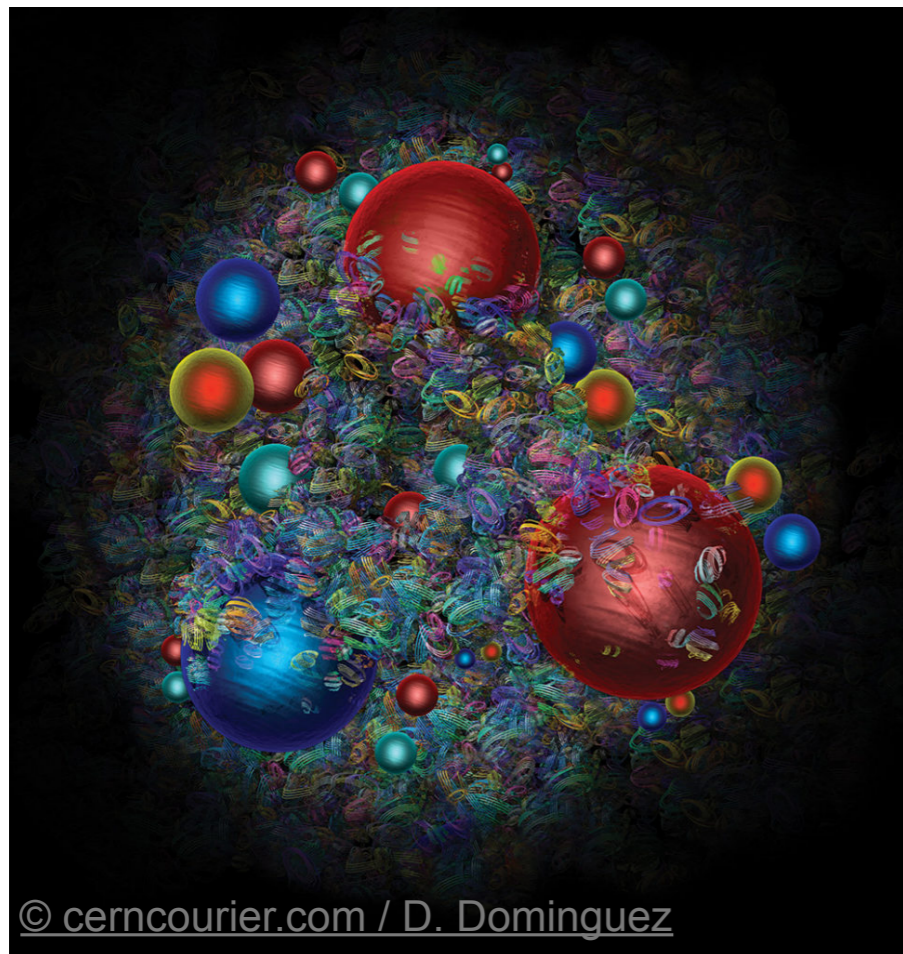


# Determination of $\alpha_S$ at NNLO QCD Using H1 Jet Cross Section Measurements



- Motivation – QCD,  $\alpha_S$  and the RGE
- H1 jet cross section measurements in DIS
- Methodology
- Extraction of the strong coupling constant  $\alpha_S$   
[Eur.Phys.J.C 77 \(2017\), 791](#)



EPS 2019  
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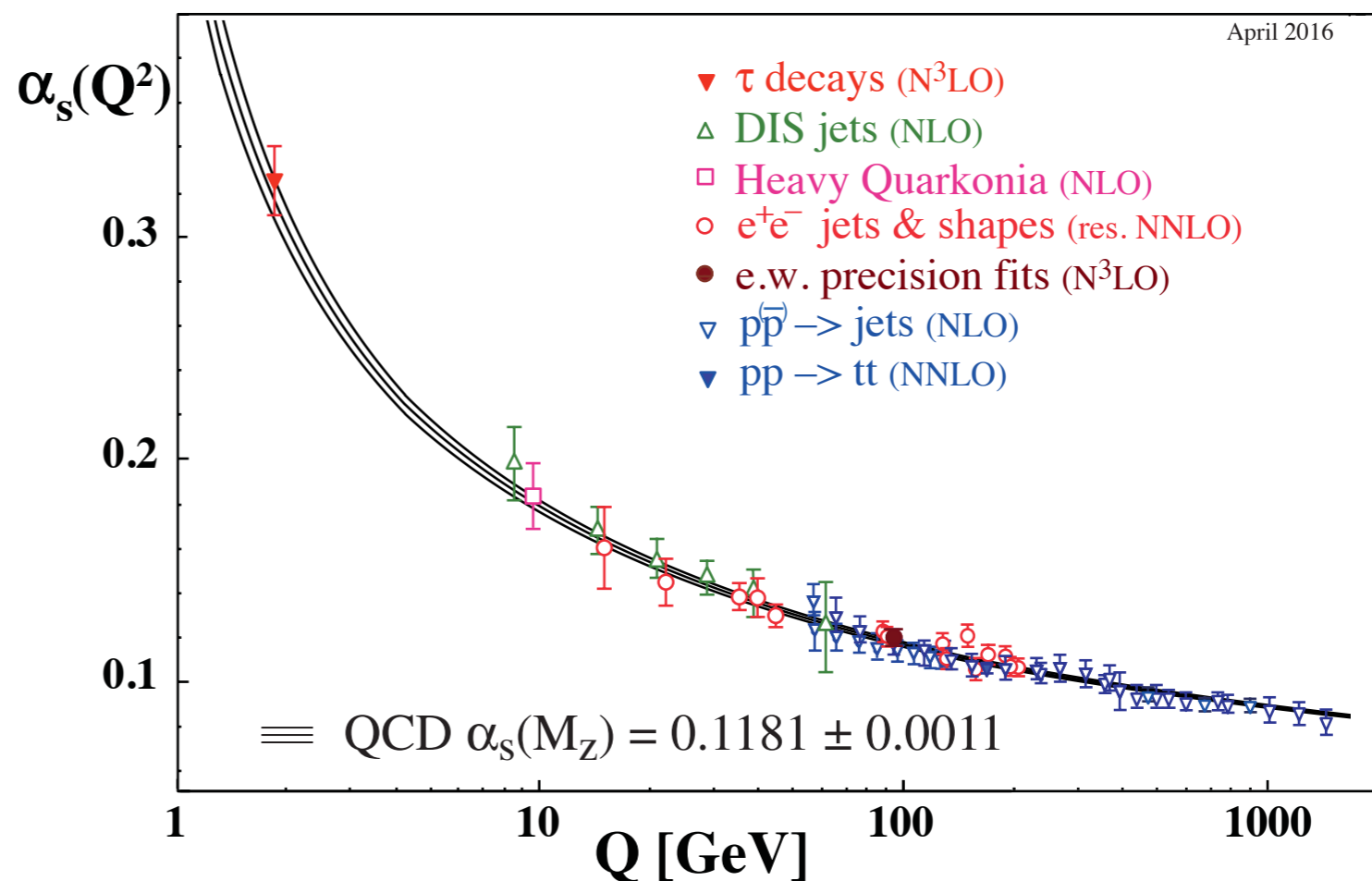
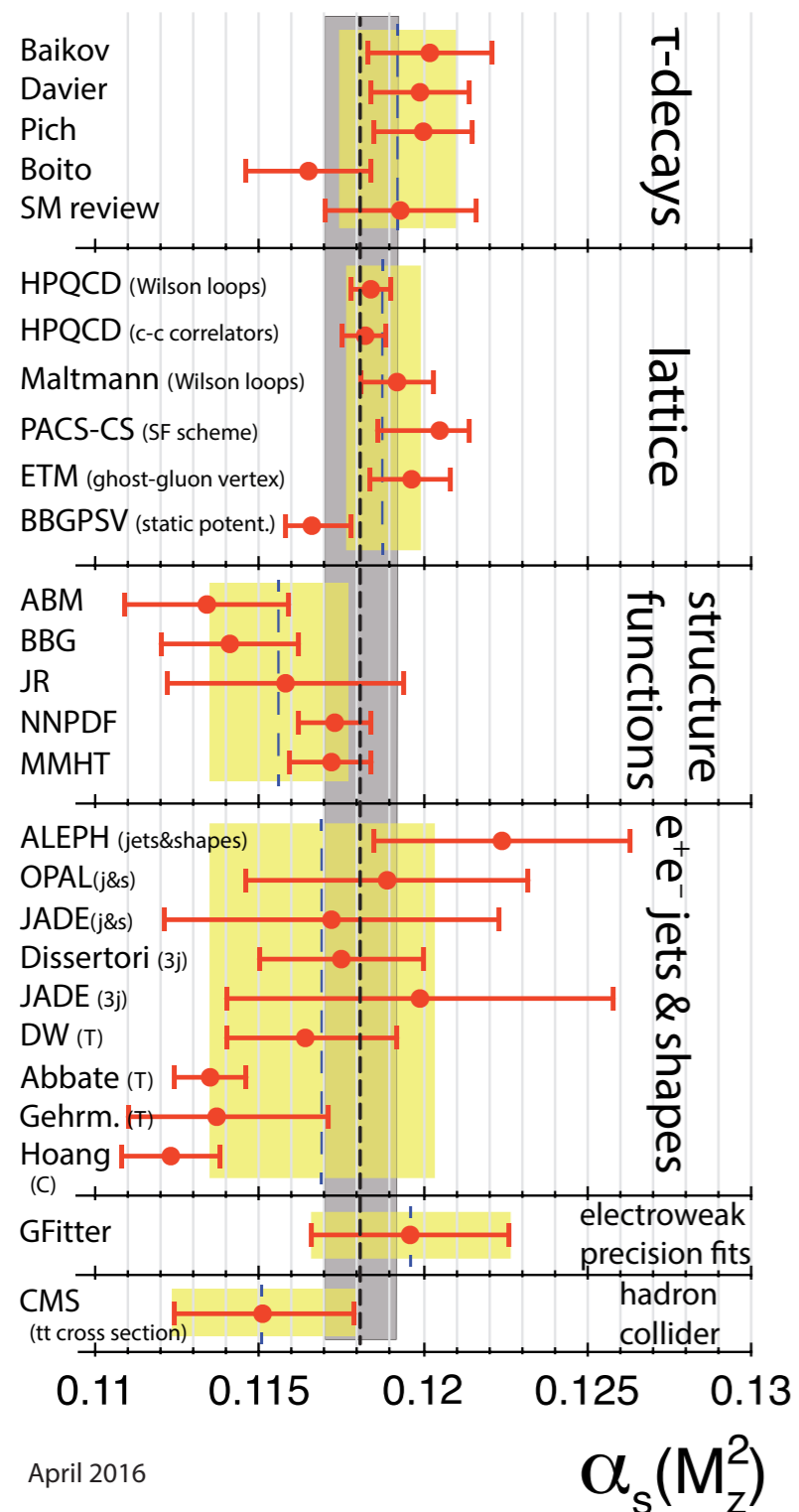


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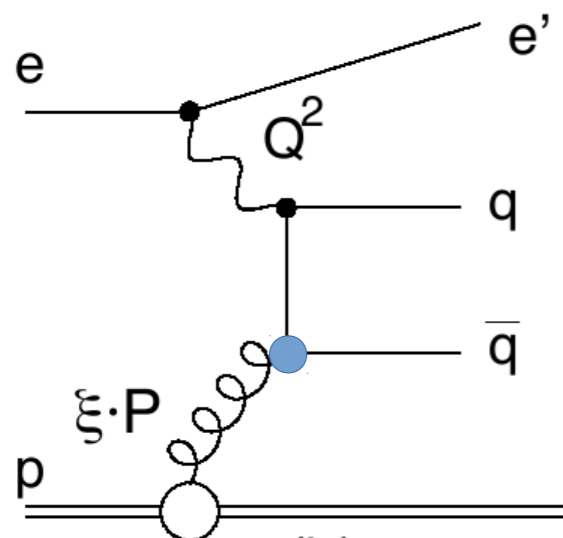


- Quantum Chromodynamics (QCD) is the theory of the strong interaction
- Renormalization group equation (RGE) encodes the dependence of the coupling parameter on the energy scale  $\mu$  (=running)
- Values of  $\alpha_s(\mu)$  are not predicted

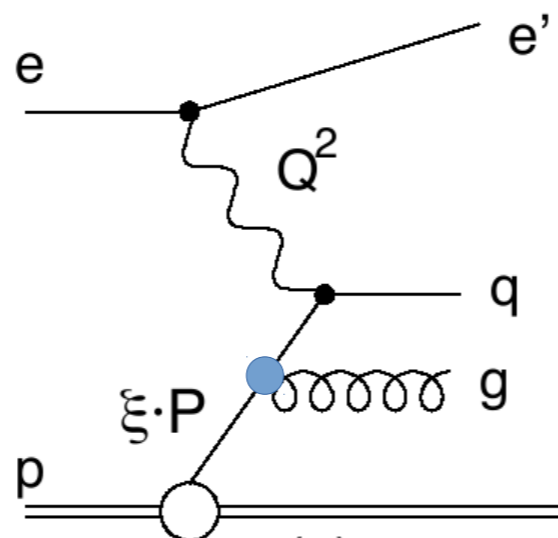


QCD Tests probe two aspects:

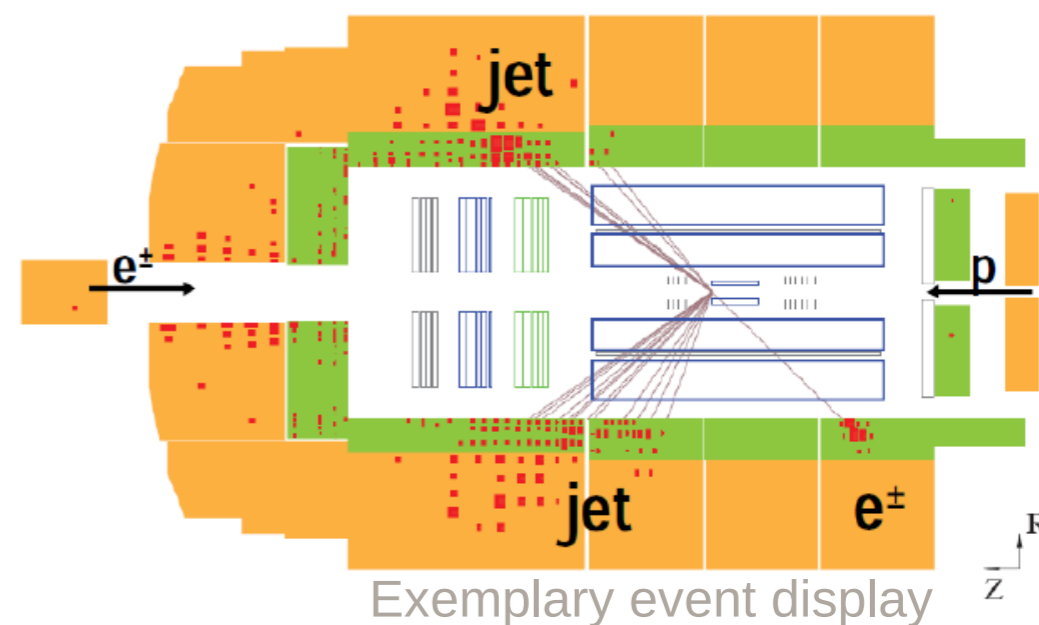
- determination of the value at some fixed scale  $\alpha_s(\mu = M_Z)$
- scale dependence of  $\alpha_s(\mu)$



Boson-gluon fusion



QCD Compton



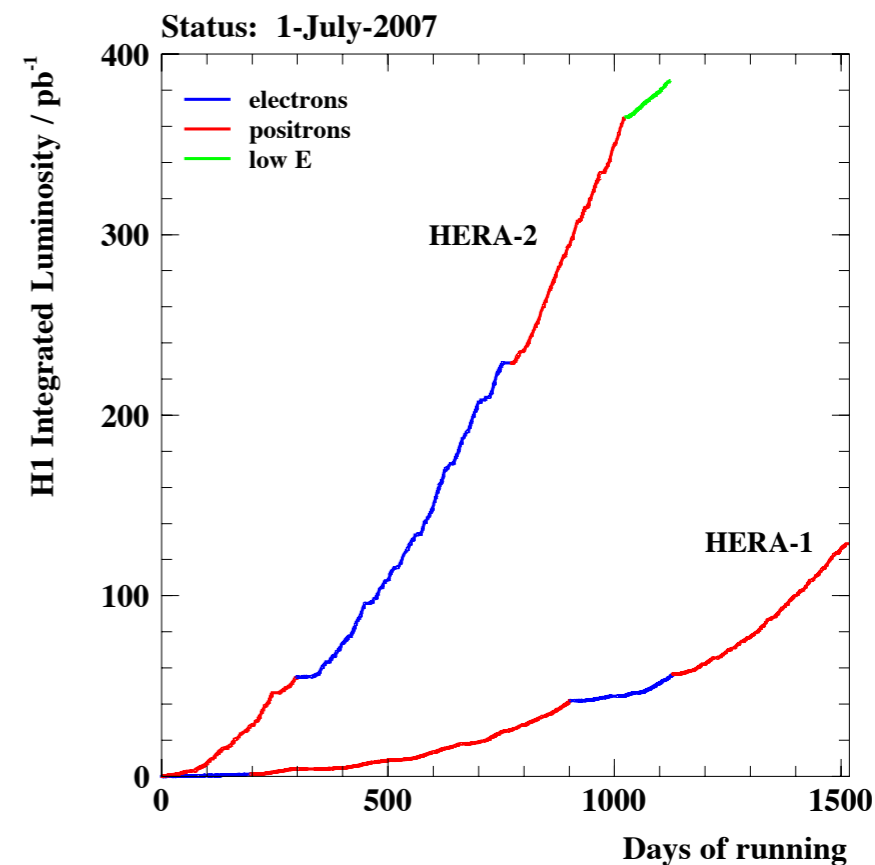
- Dependence of  $\alpha_s$  in matrix element and in PDFs
- Dominant sensitivity is from matrix element

### HERA-I operation 1993-2000

$E_e = 27.6$  GeV  
 $E_p = 820 / 920$  GeV  
 $\sqrt{s} = 301$  GeV &  $\sqrt{s} = 318$  GeV  
 $\int \mathcal{L} \sim 110$  pb<sup>-1</sup> per experiment

### HERA-II operation 2003-2007

$E_e = 27.6$  GeV  
 $E_p = 920$  GeV  
 $\sqrt{s} = 318$  GeV  
 $\int \mathcal{L} \sim 330$  pb<sup>-1</sup> per experiment  
 Longitudinally polarised leptons





## Inclusive jet cross sections

- $d\sigma/dQ^2 dP_{T,jet}$
- HERA-I & HERA-II
- low- $Q^2$  ( $<100 \text{ GeV}^2$ ) and
- high- $Q^2$  ( $>150 \text{ GeV}^2$ ) regions

## Selections

- kt-algorithm,  $R=1$
- $-1.0 < \eta_{lab} < 2.5$
- $P_T$  ranges from 4.5 to 50 GeV (Breit frame)

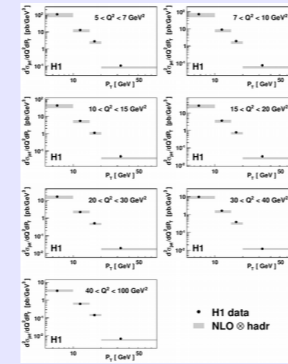
## Dijet cross sections

- $d\sigma/dQ^2 d\langle P_T \rangle$
- HERA-I & HERA-II
- low- $Q^2$  ( $<100 \text{ GeV}^2$ ) and
- high- $Q^2$  ( $>150 \text{ GeV}^2$ ) regions

## Selections

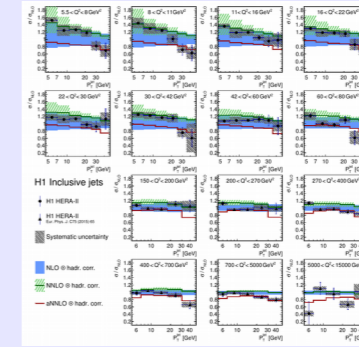
- $\langle P_T \rangle$  greater than 5, 7, or 8.5 GeV (Breit frame)
- $P_{T,jet}$  greater than 4, 5, 7 GeV
- asymmetric jet  $P_T$  cuts
- $M_{12}$  cut applied in two cases
- All data sets used in  $\alpha_s$  extractions
- Well understood data & exptl. uncertainties
- New NNLO theory now available

## HERA-I low- $Q^2$



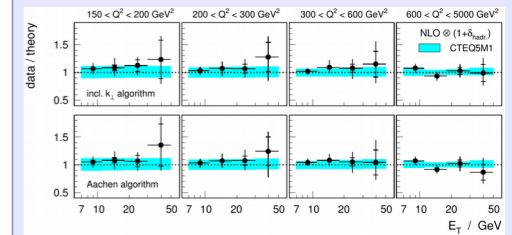
*Eur.Phys.J.C67 (2010) 1*

## HERA-II low- $Q^2$



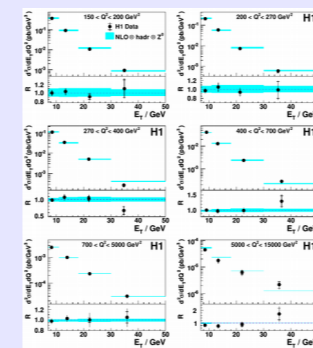
*arXiv:1611.03421*

## 300 GeV high- $Q^2$



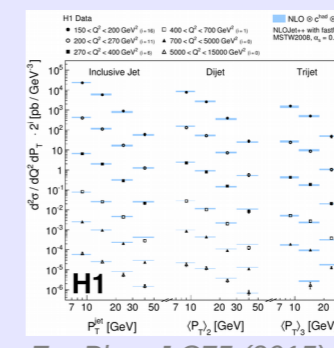
*Eur.Phys.J.C19 (2001) 289*

## HERA-I high- $Q^2$



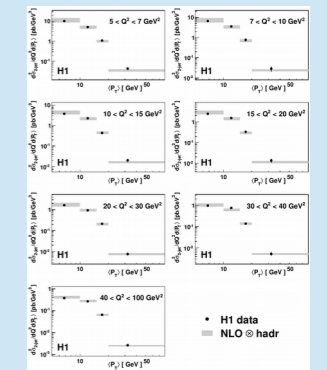
*Phys.Lett.B653 (2007) 134*

## HERA-II high- $Q^2$



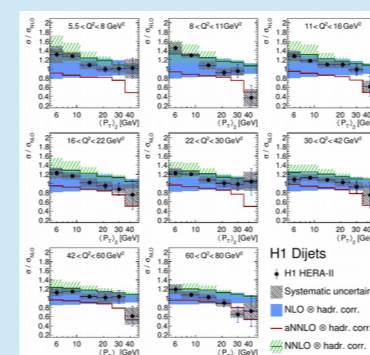
*Eur.Phys.J.C75 (2015) 2*  
*arXiv:1611.03421*

## HERA-I low- $Q^2$



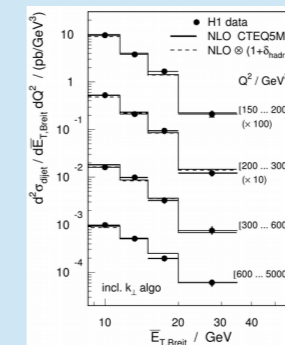
*Eur.Phys.J.C67 (2010) 1*

## HERA-II low- $Q^2$



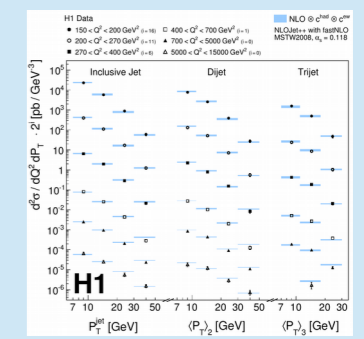
*Eur.Phys.J. C77 (2017) 215*

## 300 GeV high- $Q^2$

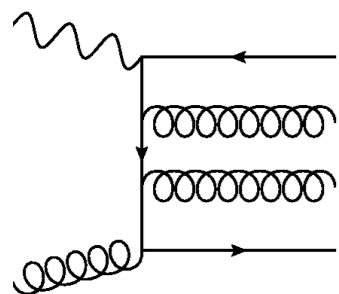


*Eur.Phys.J.C19 (2001) 289*

## HERA-II high- $Q^2$



*Eur.Phys.J.C75 (2015) 2*

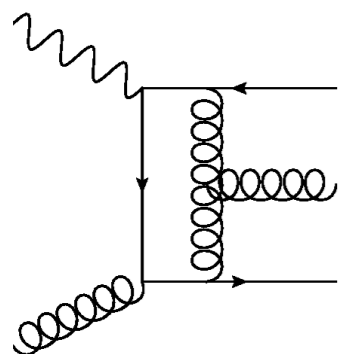


Double-real

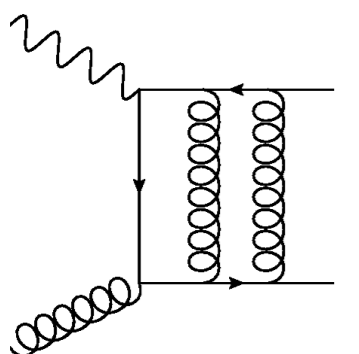
$$d\sigma_{NNLO}^{RR,S} \approx \underbrace{X(\{p_X\})}_{\text{antenna}} \overbrace{d\Phi_3(\{p_X\})}^{\text{Antenna PS}} \times \underbrace{|\mathcal{M}(\{\tilde{p}_m\})|^2}_{\text{reduced ME}} \overbrace{d\Phi_m(\{\tilde{p}_m\})}^{\text{reduced PS}} \times \underbrace{\mathcal{J}(\{\tilde{p}_m\})}_{\text{jet function}}$$

### Antenna subtraction

- Cancellation of IR divergences with local subtraction terms
- Construction of (local) counter terms
- Move IR divergences across different phase space multiplicities



Real-virtual



Double-virtual

$$\text{Take } \mu_R = \mu_F = \sqrt{Q^2 + P_T^2}$$

simple form & non-zero as  $Q$  or  $P_T \rightarrow 0$

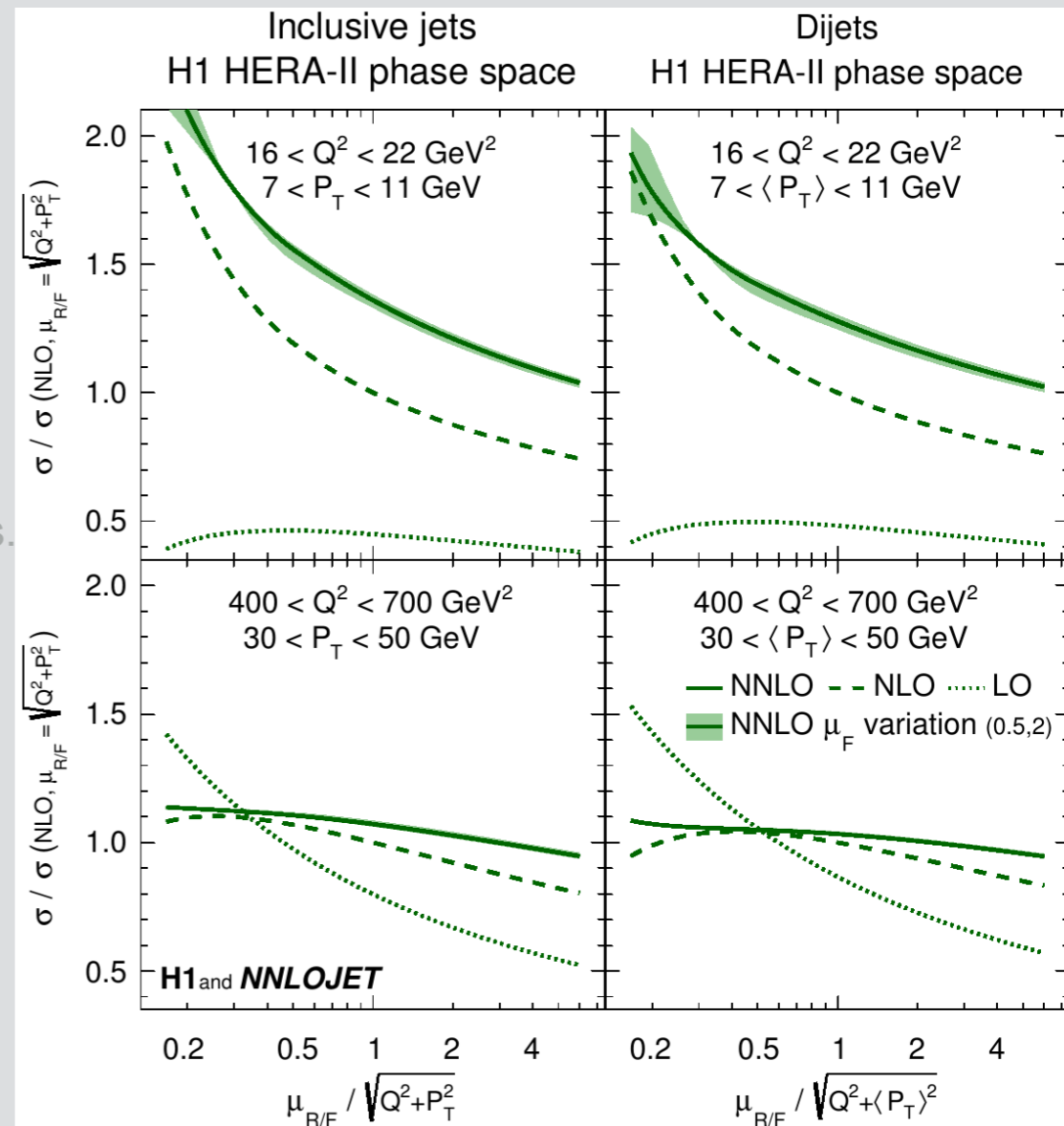
### Simultaneous variation of $\mu_R$ and $\mu_F$

At low scales:

- Significant NNLO k-factors
- Incl. jets with higher scale dependence than dijets

At higher scales

- NNLO with reduced scale dependence
- $\mu_F$  dependence very small



$\alpha_s$  determined in  $\chi^2$  minimisation method

Take experimental & theoretical uncertainties into account

$$\chi^2 = \sum_i \sum_j (\log \zeta_i - \log \sigma_i) (V_{\text{exp}} + V_{\text{had}} + V_{\text{PDF}})^{-1}_{ij} (\log \zeta_j - \log \sigma_j)$$

NNLO theory has  $\alpha_s$  dependence in PDFs and hard M.E.  
PDF piece accounted for in DGLAP evolution using  $\mu_F = 20$  GeV  
 $\mu_F = 20$  GeV is typical  $\mu_F$  of data

$\sigma$  = predictions

$\zeta$  = measurements

$V$  = Covariance matrices  $\rightarrow$  rel unc.

- $V_{\text{had}}$  = hadronisation corrections
- $V_{\text{PDF}}$  = PDFs
- $V_{\text{exp}}$  = Experimental uncertainties

Perform fit to all H1 data:

- All inclusive jet data sets (137 data points)
- All dijet data sets (103 data points)
- All H1 jet data taken together (denoted as 'H1 jets')  
(exclude HERA-I dijet data as correlations to inclusive jets are not known)

All H1 Inclusive jets & Dijets data compared to NNLO predictions

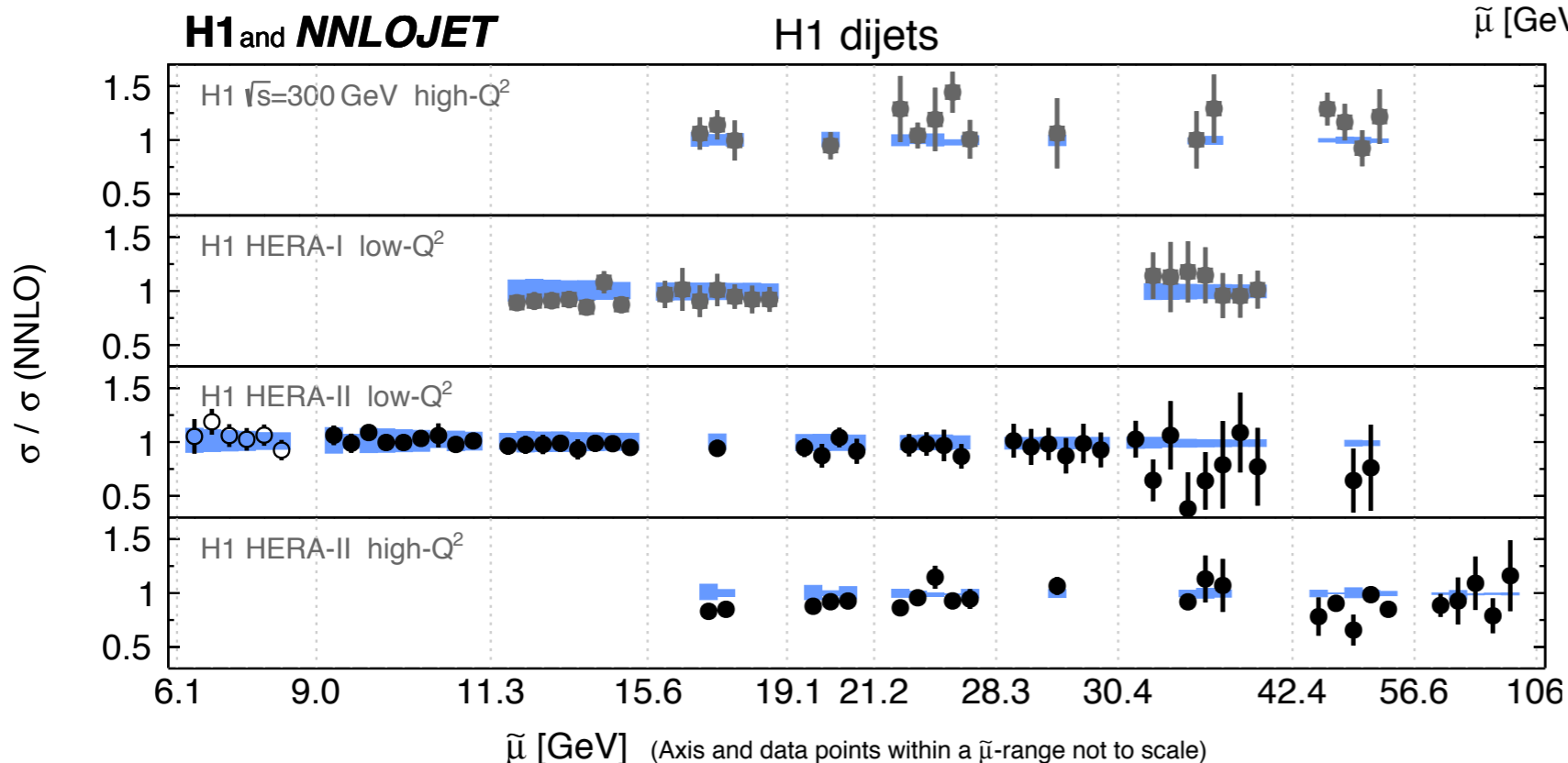
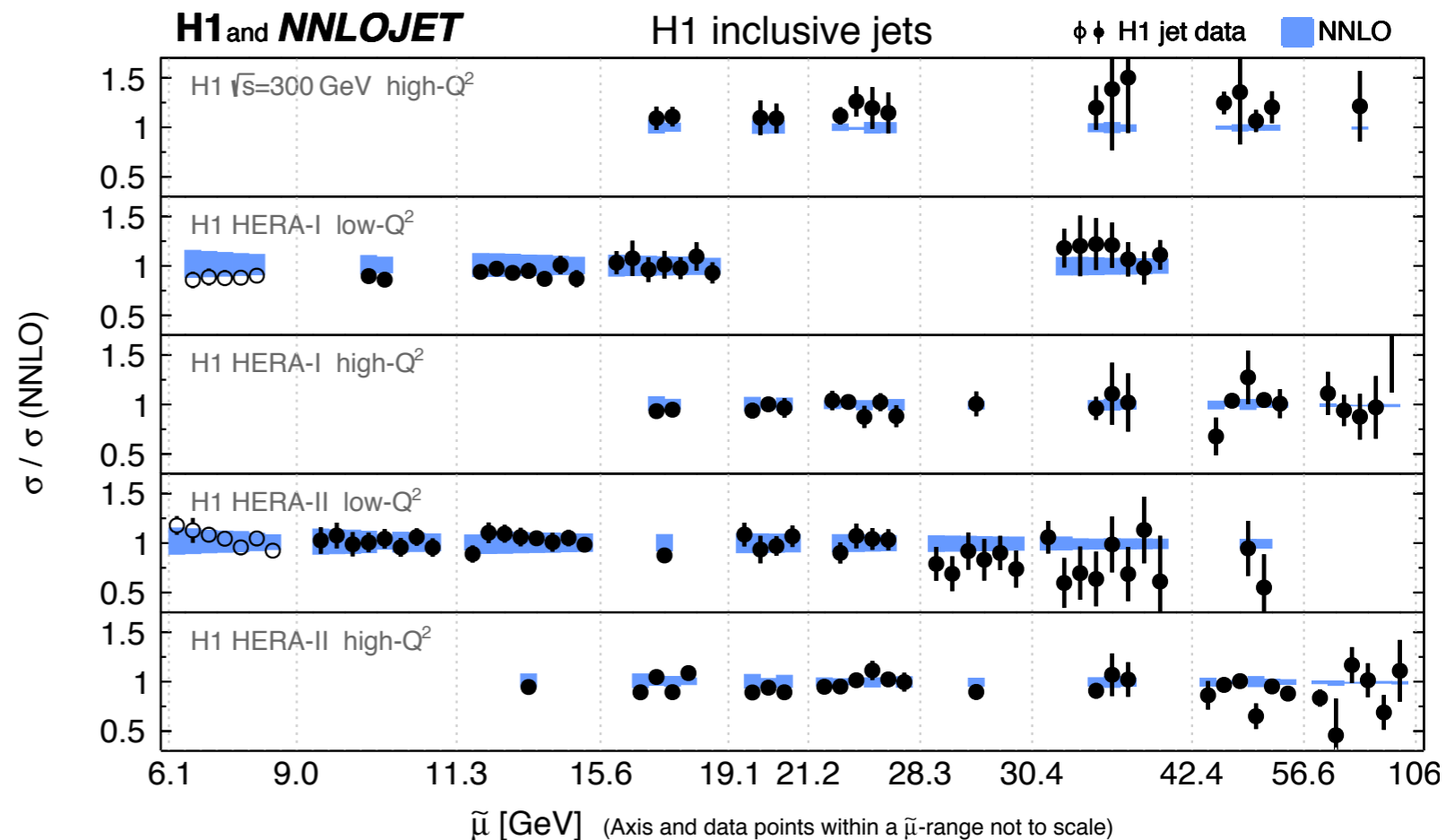
Define for each measurement bin

$$\tilde{\mu} = \sqrt{Q_{ave}^2 + P_{T,ave}^2}$$

Average of upper and lower bin edge

Data with  $\tilde{\mu} > 28$  GeV :

- independent of  $\alpha_s$  used in PDF
- reduced scale dependence



- NNLO describes data very well
- Also justified by good  $\chi^2$  values
- Great success of pQCD

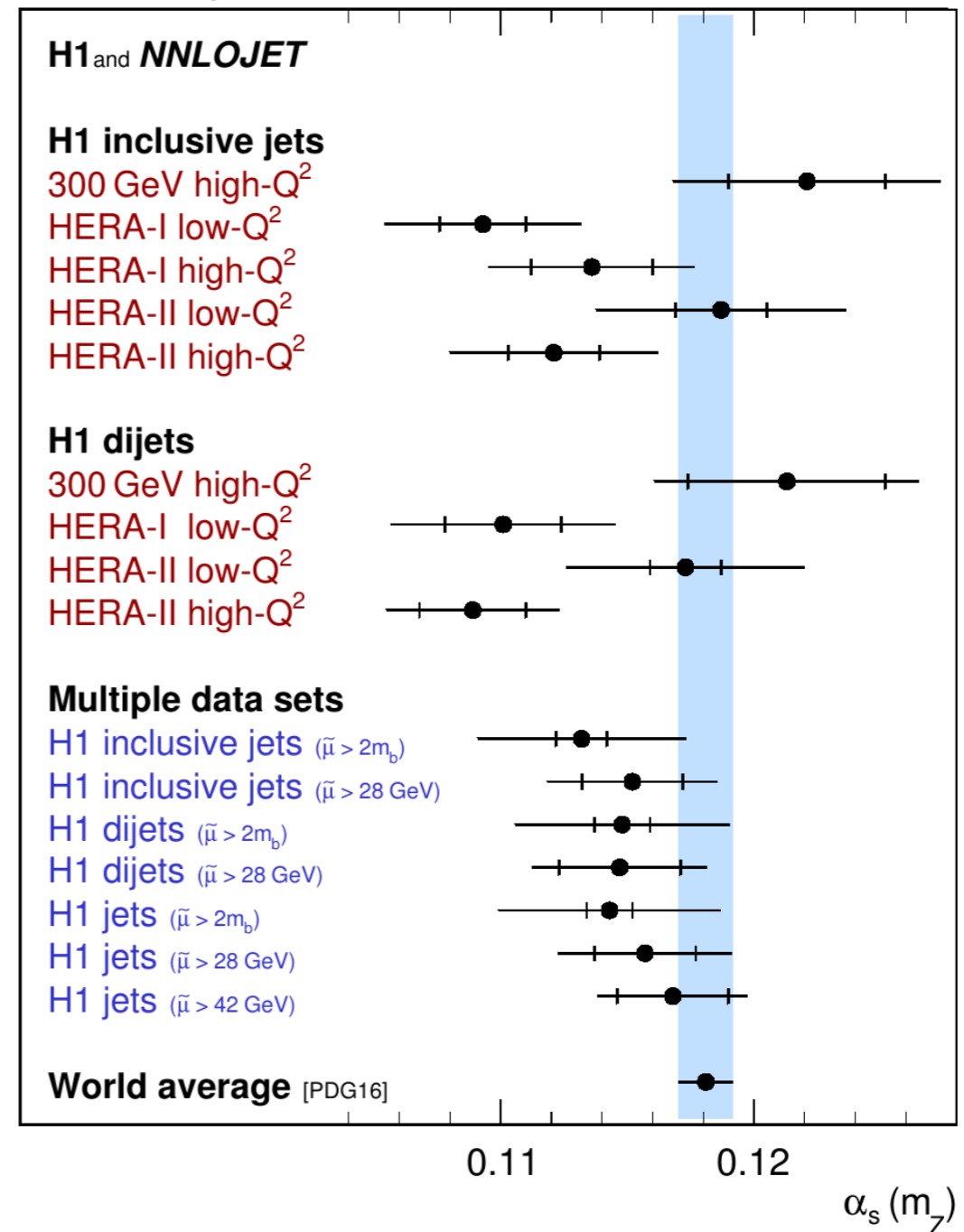
## $\alpha_s$ from individual data sets

- High experimental precision
- Scale uncertainty is largest error
- All fits have good  $\chi^2 \rightarrow$  consistency of data

## Combined $\alpha_s$ from all data sets

- Inclusive & dijet data
- $\tilde{\mu} > 28$  GeV  $\rightarrow$  91 data points
- All fits have good  $\chi^2 \rightarrow$  consistency of data
- Moderate exptl. precision
- Dominated by scale uncertainty
- Small PDF uncertainties

$\alpha_s$  results from H1 jet data in NNLO



$$\alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (27)_{\text{scale}}$$

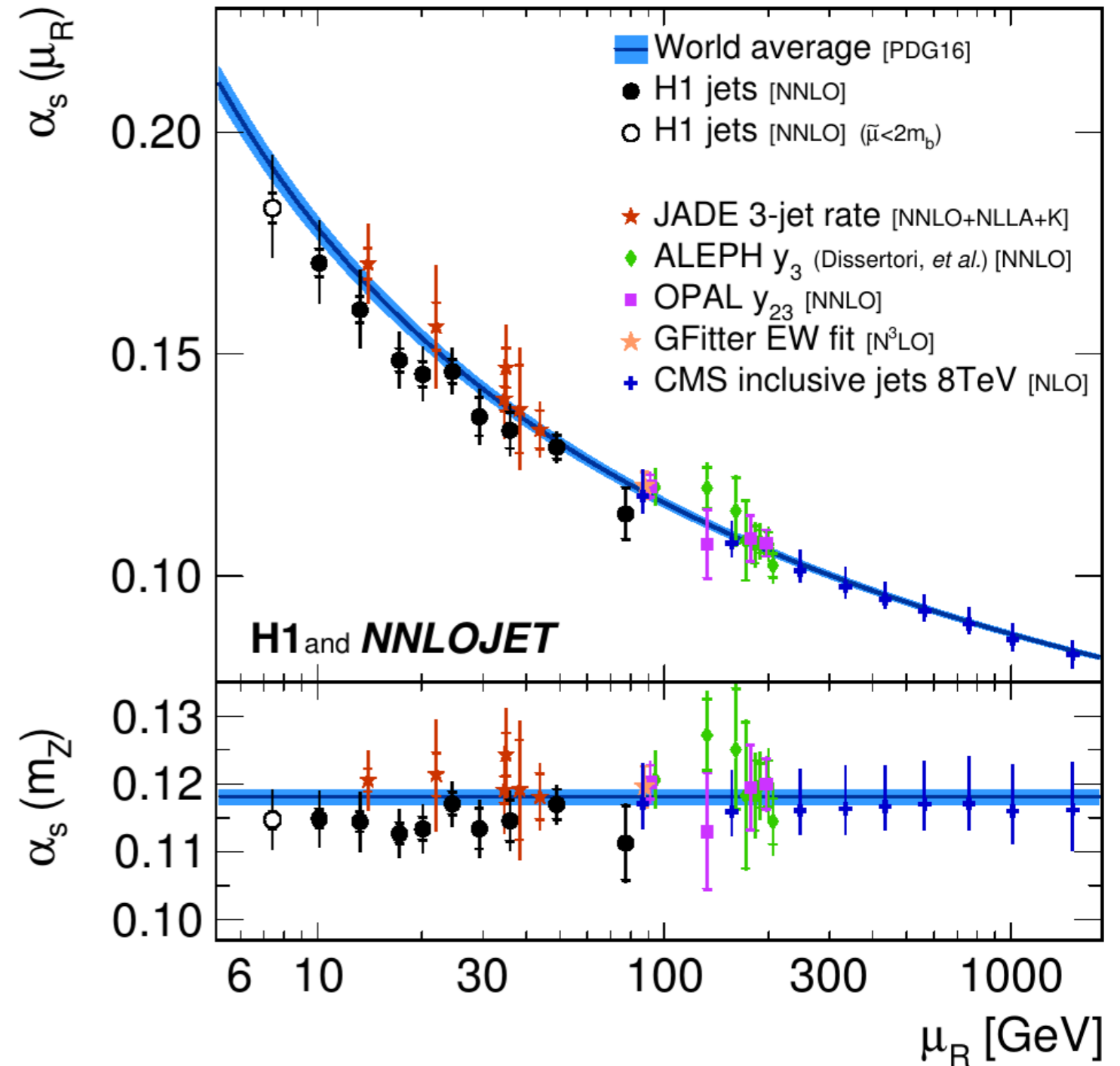


## Test running of strong coupling

- Perform fits to data at similar scale
- Assumes running valid in limited interval range
- All fits have good  $\chi^2$

## Results

- Consistency with expectation at all scales
- Scale uncertainty dominates at lower  $\mu$
- Consistency of inclusive jets and dijets





## Inclusive Neutral and Charged Current DIS data

Data set [ref.]	Lepton type	$\sqrt{s}$ [GeV]	$Q^2$ range [GeV <sup>2</sup> ]	NC cross sections	CC cross sections	Lepton beam polarisation
Combined low- $Q^2$ [64]	$e^+$	301,319	(0.5) 12 – 150	✓	–	–
Combined low- $E_p$ [64]	$e^+$	225,252	(1.5) 12 – 90	✓	–	–
94 – 97 [61]	$e^+$	301	150 – 30 000	✓	✓	–
98 – 99 [62, 63]	$e^-$	319	150 – 30 000	✓	✓	–
99 – 00 [63]	$e^+$	319	150 – 30 000	✓	✓	–
HERA-II [65]	$e^+$	319	120 – 30 000	✓	✓	✓
HERA-II [65]	$e^-$	319	120 – 50 000	✓	✓	✓

### Simultaneous PDF and $\alpha_S$ fit

PDFs mostly determined from H1 inclusive DIS data

### Perform H1 alone PDF fit: H1PDF2017

Use all H1 inclusive DIS data

Use all H1 normalised jet cross section data  
→ 1529 data points

$$\sigma_i = \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}_{i,k}(x, \mu_R, \mu_F) \cdot C_{\text{had},i}$$

$\hat{\sigma}_{i,j}$  = partonic cross section

$C_{\text{had},i}$  = hadronisation corrections

PDFs are parameterised as

$$xf(x)|_{\mu_0} = f_A x^{f_B} (1-x)^{f_C} (1 + f_D x + f_E x^2)$$

### Normalised jet cross sections

Jet cross sections normalised to inclusive DIS  
Correlations of jets and inclusive DIS cancel

## Inclusive Jets and Dijets data

Data set [ref.]	$Q^2$ domain	Inclusive jets	Dijets	Normalised inclusive jets	Normalised dijets	Stat. corr. between samples
300 GeV [17]	high- $Q^2$	✓	✓	–	–	–
HERA-I [23]	low- $Q^2$	✓	✓	–	–	–
HERA-I [21]	high- $Q^2$	✓	–	✓	–	–
HERA-II [15]	low- $Q^2$	✓	✓	✓	✓	✓
HERA-II [15, 24]	high- $Q^2$	✓	✓	✓	✓	✓

## Result for PDFs

Set of PDFs determined with high precision  
(even with  $\alpha_s$  a free fit parameter)  
→ precision is competitive with global PDF fitters

Gluon at lower x-values tends to be higher  
→ now typically favoured by small-x resummed PDFs

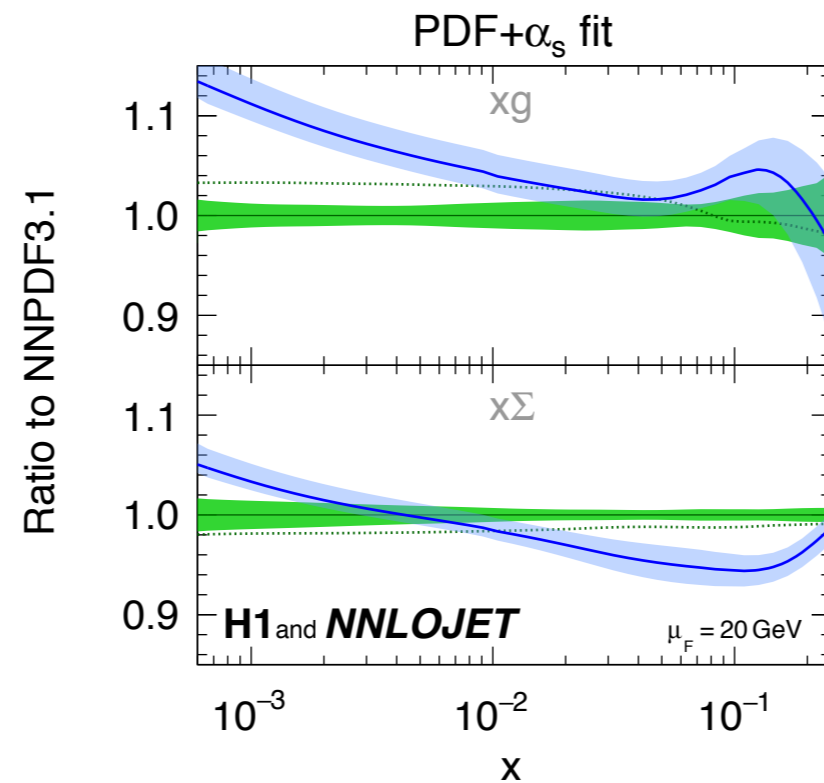
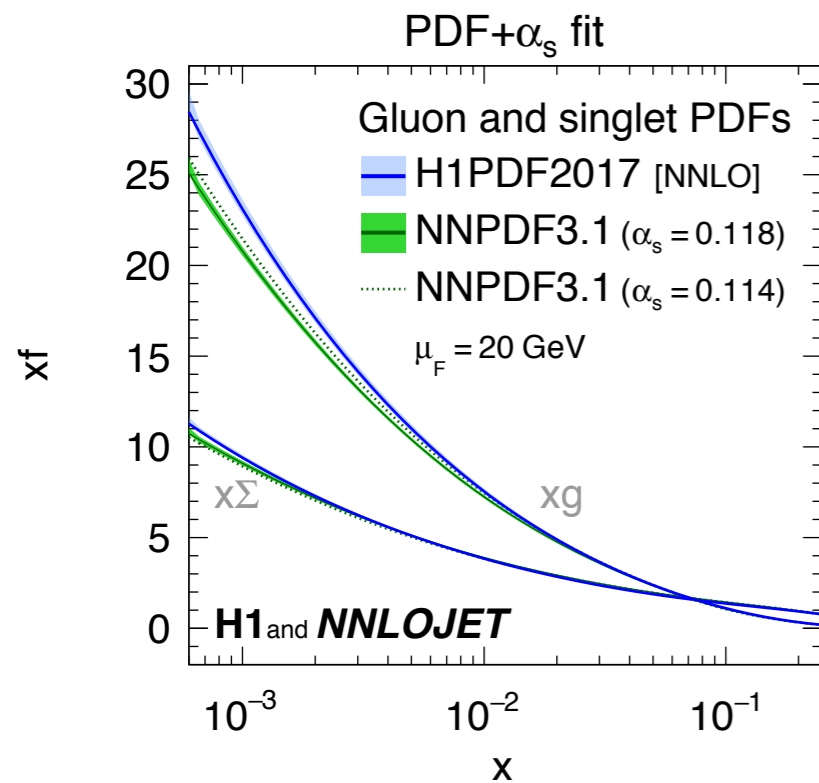
## PDF + $\alpha_s$ fit

$$\chi^2/\text{ndf} = 1.01$$

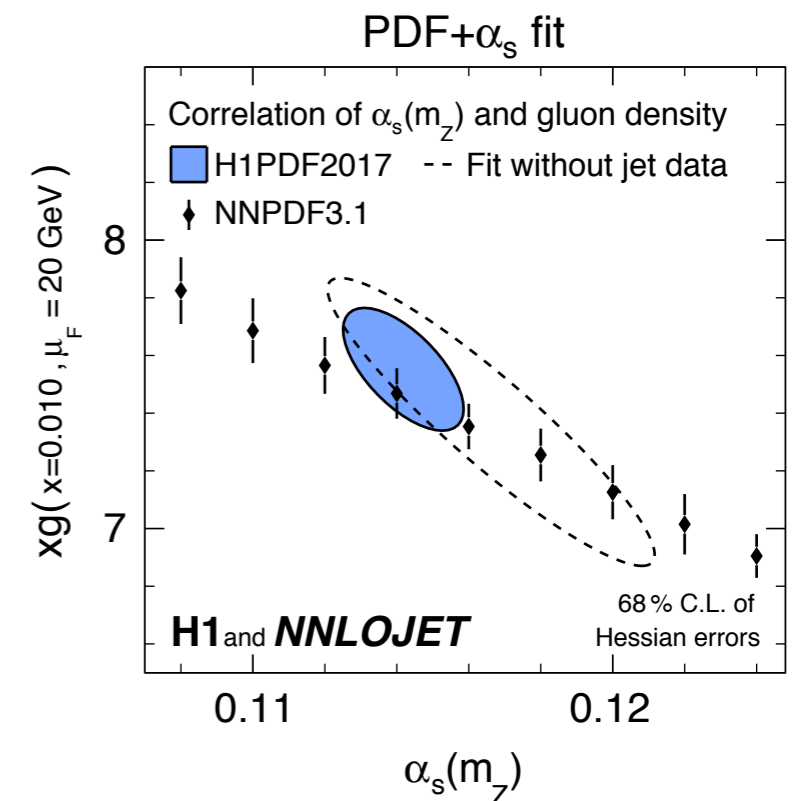
Simultaneous fit with H1 jet data

→ precise determination of the gluon PDF and  $\alpha_s$

## Comparison of H1PDF2017 with NNPDF3.1



## Correlation of $\alpha_s$ with gluon



## Simultaneous PDF and $\alpha_s$ fit

$$\alpha_s(m_Z) = 0.1142 (11)_{\text{exp, had, PDF}} (2)_{\text{mod}} (2)_{\text{par}} (26)_{\text{scale}}$$

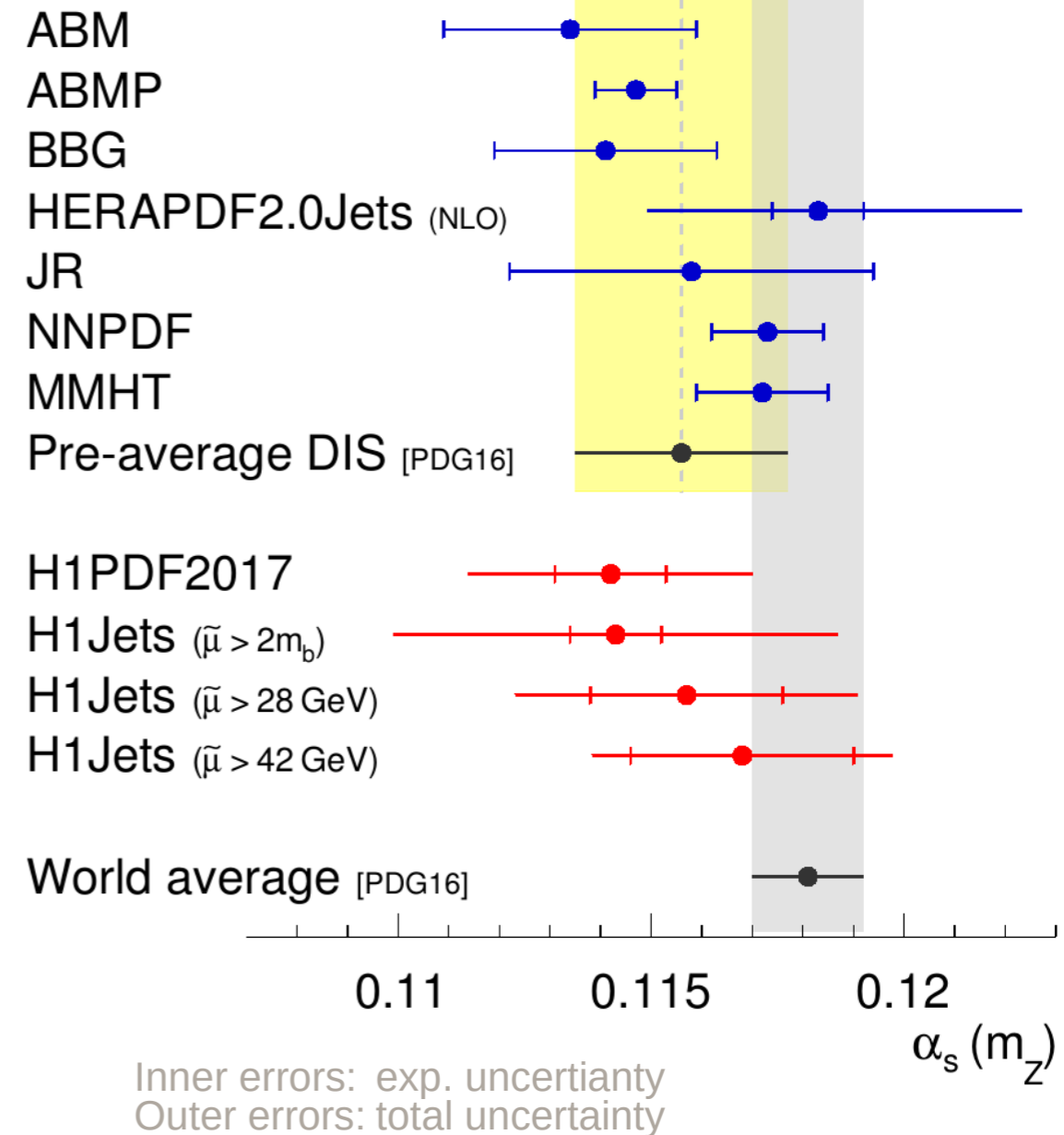
- High experimental precision
- Moderate theory uncertainty from NNLO

## Comparison

- Higher precision than most comparable determinations
  - PDF groups commonly determine exp. uncertainties only
  - We further estimate scale uncertainties
- All H1 results consistent
- Results competitive with world average
- All results from DIS data typically lower than world average

## $\alpha_s$ determinations in NNLO

H1 and NNLOJET



- All H1 jet data compared with NNLO predictions
- NNLO theory provides improved description w.r.t. NLO
- Quantitative comparisons of all data
- NNLO predictions studied in great detail

NNLO used for determination of  $\alpha_s(m_Z)$  in two methods:

$$\alpha_s \text{ fit: } \alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (27)_{\text{scale}}$$

$$\text{PDF and } \alpha_s \text{ fit: } \alpha_s(m_Z) = 0.1142 (11)_{\text{exp,had,PDF}} (2)_{\text{mod}} (2)_{\text{par}} (26)_{\text{scale}}$$

High experimental and theoretical precision achieved

- NNLO predictions for jets are used for PDF fits for the first time
- Successful determination of gluon-density and  $\alpha_s(m_Z)$  simultaneously
- Competitive precision of PDFs and  $\alpha_s(m_Z)$
- H1PDF2017 available in LHAPDF

Special thanks to **NNLOJET** team for fruitful collaboration of theoreticians and experimentalists

Extracted  $\alpha_S$  values

## Study scales calculated from $Q^2$ and $P_T$

' $P_T$ ' refers to:  $P_{Tjet}$  or  $\langle P_T \rangle$

### $\alpha_s$ results and $\chi^2$ values

- Spread of results covered by scale uncertainty
- $\chi^2$  values are similar for different choices  
→ NNLO with small 'scale dependence'

### NLO matrix elements

Large scale uncertainty

Relevant dependence of result on scale choice

- Mainly larger  $\chi^2$  values than NNLO
- Larger fluctuation of  $\chi^2$  values than NNLO

