice of input data sets, PDF parameterisations, model parameters, fit methodology, etc...
- Though: different PDFs appear to be quite consistent

**Choice of  $\alpha_s$  for PDF determination**

- $\alpha_s^{\text{PDF}}(m_Z)$  important input parameter to PDF fit
- Small correlation with fitted results

**Our (main)  $\alpha_s$  result**

- almost independent on PDF assumptions



# Scale dependence

## Test running of strong coupling

- Perform fits to groups of data points at similar scale
- Assume running to be valid within the limited range covered by interval

## Results

- All fits have good  $\chi^2$
- Consistency of inclusive jets and dijets
- Consistency with expectation at all scales
- Scale uncertainty dominates at lower  $\mu$

